Lesson Plan – Analyzing and Curve Fitting a Graph

Kenneth Patterson

Gwinnett School of Science Math & Technology
Lawrenceville, GA

Georgia Institute of Technology STEP-UP Summer 2015

Subject: Physics
Grade: 9-11
Suggested Time: 1 90 minute period

Topic: Graphical Analysis and Curve Fitting

Problem / Essential Questions:

How can we gain useful information from a graph alone?

Let's go beyond "$y = mx + b$". What are the physical significance of coefficients in a model?

Abstract:

An experimental data set is best interpreted using a mathematical model, and graphical methods such as line of best fit or curve fitting can be used to find the parameters of that model. Students in this lesson will form a graphical mathematical model of mass of a tile vs. area, and use that model to make predictions. Students will first manually gather, plot, and graph data by hand. They will then practice using the LoggerPro software to perform the same task on a computer.

Alignment with State and Local Standards:

Georgia Performance Standards

SCSh3. Students will identify and investigate problems scientifically.

c. Collect, organize and record appropriate data.
d. Graphically compare and analyze data points and/or summary statistics.

SCSh4. Students will use tools and instruments for observing, measuring, and manipulating scientific equipment and materials.

a. Develop and use systematic procedures for recording and organizing information.
b. Use technology to produce tables and graphs.
c. Use technology to develop, test, and revise experimental or mathematical models.

SP1. Students will analyze the relationships between force, mass, gravity, and the motion of objects.

a. Calculate average velocity, instantaneous velocity, and acceleration in a given frame of reference.
c. Compare graphically and algebraically the relationships among position, velocity, acceleration, and time.

Objectives:

- To relate density of an unknown substance to the slope of mass vs. area graph
- To examine the uses and importance of graphical curve-fitting in modern scientific and engineering fields

Anticipated Learner Outcomes:

Students will be able to

- Fit a proper curve to a data sets
- Linearize a curved plot of data
- Calculate physical properties from plot parameters (Slopes, constants, intercetps)

Background:

Utilizing mathematical models of physical systems to describe and predict events is one of the key ways that science and engineering work. However, students often have difficulty understanding this “model-based” way of thinking, often choosing to view models as a set of equations handed out by the teacher instead. This lesson aims to be the first building block in dispelling the non-model method of thinking.

First, this lesson will demonstrate that a mathematical function can be inferred from a set of data points, and that the coefficients of this function (or “parameters”) have physical meaning. By plotting the mass of a floor tile as a function of area, students will explore the visual meaning of density as the slope of a graph. Second, this lesson exposes students to the concept of scientific “error” not as a mistake performed by the experimenters, but as a limit of the precision in the measurements made. Too often students will justify experimental “error” by claiming that a mistake was made, a variable was miscalculated, or simply the empty phrase: “human error.” This lesson uses the y-intercept of a student-drawn graph to show that experimental error simply means that the precision of the measuring device is limited in some way. The y-intercept of a mass vs. area plot should be zero, representing zero mass of course, however, most students will discover that their y-intercept is slightly nonzero. It is important to stress that this error is not due to a mistake, but to do the limited precision of the tools used to make a measurement during the experiment.

Finally, the skill of linearizing a set of data is becoming more and more common in AP physics assessments. Linearization can be used when the data fits the form

$$f(y) = m * g(x) + b$$

where m and b are the familiar slope and y-intercept, and f(y) and g(x) are some easily calculated manipulation of y or x. For example, for the equation of constant acceleration $x = (1/2)a*t^2$, the $f(x)$ would be x, and $g(t)$ would be t^2. To linearize the graph, plot x as a function of t^2, and $(1/2)a$ will be the slope.
Linearization and its importance in graphing

Microsoft Excel is a great general curve fitting tool, but it can only fit linear, polynomial, exponential, power, and logarithmic functions innately. For a more specific classroom scientific use, this lesson uses the Vernier LoggerPro software which can fit a wider variety, and can fit custom functions (up to 6 parameters). Software is available from http://www.vernier.com.

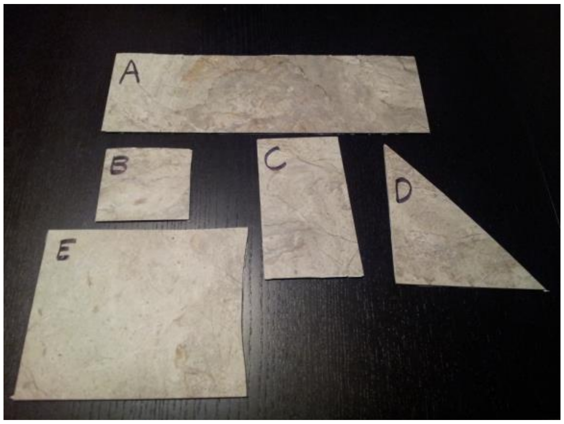
Teachers looking for additional resources and ideas can also check out this Vernier packet on model-based curve fitting:
http://physics.dickinson.edu/~wp_web/wp_resources/wp_downloads/PVA_AppendixC.pdf

Materials – At least one per group of:

- Device running LoggerPro software
- Mass scale
- Set of 4-5 vinyl flooring tiles of known area
- 1 linoleum tile of unknown mass (alternatively: 1 tile of known mass but unknown area)

Plan:

1. **Preparation:**
   Cut flat, non-sticky vinyl flooring tiles into various simple small shapes, these shapes MUST be basic enough that students can calculate their area using simple geometry. Label the shapes, and split them up among student groups. (It is not necessary for each group to have identical shapes, they will all find the same density!)



Cut at least 1 large "mystery" tile, again with an area that is easily measurable by students, and label it the unknown mass.

2. **In-Class Activity**
   Hand out the accompanying "Mystery Tile" handout (see appendix) to each group, along with supplies and measurement devices. Students should spend an entire class period on the activity.

   During the activity, discuss the physical meaning of coefficients (the slope as density), and the precision of the mass scale and rulers. Be sure to explicitly discuss how "error" arises from imprecision, and not from mistakes!

3. **Using LoggerPro**
   This lesson is designed for students who have access to either a school computer or a home device capable of running LoggerPro. Have students repeat their hand-drawn linear fit in LoggerPro, and have them fit their line using the software (This lesson can also be modified to use Excel or a graphing calculator).

   Attached in the appendix is a file with three sample data sets that can be opened in LoggerPro. These data sets represent quadratic, inverse, and Gaussian fits. Students can download the file or files, and practice fitting these data, or come up with their own! The skills used in this lesson are designed to be used again throughout science labs during the year.

Assessment:

Assessment I
Assessment I is a classwork or homework worksheet (see appendix). This assessment involves students analyzing data sets on their own. Instructions and files are attached below.

Assessment II
Assessment II is a rubric to evaluate students' graphs performed while completing the Mystery Tile lab activity. Rubric is attached below.

Summary:

This lesson is designed to be an introductory lesson to the skills of graphing a set of data and finding and *using* the line of best fit. It is critical that students begin thinking about coefficients as having a physical meaning, rather than just numbers in an algebraic equation. This lesson teaches lab skills that will be needed throughout the year. Skills practiced in this lesson need to be used and reused in subsequent physics labs in order to have any meaning.

APPENDIX

I. In-Class Activity Handout for Students

II. Step-by-step instructions on How-to Use LoggerPro

III. Graphing Rubric

IV. Curve Fitting and Linearization Homework (Requires LoggerPro and attached .cmbl files)

V. Powerpoint slides for teachers

Mystery Plot

INTRODUCTION PROBLEM

Can you accurately predict the mass of a vinyl tile without using a scale

What if I gave you other vinyl tiles, and let you measure those?

TASK(S)

- Measure and record data
- Graph data onto a chart
- Estimate a line of best fit
- Use line of best fit as a model to predict future results

ACTIVITY/PROCESS

1. As a group, examine your fragments of floor tile. Determine the quantities you want to measure. What tools do you need to make these measurements and how precise are these tools?
2. Identify the independent and dependent variable.
3. In your notebook, construct a data table that will include all your measurements and calculations.

Mystery Plot

4. On paper, with a ruler and your completed data table, make a GRAPH of your measurements. Make sure the graph is as large as 1 page! Plot your data points accurately on the graph.
5. Draw in a line of best fit, using a straightedge. Don't just randomly guess, data points should be as close to the line as possible!
6. What is your value for slope? For y-intercept? What is the physical meaning of these coefficients?

MATERIALS

The following materials will be available for you to use. Additional materials may be requested from the instructor on an as-needed basis.

- Device running LoggerPro software
- Mass scale
- Rulers
- Markers and graph paper
- Set of 4-5 linoleum tiles of known area
- 1 linoleum tile of unknown mass

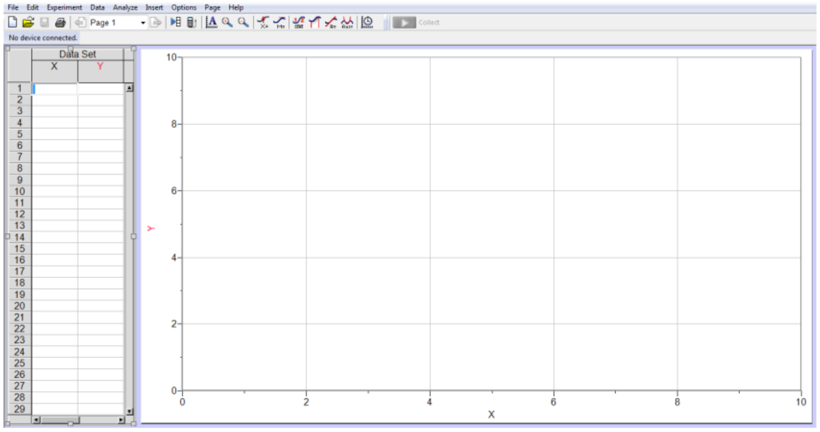
EVALUATION

You will be evaluated on the following criteria:

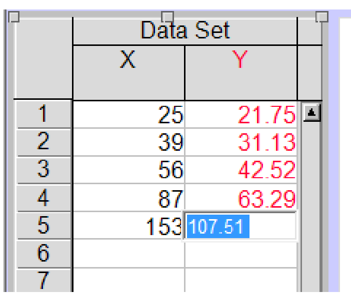
- Graph must be full-page!
- Axes labeled and graph titled
- Data points plotted accurately
- Line of best fit PROPERLY estimated
- Slope measured from line (Hmm, how do you measure slope if all you have is a line on graph paper?)
- Analysis and meaning of coefficients (slope and y-intercept)
- Prediction of mystery tile based on your model

Instructions for using Logger Pro
Basics for Curve Fitting and Data Analysis

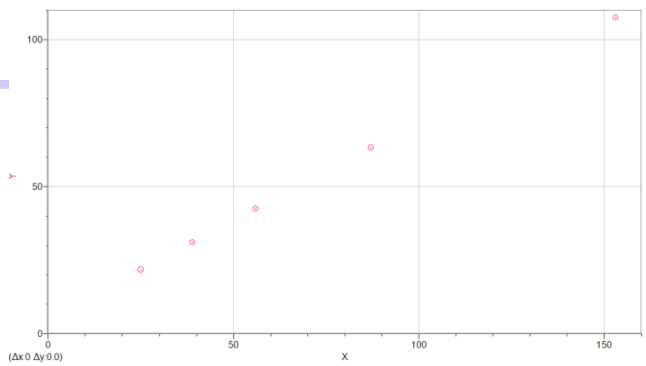
1. **Logger Pro appears with a blank data table and graph on startup.**



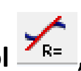
2. **To begin, enter your data into the data table manually. LoggerPro automatically begins plotting points.**

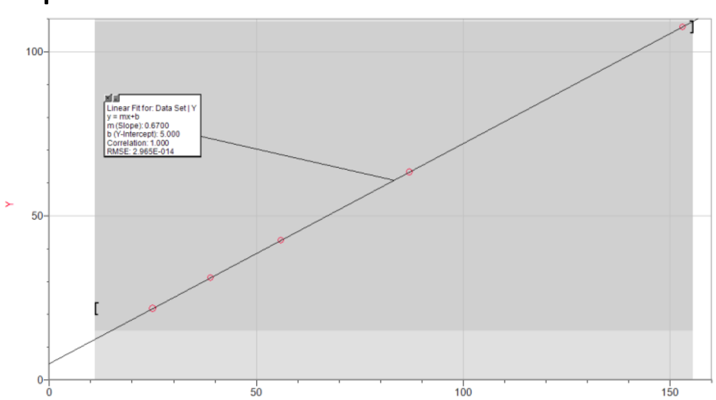


	Data Set X	Data Set Y
1	25	21.75
2	39	31.13
3	56	42.52
4	87	63.29
5	153	107.51
6		
7		



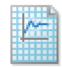
3. **Click and drag a box around data points you want to select to use.**

4. **Performing a Linear Fit can be done by using the Linear Fit tool**  **, or under the "Analyze" drop down menu.**



Your equation and coefficients are all displayed. A Linear Fit is easy! Now let's try something more challenging.

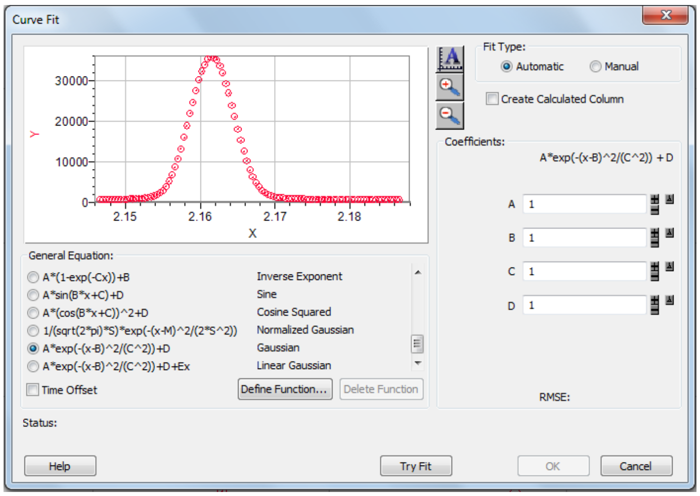
Part 2 – Using the Curve Fit Window in LoggerPro



Gaussian1.cmbl

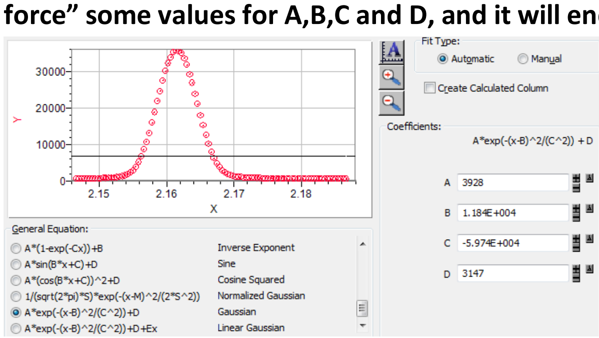
5. Load "Gaussian1.cmbl" into LoggerPro.

6. Open the Curve Fit window  **.**

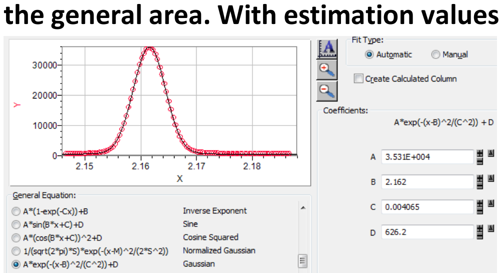


7. This window provides many more curve fitting options than Microsoft Excel. Scroll down to the "Gaussian" function. This function is of the form: $f(x) = a\exp\left(-\frac{(x-b)^2}{2c^2}\right)$**, where A is the amplitude of the peak, B is the position of the center of the peak, and C is the width of the peak.**

8. If you press the "Try Fit" button without any estimation, your computer will try to "brute force" some values for A,B,C and D, and it will end up like this . . .

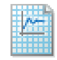


9. Not a fit at all! The computer needs some input to narrow the range of values it attempts, so type in your own best guess for A, B, C, and D. These guesses do not have to be precise, just in the general area. With estimation values plugged in, attempt "Try Fit" again:



Success!

Part 3 – Linearizing a Graph in LoggerPro

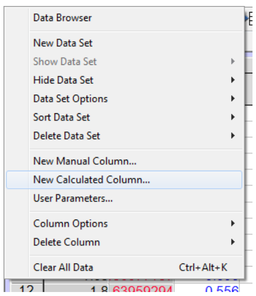


SampleDataSet2.cmbl

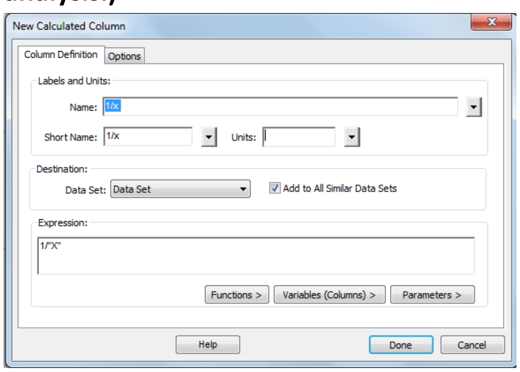
10. Load "SampleDataSet2.cmbl" into LoggerPro.

11. This is an inverse data set of the form y = A/x.

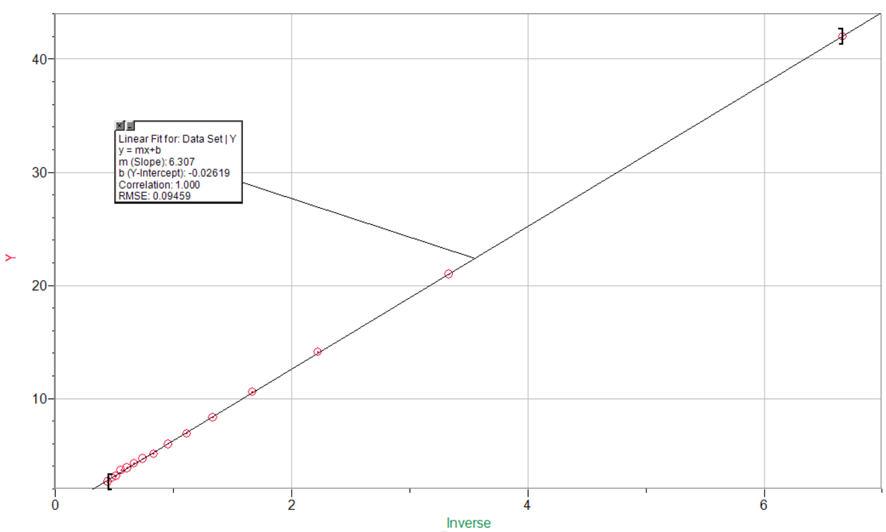
12. To linearize the graph, 1/x needs to be plotted on the horizontal axis. Go to Data > New Calculated Column.



13. Enter the options as below for the calculated column (note that this tool is very powerful for analysis.)



14. Click on the horizontal axis label, and select "1/x". You can now fit a linear curve to determine coefficients.



Category	**Rating Criteria**	**Points Earned**	**Points Possible**
Variables	_ Correct variable is graphed on the X axis. _ Correct variable is graphed on the Y axis.		2
Plot Area/Layout	_ Variable with the greatest range is graphed along the widest edge of the graph paper. _ Numerical data is plotted proportionally so that graph occupies maximum space. _ Axis increments (intervals) are equally spaced. _ Axis increments (intervals) reflect precision in experiment.		4
Axis Identification	_ Each axis is correctly labeled/titled. _ Each axis title/label is centered and well-placed. _ Each axis label/title includes appropriate measurement units for the variable. _ Major gridlines are clearly and neatly numbered.		4
Graph Data	_ Data points are accurately graphed and reflect experimental data. _ Data points are identified with an appropriately-sized marker. _ Data line is a smoothed, best-fit line drawn with a straight edge. _ Data line is appropriate, best-fit line.		4
Graph Title	_ A title is recorded in an open area above the graph. _ The subject of the graph can be identified from its title. (The title does not merely reflect the axis labels.)		2
Conventions	_ There are no spelling or grammatical errors.		1
	TOTAL POINTS		

Name________________________________ Date___________ Period_______

Download or save the following 3 data sets:

SampleDataSet1.cmbl SampleDataSet2.cmbl Gaussian1.cmbl

Questions for SampleDataSet1:

1. Load the .cmbl file into LoggerPro, and use the Curve Fitting menu to try out some fits! Which type of function fits this data set best? How do you know?

2. Write out the equation for the curve of best fit. What are the parameters?

Questions for SampleDataSet2:

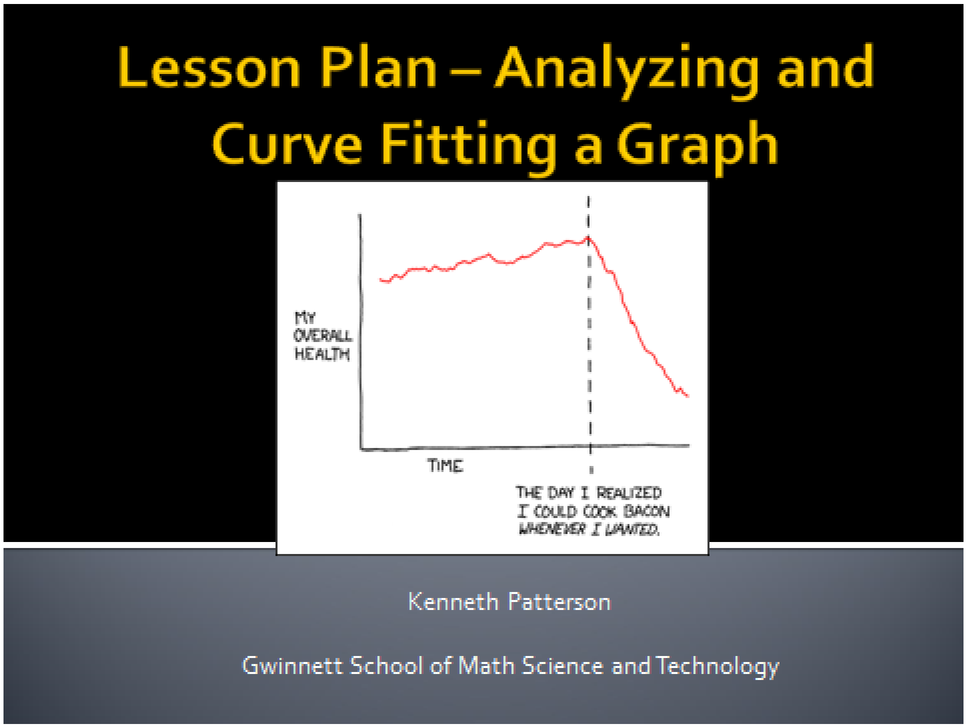
1. Load the .cmbl file into LoggerPro, and use the Curve Fitting menu to try out some fits! Which type of function fits this data set best? How do you know?
2. Write out the equation for the curve of best fit. What are the parameters?
3. Now try Linearizing this set! Use "New Calculated Column" to calculate new values using the function you just determined for this data set.
4. Compare the linear slope of the linearized graph to the parameter in question 2. What do you notice?

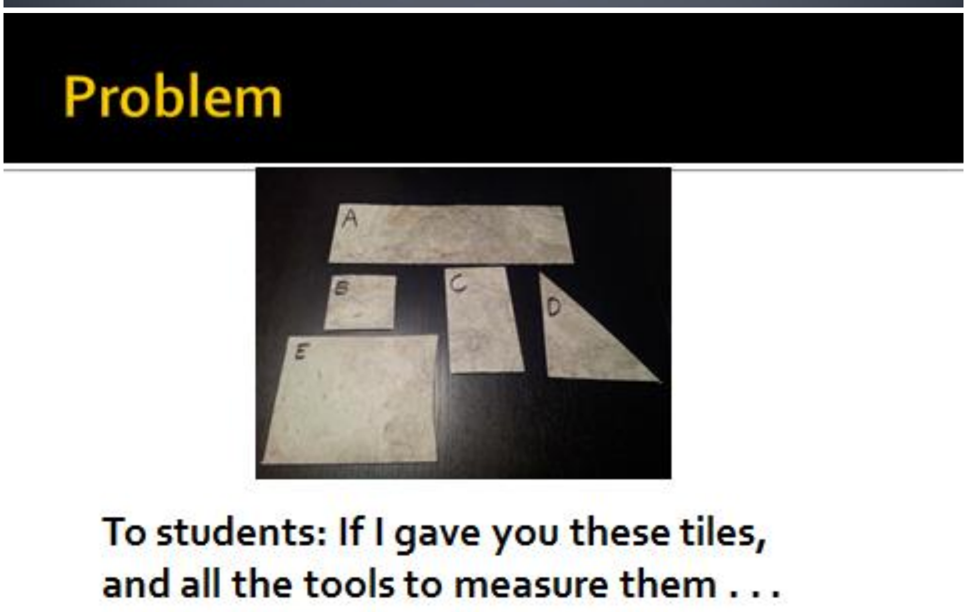
Questions for Gaussian1:

1. This data set will use a Gaussian curve fit. Select the function and try the fit. You should see that the program is overwhelmed with too many parameters and cannot provide a good fit.

2. You will have to manually input parameters for A, B, C, and D, by estimating values from the graph.

A is the amplitude (height) of the peak
B is the position of the center of the peak
C is the width of the peak
D is the y-intercept (you may leave D as 0)


Lesson Plan – Analyzing and Curve Fitting a Graph
MY OVERALL HEALTH
TIME
THE DAY I REALIZED I COULD COOK BACON WHENEVER I WANTED.
Kenneth Patterson
Gwinnett School of Math Science and Technology


Problem
A
B
C
D
E
To students: If I gave you these tiles, and all the tools to measure them . . .

Problem

- Could you tell me the mass of this unknown tile WITHOUT using a scale?

Materials

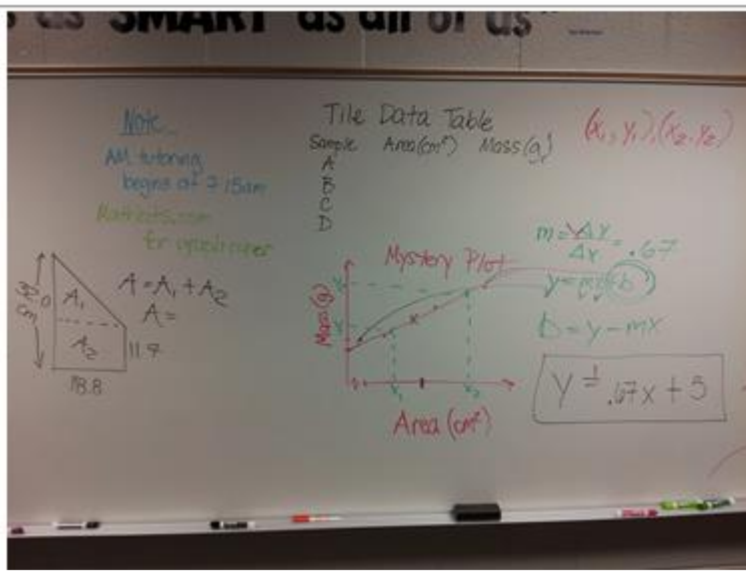
- 5 known mass/area vinyl tiles
- 1 unknown vinyl tile
- 1 mass scale
- LoggerPro
- Graph paper and rulers!

Standards

Georgia Performance Standards

- **SCSh3. Students will identify and investigate problems scientifically.**
- c. Collect, organize and record appropriate data.
- d. Graphically compare and analyze data points and/or summary statistics.
- **SCSh4. Students will use tools and instruments for observing, measuring, and manipulating scientific equipment and materials.**
- a. Develop and use systematic procedures for recording and organizing information.
- b. Use technology to produce tables and graphs.
- c. Use technology to develop, test, and revise experimental or mathematical models.

Plan – Step 1



Plan – Step 2

Area (cm^2)	Mass (g)
25	21.75
39	31.13
56	42.52
87	63.29
153	107.51

- Students measure and plot data by hand (bust out the graph paper)!
- Common mistake: Students drawing lines of best fit through points or through origin

Plan – Step 3

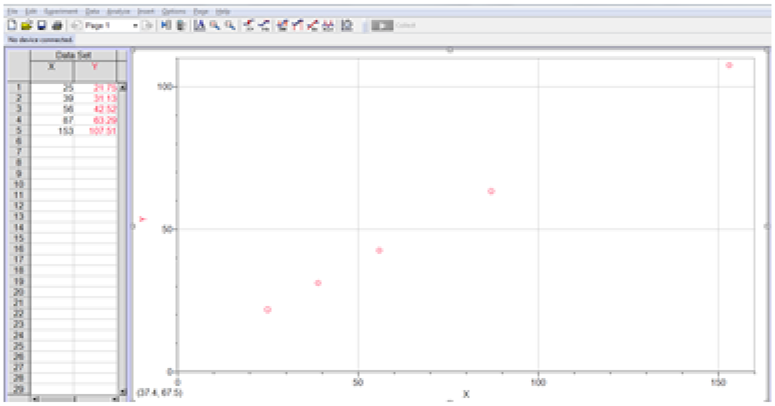
- Measure slope directly from graph paper
- Write equation in form:
  - Mass = density * area + b

- Ask: "What does *b* represent in this form?"

  The goal of a science class is to find the physical meaning of coefficients, not just "it is the y-intercept"

Plan – Step 4

- Repeat analysis, this time in LoggerPro!



Plan – Step 5

- Sample data sets included in lesson (incl. data sets from summer research experience)
- Testing different functions and parameters in Logger Pro
- Practice problems on linearization, sample data sets, Gaussians

Big Ideas

- Key word is MODEL – The line is not some physical law, but a relationship that can make predictions with known error

- This lesson is all about lab skills. The skills and software used here need to be reused and revisited throughout the year, else they have no meaning!