Lesson Plan: Introduction to Quantum Mechanics via Wave Theory and the Photoelectric Effect

Will Stoll, Norcross High School

Problem: To understand the basic principles of Quantum Mechanics through an extension of wave theory and the experimental results of the photoelectric effect.

Abstract: Students will be introduced to quantum mechanics by extending wave theory. By looking at a traveling wave and through a brief overview of Fourier Transforms in complex waves the basic tenants of the uncertainty principle will be shown. Standing waves and their boundary conditions will be considered to show the quantized nature of particles. The historical development of quantum mechanics will be shown through Planck’s work with blackbodies and then the photoelectric effect will be considered in detail. Following this, students will conduct a virtual lab on the photoelectric effect generating Planck’s constant.

Standards Alignment:
International Standards: International Baccalaureate – (*) denotes option for Standard Level

4.4 Wave characteristics
4.4.1 Describe a wave pulse and a continuous progressive (travelling) wave.

4.5 Wave properties
4.5.1 Describe the reflection and transmission of waves at a boundary between two media.
4.5.6 State and apply the conditions for constructive and for destructive interference in terms of path difference and phase difference.

8.4 Solar Power
8.4.12 Distinguish between a photovoltaic cell and a solar heating panel.

11.1 (A.1) Standing (stationary) waves
11.1.1 (A.1.1) Describe the nature of standing (stationary) waves.
11.1.2 (A.1.2) Explain the formation of one-dimensional standing waves.
11.1.3 (A.1.3) Discuss the modes of vibration of strings and air in open and in closed pipes.
11.1.4 (A.1.4) Compare standing waves and travelling waves.

13.1 (B.1) Quantum physics
13.1.1 (B.1.1) Describe the photoelectric effect.
13.1.2 (B.1.2) Describe the concept of the photon, and use it to explain the photoelectric effect.
13.1.3 (B.1.3) Describe and explain an experiment to test the Einstein model.
13.1.4 (B.1.4) Solve problems involving the photoelectric effect.

The wave nature of matter
13.1.5 (B.1.5) Describe the de Broglie hypothesis and the concept of matter waves.
13.1.6 (B.1.6) Outline an experiment to verify the de Broglie hypothesis.
13.1.7 (B.1.7) Solve problems involving matter waves.
13.1.11 (B.1.11) Explain the origin of atomic energy levels in terms of the “electron in a box” model.
13.1.12 (B.1.12) Outline the Schrödinger model of the hydrogen atom.
IV. Waves and Optics - Wave Motion
1. Traveling waves – Students should understand the description of traveling waves, so they can:
   a) Sketch or identify graphs that represent traveling waves and determine the amplitude, wavelength, and frequency of a wave from such a graph.
   b) Apply the relation among wavelength, frequency, and velocity for a wave.
   d) Describe reflection of a wave from the fixed or free end of a string

2. Wave propagation
b) Students should understand the inverse-square law, so they can calculate the intensity of waves at a given distance from a source of specified power and compare the intensities at different distances from the source.

3. Standing waves - Students should understand the physics of standing waves, so they can:
   a) Sketch possible standing wave modes for a stretched string that is fixed at both ends, and determine the amplitude, wavelength, and frequency of such standing waves.

4. Superposition - Students should understand the principle of superposition, so they can apply it to traveling waves moving in opposite directions, and describe how a standing wave may be formed by superposition.

V. Atomic and Nuclear Physics
A. Atomic physics and quantum effects
1. Photons, the photoelectric effect, Compton scattering, x-rays
   a) Students should know the properties of photons, so they can:
      (1) Relate the energy of a photon in joules or electron-volts to its wavelength or frequency.
      (2) Relate the linear momentum of a photon to its energy or wavelength, and apply linear momentum conservation to simple processes involving the emission, absorption, or reflection of photons.
      (3) Calculate the number of photons per second emitted by a monochromatic source of specific wavelength and power.
   b) Students should understand the photoelectric effect, so they can:
      (1) Describe a typical photoelectric-effect experiment, and explain what experimental observations provide evidence for the photon nature of light.
      (2) Describe qualitatively how the number of photoelectrons and their maximum kinetic energy depend on the wavelength and intensity of the light striking the surface, and account for this dependence in terms of a photon model of light.
      (3) Determine the maximum kinetic energy of photoelectrons ejected by photons of one energy or wavelength, when given the maximum kinetic energy of photoelectrons for a different photon energy or wavelength.
      (4) Sketch or identify a graph of stopping potential versus frequency for a photoelectric-effect experiment, determine from such a graph the threshold frequency and work function, and calculate an approximate value of $h/e$.

   Wave-particle duality- Students should understand the concept of de Broglie wavelength, to
   a) Calculate the wavelength of a particle as a function of its momentum.
   b) Describe the Davisson-Germer experiment, and explain how it provides evidence for the wave nature of electrons.

1. Design experiments
2. Observe and measure real phenomena
State Standards – Georgia Science Standards

SCSh3. Students will identify and investigate problems scientifically.
   a. Suggest reasonable hypotheses for identified problems.
   b. Develop procedures for solving scientific problems.
   c. Collect, organize and record appropriate data.
   d. Graphically compare and analyze data points and/or summary statistics.
   e. Develop reasonable conclusions based on data collected.

SCSh4. Students will use tools and instruments for observing, measuring, and manipulating scientific equipment and materials.
   a. Develop and use systematic procedures for recording and organizing information.
   b. Use technology to produce tables and graphs.
   c. Use technology to develop, test, and revise experimental or mathematical models.

SP4. Students will analyze the properties and applications of waves.
   a. Explain the processes that results in the production and energy transfer of electromagnetic waves.
   b. Experimentally determine the behavior of waves in various media in terms of reflection, refraction, and diffraction of waves.
   c. Explain the relationship between the phenomena of interference and the principle of superposition.

SP5. Students will evaluate relationships between electrical and magnetic forces.
   a. Describe the transformation of mechanical energy into electrical energy and the transmission of electrical energy.
   b. Determine the relationship among potential difference, current, and resistance in a direct current circuit.

SP6. The student will describe the corrections to Newtonian physics given by quantum mechanics and relativity when matter is very small, moving fast compared to the speed of light, or very large.
   a. Explain matter as a particle and as a wave.
   b. Describe the Uncertainty Principle.

Objectives:
The student will understand the basic concepts of quantum mechanics.
The student will be able to connect wave theory with quantum mechanics.
The students will be aware of the historical development of quantum mechanics from short comings in Newtonian physics.
The student will understand how the photoelectric effect demonstrates the wave-particle duality of light.
The student will demonstrate analytical skills in solving problems based on the relations of the photoelectric effect.
The student will correctly generate and analyze data to graphically determine relationships.

Anticipated Learner Outcomes:
The student will be able to define the basic concepts of quantum mechanics – uncertainty principle and quantized energy.
The student will be able to explain the photoelectric effect and identify the relationships between the intensity of light, frequency of light, and energy.
The student will derive the Planck’s constant from data from a photoelectric simulation. The student will be able to discuss the implications of generating Planck’s constant from this experiment.

Materials & Supplies:
- Computer Access
- Rope or String
- Photoelectric Setup

Plan:
Day 1: Present
- Wave Review
- Development of the uncertainty principle from Fourier Analysis of Complex Waves
Reinforcement:
  - In groups of 2 have students go to:
  - Complete the questions on Fourier Making Waves Simulation Activity (Appendix I)

Day 2
- Development of quantized states from boundary condition of a standing wave.
Demonstrations:
  - With a rope of long spring demonstrate how a wave reflects at a fixed boundary and then generate a standing wave.
Reinforcement:
  - In groups of 2 have students go to:
  - Complete the questions on Wave Simulation Activity (Appendix II)

Day 3: Present
- Planck’s work with Blackbody Radiation
- Historical development of Photoelectric Effect
- Einstein’s solution to the Photoelectric Effect
- Photoelectric Effect Experiment and Results
Demonstrations:
  - Embedded videos and applets in PPT
Reinforcement:
  - Problems on the Photoelectric Effect Worksheet (Provided in Appendix III)

Day 4 & 5: Photoelectric Effect Experiment
In groups of two have students complete the Photoelectric Virtual Lab (Appendix IV). Using computer simulations the students will investigate the photoelectric effect. While students are completing their lab, have three groups (6 students), join you at the Photoelectric Effect Setup. Have each group generate a set of data points from a couple different frequencies of light. Record this data and then call the next three groups to complete the same exercise but with different frequencies of light. Continue until all the students have rotated through. Either post the
recorded data online or in on an overhead chart, have the students record the data and for homework plot the data, generate a best fit line, and calculate the percent error. The students are to hand this in with their completed Photoelectric Virtual Lab

**Assessment:** See attached handouts
Appendix I

Physics: Fourier Making Waves Simulation Activity

**Learning Goals:** Students will apply their understanding about wave characteristics and superposition to match wave functions.


1. Use the Level 1 for the beginning of the competition. Take turns:
   Press *New Game*, think about what should work, and then type in your guess in the Amplitude box.

   **Keep score:** -5 points for correct on first try, -3 for second and -1 for third.

<table>
<thead>
<tr>
<th>Name</th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
<th>Game 4</th>
<th>total</th>
</tr>
</thead>
</table>

2. In Level 3, you have to choose one of eleven harmonics. Take turns:
   Press *New Game*, decide which harmonic to use, think about what should work, and then type in their guess in the Amplitude box.

   **Keep score:** -10 points for correct on first try, -5 for second and -1 for third.

<table>
<thead>
<tr>
<th>Name</th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
<th>Game 4</th>
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</tr>
</thead>
</table>

3. Play the game, and see how high a level you can go before you can’t match the wave. How high did you make it? Comment here on what you learned in trying to match the wave form.

4. Next, go to Level and select preset. Complete each of the preset wave functions. Use hints if necessary. What trends did you observe in the amplitude and the number of terms required to make the more complex waves? Why do you think this is?

5. Finally, click on the Discrete to Continuous Tab. Vary the wave packet width ($\sigma_k$), describe the effect on both the amplitude and the sum displays. Reviewing your notes how does this tie in with the uncertainty principle.

Source: Trish Loeblein, PhET and Evergreen HS (7/08)
Appendix II

Physics: Standing-Wave Simulation Activity

Objectives:
- To observe and describe interference when a wave is reflected back along the rope.
- To create a standing wave.

1. Open http://phet.colorado.edu/simulations/stringwave/stringWave.swf

2. Change the option from manual to pulse. Play with the simulation for a few minutes. You will introduce more than one wave into the string by pressing “pulse”. Which variables change the shape of the wave?

3. Now hit the “reset” button. Make the following changes.
   * Keep the option at pulse.
   * Change the end of the rope to fixed end.
   * Set the damping to zero.
   * Set the tension to high.
   How does the pulse width affect the shape of the wave?

4. Send one wave down the string. Describe the wave that is reflected back from the fixed end.

5. Hit reset. Send two waves down the string. Pause the waves when they interfere completely. Describe the wave that results and the type of interference that occurs.

6. Change the end of the rope to “loose end” and send one wave down the string. Describe the wave that is reflected back from the loose end.

7. Hit reset. Send two waves down the string. Pause the waves when they interfere completely. Describe the two waves that can result and the type of interference that occurs.

8. From your knowledge or previous notes define the following.
   a. node
   b. antinode
10. Can you create a standing wave on the simulation program? Hint: Change the end of the rope to “fixed end”. Explain what you did below and show your instructor the standing wave you created.

Source: Messieurs Graham and Gaines February, 2007
Appendix III
Problems on Photoelectric Effect

Equations:
\[ \nu = f \times \lambda \]  \( \nu = \) speed of light in m/s \( f = \) frequency in Hertz (Hz = \#waves / second)
\[ \lambda = \text{Wavelength in meters} \]
\[ E = h \times f \]  \( E = \) energy in Joules
\[ \frac{1}{2}mv^2 = hf - \phi \]  \( v = \) the velocity of the electron in m/s \( m = \) mass of the electron (look up in your book)
\[ h = 6.63 \times 10^{-34} \text{ J s} \]  \( h = \) Planck’s constant
\[ f = \text{the frequency of the light in Hz} \]
\[ \phi = \text{the work function in Joules (which is the minimum amount of energy needed to ‘kick’ out an electron)} \]

1. How much energy does a photon with a frequency of \( 2.5 \times 10^{14} \) Hz have?

2. If this were the minimum energy required to emit photoelectrons from a metal, what is the work function of the metal? (this is kind of a trick question)

3. If the frequency of the photons shining on the metal is increased to \( 4.5 \times 10^{15} \) Hz, how fast are the electrons that are kicked out moving?

4. If the work function for a copper plate is \( 4.65 \times 10^{-19} \) J, what is the minimum frequency required to kick out electrons?

5. If the electrons are traveling at \( 3.7 \times 10^5 \) m/s, what frequency light is shining on the copper plate?

6. If \( 1.1 \times 10^{15} \) Hz frequency light shines on a magnesium plate which has a work function of
3.6*10^{-19} \text{ J} and electrons kick out at a speed of 9.00*10^5 \text{ m/s}, what is the mass of this electron?

7. What happens to the photoelectric equation if the work function is greater than the energy of the photon (that is, if $\phi > h\cdot f$)?

8. How much energy does a photon with a frequency of 6.8*10^{13} \text{ Hz} have?

9. If a metal plate has a work function of 2.99*10^{-19} \text{ J}, how fast are electrons moving that are struck by photons of frequency 5.5*10^{14} \text{ Hz}?
Appendix IV

Photoelectric Virtual Lab

Name: ____________________________
Partner’s Name: ____________________

Instructions:
Log onto a computer and go to the following site:
http://www.lon-capa.org/~mmp/kap28/PhotoEffect/photo.htm

Part A:
• Set the voltage to 0. Vary the wavelength. Observe the onset of the photo-current. What are the biggest wavelengths for the two materials, for which you still detect photo-current? (Record in the table below)

<table>
<thead>
<tr>
<th>Material</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium</td>
<td></td>
</tr>
<tr>
<td>Cesium</td>
<td></td>
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</tbody>
</table>

• Set the voltage to 0. Go to a wavelength bigger than the onset point and vary the intensity of the light. Is there any photo-current? What if you raise the voltage? Why might this be?

• Go to a wavelength smaller than the onset value and vary the voltage. Describe the resulting photo-current. Why might this be?

Part B:
Go to the following site:
Click the Run Now! button
1. Freely investigate the simulation
2. What variables can you change in this simulation?

3. Have the light source turned on and battery set to zero volts. Adjust the color of the source until electrons just begin to be ejected from the sodium surface. Note their speed. As you shorten the wavelength of the light source, what change did you notice about electron speed? Explain the cause of the change.
4. From your knowledge of light energy, electrons, wavelength, and frequency, predict the graph of the electron energy as function of wavelength by sketching.

5. Use the simulation to do a controlled experiment to verify your prediction. Beginning with the plates made of sodium, keep all the parameters constant except for the color (frequency) of the light. You can then enable the graph of the electron energy vs. light frequency. Sketch the result below.

Part C:
Go to Photoelectric Stimulation

Background: In this simulation of a photoelectric effect experiment, light of a particular frequency shines on a metal plate. If the energy of the photons is larger than the work function of the metal, electrons are ejected. By graphing the maximum kinetic energy (KE_max) of the electrons as a function of frequency, you can determine Planck’s constant from the slope and the work function from the y-intercept.

Procedure:
1. Choose the material to be used as the plate. Set the voltage to zero, and start at a low frequency. If the current is zero, the frequency is below the threshold frequency required to eject electrons. Press the “Plot data” button to record the voltage (zero) as a function of frequency.

2. Increase the frequency, recording points on the graph at a few frequencies. When you increase the frequency above the threshold, you will see a non-zero current reading. This means electrons are being ejected from the metal. To determine their maximum kinetic energy (KE_max), slowly increase the battery voltage until the current returns to zero. Press the “Plot data” button to record the maximum kinetic energy for this frequency.

3. Plot a few more points, increasing the frequency and finding the maximum kinetic energy each time by finding the minimum battery voltage required to bring the current to zero.

4. When you have several points for frequencies above the threshold frequency, press the “Fit the data” button. Your points should follow a straight line, because the equation governing the process is KE_{max} = hf - W, where h = Planck’s constant, f = frequency, and W is the work function of the metal.
With $KE_{\text{max}}$ on the y-axis and $f$ on the x-axis, what should the slope of the line be equal to?

What should the y-intercept be equal to?

Note that the fit only includes the data points with non-zero $KE_{\text{max}}$ values. Why are the others ignored?

5. When you are satisfied with your graph and the values for the first material, choose a new material for the plate and repeat the process.

What are the similarities between the two graphs?

What are the differences?

Questions:

1. To explain the photoelectric effect Einstein used the photon theory, which predicts that increasing the intensity of the light has no impact on the maximum kinetic energy of the emitted electrons. This contrasts with the wave theory of light, which predicts that increasing the intensity increases $KE_{\text{max}}$. In the simulation, does changing the intensity affect $KE_{\text{max}}$?

2. The y-axis of the graph shows the maximum kinetic energy of the electrons, which is numerically equal to the stopping potential (the minimum battery voltage required to reduce the current to zero). Why are these values numerically equal? (Note the units used for $KE_{\text{max}}$.)

3. What does the x-intercept of the graph represent?

4. If light in just the visible spectrum was used to illuminate the plates, from which materials (aluminum, silver, or sodium) would electrons be ejected?

5. Does the number of points on your graph affect the accuracy of the values of Planck’s constant and work function you determined using the simulation?

6. The method suggested to carry out the experiment, recording the minimum voltage required to reduce the current to zero, introduces a small systematic error by consistently obtaining values for $KE_{\text{max}}$ that are close to, but generally a little larger than, the actual $KE_{\text{max}}$ values. For instance, if $KE_{\text{max}}$ were actually 4.14 volts, you would record it as 4.2 volts because of the limitations of the simulation. Is this systematic error likely to have more impact on the slope of the graph or on the y-intercept? How does the systematic error affect the work function determined from your graph, for instance?