INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 22 numbered problems. Note that Problem 2 occupies 2 pages, Problem 8 occupies 2 pages, Problem 12 occupies 2 pages, Problem 22 occupies 2 pages. Including the cover sheet, you should have 58 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!!

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 14 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:
   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in EVERY sheet you are turning in.
   (c) On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.
   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

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1 of 58
A. (3 points) A function $F(A,B,C,D)$ is defined by the Karnaugh map below (x=“don’t care”). List ALL of the prime implicants of this function in algebraic form and circle them in the map. For each prime implicant, circle “ess” if it is an essential prime implicant of this function. (You may not need all of the lines.)

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```

Prime implicants:

__ess__ __ess__ __ess__ __ess__

B. (1 points) Write a minimal sum-of-products (SOP) expression for this function.

$$F(A, B, C, D) = \text{expression here}$$

C. (3 points) In the space below, draw the schematic diagram for a MINIMAL circuit implementing $F(A,B,C,D)$ using only NAND gates and INVERTERs. Assume that input variables are available only in uncomplemented form.

D. (3 points) In the space below, draw the schematic diagram for a MINIMAL circuit implementing $F(A,B,C,D)$ using only NOR gates and INVERTERs. Assume that input variables are available only in uncomplemented form. A copy of the Karnaugh is provided for your optional use.
A. (3 points) A function $F(A,B,C,D)$ is defined by the Karnaugh map below ($x$ = “don’t care”). List ALL of the prime implicants of this function in algebraic form and circle them in the map. For each prime implicant, circle “ess” if it is an essential prime implicant of this function. (You may not need all of the lines.)

Prime implicants:
- $\overline{A} \overline{D}$
- $A \overline{C}$
- $B \overline{C}$
- $\overline{C} D$
- $A \overline{B}$
- $A \overline{B} \overline{D}$
- $B \overline{C}$
- $B C \overline{D}$
- $C D$

B. (1 point) Write a minimal sum-of-products (SOP) expression for this function.

$$F(A, B, C, D) = \overline{A} D + \overline{A} C + \overline{A} B C \quad \text{OR} \quad \overline{A} D + A \overline{C} + B C \overline{D}$$

C. (3 points) In the space below, draw the schematic diagram for a MINIMAL circuit implementing $F(A,B,C,D)$ using only NAND gates and INVERTERS. Assume that input variables are available only in uncomplemented form.

D. (3 points) In the space below, draw the schematic diagram for a MINIMAL circuit implementing $F(A,B,C,D)$ using only NOR gates and INVERTERS. Assume that input variables are available only in uncomplemented form. A copy of the Karnaugh is provided for your optional use.

$$F = (B + D) (\overline{A} + \overline{C}) (A + C + D)$$
Problem 2 (Core: DSP - ECE 2026)

Problem XX (AREA) Code Number:

Problem - Ideal C-to-D Converter LTI System $H(z), H(e^{j\omega}), h[n]$ Ideal D-to-C Converter

The block diagram above defines a system for discrete-time filtering of continuous-time signals. Note: all parts of this question can be worked independently.

(a) Suppose that the discrete-time LTI system is a bandpass filter whose (causal) finite-length impulse response has the following form:

$$h[n] = \begin{cases} 
0 & n < 0 \\
e^{0.6\pi n} & 0 \leq n \leq 9 \\
0 & n > 9 
\end{cases}$$

and whose frequency response magnitude $|H(e^{j\omega})|$ is shown below.

Determine all the poles and zeros of the system. Give your answer as pole-zero plot plot. Label carefully the locations of the poles and zeros; also note multiplicities if appropriate.
(b) Suppose that a different system function for the LTI system is

\[ H(z) = \frac{10z^{-3} - 5z^{-4}}{4 + 3z^{-1}} \]

Determine the output \( y[n] \) when the input signal is

\[ x[n] = 7(0.5)^n u[n] \]

Simplify your answer as much as possible.

\[ y[n] = \]
SOLUTION

The block diagram above defines a system for discrete-time filtering of continuous-time signals.

Note: all parts of this question can be worked independently.

(a) Suppose that the discrete-time LTI system is a bandpass filter whose (causal) finite-length impulse response has the following form:

\[ h[n] = \begin{cases} 
0 & n < 0 \\
\exp(j0.6\pi n) & 0 \leq n \leq 9 \\
0 & n > 9 
\end{cases} \]

and whose frequency response magnitude \(|H(e^{j\omega})|\) is shown below.

Determine all the poles and zeros of the system. Give your answer as pole-zero plot.
Label carefully the locations of the poles and zeros; also note multiplicities if appropriate.

\[ H(z) \text{ is } 9\text{th order. Nine poles at } z = 0; \text{ nine zeros on unit circle at angles that are multiples of } 0.2\pi \text{ rad, which is } 36^\circ, \text{ except for the angle } 0.6\pi. \]

\[ \text{zeros at } z = e^{0.2\pi \ell}, \quad \ell = 0, 1, 2, \bullet, 4, 5, 6, 7, 8, 9 \]
Problem 2 (Core: DSP - ECE 2026) Solution

(b) Suppose that a different system function for the LTI system is

\[ H(z) = \frac{10z^{-3} - 5z^{-4}}{4 + 3z^{-1}} \]

Determine the output \( y[n] \) when the input signal is

\[ x[n] = 7(0.5)^n u[n] \]

Simplify your answer as much as possible.

\[ y[n] = (17.5)(-0.75) (n-3) u[n-3] \]

(c) Now consider the C-to-D converter where \( x(t) \) is sampled at a rate of \( f_s \). Assume that the result after sampling is a periodic signal \( x[n] \) whose period is 100. After the discrete-time signal \( x[n] \) is analyzed with a 100-point DFT, we find that the DFT \( X[k] \) has only four nonzero values with the following magnitudes:

\[ |X[20]| = 300, \ |X[40]| = 300, \ |X[60]| = 300, \ |X[80]| = 300 \]

when the input is \( x(t) = A \cos(4000\pi t + 0.3\pi) + B \cos(7000\pi t - 0.2\pi) \).

Determine one sampling rate \( f_s \) of the C-to-D converter (in Hz) such that this is true.

Note: Since the signal \( x[n] \) repeats with a period of 100, any arbitrary section of 100 points analyzed with a 100-point FFT will give the same magnitudes, \( |X[k]| \).

There are many possible answers: \( f_s = 2500, 1250, 625, \ldots \)
Consider a doubly linked list that is implemented using the following struct definitions.

```c
typedef struct node_t {
    int            data;
    struct node_t* prev;
    struct node_t* next;
}Node;

typedef struct dll_t {
    struct node_t* head;
    struct node_t* tail;
} DLList;
```

**Part A.** Suppose the C function `Find_Node` takes an integer `k` and a pointer to a sorted doubly linked list `dll` which might be empty. Its nodes are sorted in order of increasing `data` values. Complete the function to efficiently search `dll` for a node with `k` as its `data` value (use the fact that it is sorted). Return a pointer to that node if found or return `NULL` otherwise.

```c
Node* Find_Node(int k, DLList* dll){
    // Implementation goes here
}
```

**Part B.** Complete the procedure `Delete_Node` below which takes a pointer to a `Node` (which may be `NULL`) and deletes it from the doubly linked list `dll`, deallocating the node’s memory. Be sure to update the `head` and `tail` of `dll` if necessary. Also, be sure to guard against dangling pointers.

```c
void Delete_Node(Node* n, DLList* dll){
    // Implementation goes here
}
```
Consider a doubly linked list that is implemented using the following struct definitions.

```c
typedef struct node_t {
    int data;
    struct node_t* prev;
    struct node_t* next;
} Node;

typedef struct dll_t {
    struct node_t* head;
    struct node_t* tail;
} DLList;
```

**Part A.** Suppose the C function `Find_Node` takes an integer `k` and a pointer to a sorted doubly linked list `dll` which might be empty. Its nodes are sorted in order of increasing `data` values. Complete the function to efficiently search `dll` for a node with `k` as its `data` value (use the fact that it is sorted). Return a pointer to that node if found or return `NULL` otherwise.

```c
Node* Find_Node(int k, DLList* dll) {
    Node* current = dll->head;
    while (current && (current->data <= k)) {
        if (current->data == k)
            return current;
        current = current->next;
    }
    return NULL;
}
```

**Part B.** Complete the procedure `Delete_Node` below which takes a pointer to a `Node` (which may be `NULL`) and deletes it from the doubly linked list `dll`, deallocating the node’s memory. Be sure to update the `head` and `tail` of `dll` if necessary. Also, be sure to guard against dangling pointers.

```c
void Delete_Node(Node* n, DLList* dll) {
    if (n) {
        if (n->next)
            (n->next)->prev = n->prev;
        if (dll->head == n)
            dll->head = n->next;
        if (n->prev)
            (n->prev)->next = n->next;
        if (dll->tail == n)
            dll->tail = n->prev;
        n->next = NULL; n->prev = NULL;
        free(n);
    }
    return;
}
```
Problem: For the resistor, $R$, in the following figure,

i) find its resistance such that it consumes the maximum power;

ii) find the maximum power consumed by the resistor, $R$. 

![Diagram of electrical circuit with a 1 A source, two resistors of 2 Ω, and a variable resistor $R$.]
Problem: For the resistor, $R$, in the following figure,
  i) find its resistance such that it consumes the maximum power;
  ii) find the maximum power consumed by the resistor, $R$.

Solution:

$R = 4\Omega$

$P_{\text{max}} = \frac{U_R}{R} = \frac{(\frac{U}{2})^2}{R} = \frac{1}{4} W$
PROBLEM

A transmission line system has a DC source voltage $V_s = 10 \text{ V}$, $R_g = 20 \, \Omega$, $Z_0 = 100 \, \Omega$, and $R_l = 1800 \, \Omega$.

(a) If the uncharged line is suddenly connected to the source, what is the initial power expended by the DC source?

(b) What is the power expended by the DC source upon reaching the steady state when $t \to \infty$?

(c) Where did the extra power go?

(d) If a transmission line fans out to $N$ identical lines in parallel, each having the same characteristic impedance $Z_0$ as the primary line, the reflection coefficient at the end of the primary line is _______.
A transmission line system has a DC source voltage \( V_s = 10 \, \text{V} \), \( R_g = 20 \, \Omega \), \( Z_0 = 100 \, \Omega \), and \( R_L = 1800 \, \Omega \).

(a) If the uncharged line is suddenly connected to the source, what is the initial power expended by the DC source?

At initial DC excitation, the source sees \( R_G \) in series with \( Z_0 \). The power it expends is equal to

\[
P = \frac{V^2}{R} = \frac{10^2}{20 + 100} = 0.833 \, \text{Watts}
\]

(b) What is the power expended by the DC source upon reaching the steady state when \( t \to \infty \)?

At steady-state DC excitation, the source sees \( R_G \) in series with \( R_L \). The power it expends is equal to

\[
P = \frac{V^2}{R} = \frac{10^2}{20 + 1800} = 0.0549 \, \text{Watts}
\]

(c) Where did the extra power go?

This is a decrease in 0.778 W of power (a 93% drop) compared to the output of the source at initial excitation. The extra power was used to charge the line towards steady state.

(d) If a transmission line fans out to \( N \) identical lines in parallel, each having the same characteristic impedance \( Z_0 \) as the primary line, the reflection coefficient at the end of the primary line is \( \frac{(1-N)}{(1+N)} \).
Consider a DRAM circuit.

a. Draw a schematic for a 1-transistor DRAM cell and associated bit line.

b. Draw the voltages of all relevant signals during a read operation in which the cell stores a low value.

c. The cell capacitance is $C_{cell} = 10 \text{ fF}$ and the bit line capacitance is $C_{bit} = 25 \text{ fF}$. The bit line is charged to 1V and the cell is charged to 0V. How much will the bit line voltage dip during the read operation?
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   a. Draw a schematic for a 1-transistor DRAM cell and associated bit line.
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\[
\begin{align*}
U_{bg} &= U_{bt} \frac{C_{bit}}{C_{bit} + C_{cell}} = 1V \frac{25 \text{ fF}}{25 \text{ fF} + 10 \text{ fF}} = 0.7 \text{ V}.
\end{align*}
\]
The figure depicts the current density of one carrier type in a forward biased silicon pn junction versus position. The total current density is also shown. The currents and positions are drawn to scale.

![Diagram of current density](image)

**Problem**

a) Identify the carrier type of the depicted (dotted line) current density

*Explain*

b) Accurately sketch the current due to the other carrier type

*Explain*

c) What is the approximate ratio of dopant concentrations, $N_A/N_D$?

*Explain*

d) For the diffusion coefficients, is $L_n > L_p$?

*Explain*

e) Sketch the conduction band minimum $E_c$ and valence band maximum $E_v$ vs. position for a forward biased pn diode. Include the Quasi Fermi levels vs. position for both electrons and holes.
The figure depicts the current density of one carrier type in a forward biased silicon pn junction versus position. The total current density is also shown. The currents and positions are drawn to scale.

a) Identify the carrier type of the depicted (dotted line) current density.
   Explain
   The depicted current is due to holes
   The quasi-neutral regions (away from the transition region) carry current by majority carriers. The current shown is dominant in the p-type material thus it is due to holes
   Alternatively, the depicted current is shown as a minority carrier in the n-type material and again it must be holes.

b) Accurately sketch the current due to the other carrier type.
   Explain
   The sum of the electron and hole currents must be equal to the total current density
   The new (red) curve is the difference between $J_{\text{Total}}$ and $J_p$

c) What is the approximate ratio of dopant concentrations $N_A/N_D$?
   Explain
   From the figure: hole depletion region $X_p \sim 0.04 \mu m$ electron depletion region $X_n \sim 0.1 \mu m$
   And charge neutrality in the intrinsic region requires $X_p N_A = X_n N_D$
   Thus $N_A/N_D \sim 2.5$

d) For the diffusion coefficients, is $L_n > L_p$?
   Explain
   Yes. $L_n$ ($L_p$) is the diffusion length of electrons (holes) in p-type (n-type)
   From the figure, the recombination in n-type material is much faster (shorter) and hence $L_p$ is smaller than $L_n$

e) Sketch the conduction band minimum $E_c$ and valence band maximum $E_v$ vs. position for a forward biased pn diode. Include the Quasi Fermi levels vs. position for both electrons and holes.
Problem 8 (Core: MICROSYS - ECE 3040) Solution

**Problem**

Consider the incomplete full-wave rectifier circuit below, with a 60 Hz 120V RMS source connected to the transformer’s primary winding. You may assume that $R_{Load}$ is 50 Ohms, and that the ripple is negligible. You may also assume that all 4 diodes are identical, and their I/V characteristic follows the ideal diode equation with an ideality factor of 1. It is also known that the voltage delivered to the load is at most 1.5 Volts less than the peak voltage which appears across the secondary winding of the transformer. The ambient temperature is 300K.

![Diagram of full-wave rectifier circuit]

a) Observing the proper polarities, draw the missing diodes, D1, D2, D3 and D4 in the schematic above to complete the circuit.

b) What is the ratio of the number of turns on the primary winding of the transformer to the number of turns on the secondary?

c) The saturation current of diodes D1 through D4 is AT LEAST / AT MOST (circle one) how many Amps? Provide a numerical answer in addition to circling the correct inequality.
d) The breakdown voltage of diodes D1 through D4 must necessarily be AT LEAST / AT MOST (circle one) how many Volts? Provide a numerical answer in addition to circling the correct inequality.
Problem 8 (Core: MICROSYS - ECE 3040)  
Solution

a) Observing the proper polarities, draw the missing diodes, D1, D2, D3 and D4 in the schematic above to complete the circuit.

```
120 V

```

b) What is the ratio of the number of turns on the primary winding of the transformer to the number of turns on the secondary?

\[
\frac{N_2}{N_1} = \frac{7}{120} = 17.14 \approx 17
\]

c) The saturation current of diodes D1 through D4 is \( \text{AT LEAST} \) \( \text{AT MOST} \) (circle one) how many Amps? Provide a numerical answer in addition to circling the correct inequality.

\[
\begin{align*}
V_{\text{peak}} &= 7\sqrt{2} = 9.8995\text{V} \\
V_{\text{peak}} - V_{\text{Load}} &= 2V_o \leq 1.5\text{V} \\
\Rightarrow V_o &\leq 0.75\text{V} \\
I_o &\geq \frac{V_{\text{peak}} - 1.5}{V_o} = \frac{9.8995 - 1.5}{50} = 0.168\text{A} \\
I_o &= I_s \cdot \frac{V_o}{V_T} \geq 1.68\text{A} \\
I_s &\geq \frac{1.68}{-V_o/V_T} = 0.168 \cdot -0.75/0.025895 \\
I_s &\geq 4.356 \times 10^{-4}\text{A}
\end{align*}
\]
d) The breakdown voltage of diodes D1 through D4 must necessarily be [AT LEAST/AT MOST (circle one)] how many Volts? Provide a numerical answer in addition to circling the correct inequality.

\[
\text{Peak inverse voltage is}
\]

\[
V_{p(\text{max})} - V_0 \geq 9.8995 - 0.75
\]

\[
\implies V_{0R} \geq 9.1495 \text{ V}
\]
Assume a single cycle non-pipelined implementation of a processor that operates at a clock frequency of 100 MHz (clock time of 10 ns). Your job is to convert this design into a five stage pipelined implementation. You are considering two designs: M1 and M2. Shown below are the critical paths for each of the five stages for these two designs.

**a)** What is the maximum clock frequency at which M1 can operate correctly?

**b)** What is the maximum clock frequency at which M2 can operate correctly?

**c)** Which design would you recommend for highest performance?

**d)** For the recommended design, what is the expected speedup compared to the non pipelined implementation. Assume that the Instructions Per Cycle (IPC) of the pipelined machine is 0.75.
Assume a single cycle non-pipelined implementation of a processor that operates at a clock frequency of 100 MHz (clock time of 10 ns). Your job is to convert this design into a five stage pipelined implementation. You are considering two designs: M1 and M2. Shown below are the critical paths for each of the five stages for these two designs.

a) What is the maximum clock frequency at which M1 can operate correctly? 400 MHz

b) What is the maximum clock frequency at which M2 can operate correctly? 333 MHz

c) Which design would you recommend for highest performance? M1

d) For the recommended design, what is the expected speedup compared to the non pipelined implementation. Assume that the Instructions Per Cycle (IPC) of the pipelined machine is 0.75. 4*0.75=3x
A 3 phase 7.5 kV rms (line-neutral) 60 hertz source feeds two balanced three phase loads, an induction motor rated at 1 MW @ 0.9 PF, and the second a delta connected load with impedances shown.

a) Calculate the current and power factor at the source under steady state conditions.

b) Find the value of a wye connected capacitor C in microfarads needed to realize unity power factor at the source (power factor compensated case)

c) Find the percentage reduction in line losses, measured upstream of the capacitor, under normal and power factor compensated conditions.

d) If the motor draws 6X of the normal operating current at start-up with a power factor of 0.4, how much real power does the motor consume at start-up?
Problem 10 (Core: POWER - ECE 3072)  Solution

SOLUTION

\[ I_s = I_{a1} + I_{a2} \]

Y load \[ I_{ab} = \frac{\sqrt{3} \cdot 7.5\text{kV} \angle 30^\circ}{277.3 \angle 25.6^\circ} = 46.88\text{A} \angle 4.4^\circ \]

\[ I_{a1} = \sqrt{3} \cdot I_{ab} \angle -30^\circ = \sqrt{3} \cdot 46.88 \angle 25.6^\circ \]

Motor load \[ I_{a2} = \frac{1000\text{KW}}{13\text{kV} \cdot \sqrt{3} \times 0.9} = 49.36\text{A} \angle 25.6^\circ \text{ lagging} \]

Total current \[ I_s = 130.56 \angle 25.6^\circ \text{A} \quad \text{PF} = 0.9 \]

b) \[ I_{\text{reactive}} = 130.56 \times \sin 25.6^\circ = 56.4\text{A reactive} \]

Capacitor current needed = 56.4A = \[ \frac{\text{Var}}{X_c} \]

\[ X_c = \frac{7.500}{56.4} \Omega = 132.98 \Omega = \frac{1}{W_C} \]

\[ C = \frac{1}{132.98 \times 377} \approx 19.95 \mu\text{F} \]
c) Original current w/o PF correction = 130.56 A
Current after PF correction = 130.56 \times \cos(25.4°) = 117.5 A

Line losses proportional to \( I^2 \)

\[ \text{Loss reduction} = \left( \frac{117.5}{130.56} \right)^2 = 0.81 \]

\[ \text{19% reduction in line losses} \]

d) Normal Motor Current = 49.36 A
Starting Current = 6 \times 49.36 = 296.1 A
Power factor = 0.4

Real power to motor at start =

\[ \sqrt{3} \times 13 kV \times 296.1 A \times 0.4 = 2,666 \text{ MW} \]
For this problem it will be helpful to recall that an exponential random variable $X \sim \text{Exp}(\lambda)$ with rate parameter $\lambda > 0$ has a probability density function (PDF) given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

A room is lit by two light bulbs. Let $A$ and $B$ be random variables that denote the lifetimes of these two light bulbs (in units of days). Assume that $A \sim \text{Exp}\left(\frac{1}{3}\right)$, that $B \sim \text{Exp}(1)$, and that $A$ and $B$ are independent of each other.

1. What is the probability that light bulb “A” lasts longer than light bulb “B”? In other words, what is $P[B < A]$?

2. Let $Z = \max(A, B)$ be the amount of time until both light bulbs burn out and the room goes dark. What is the probability that $Z \leq 2$?

3. Again letting $Z = \max(A, B)$, what is the expected value $E[Z]$?

4. You visit the room after 2 days to find both bulbs still lit, and again after 4 days to find that only bulb $B$ has burned out. Given this information, what is the probability that bulb $B$ lasted at least 3 days?
Problem 11 (Core: DSP/TLCOM - ECE 3077) Solution

1. \[ P[B < A] = \int_{a=0}^{\infty} \int_{b=0}^{a} \frac{1}{3} e^{-a/3-b} \, db \, da \]
   \[ = \int_{a=0}^{\infty} \frac{1}{3} e^{-a/3}(1 - e^a) \, da \]
   \[ = \int_{a=0}^{\infty} \frac{1}{3} e^{-a/3} da \, - \frac{1}{3} \int_{a=0}^{\infty} e^{-4a/3} da \]
   \[ = 1 - \frac{3}{4} \cdot \frac{3}{4} \]

2. The random variable \( Z = \max(A, B) \) has cumulative distribution function (CDF) given by
   \[ F_Z(z) = P[A \leq z \text{ and } B \leq z] \]
   \[ = P[A \leq z] \cdot P[B \leq z] \]
   \[ = \left( \int_{0}^{z} \frac{1}{3} e^{-a/3} \, da \right) \left( \int_{0}^{z} e^{-b} \, db \right) \]
   \[ = (1 - e^{-z/3})(1 - e^{-z}) \]
   \[ = 1 - e^{-z/3} - e^{-z} + e^{-4z/3} \]
   for \( z \geq 0 \). Therefore, \( P(Z \leq 2) = F_Z(2) = 1 - e^{-2/3} - e^{-2} + e^{-8/3} = 0.4207 \)

3. The PDF for \( Z \) is given by the derivative of the CDF:
   \[ f_Z(z) = \begin{cases} \frac{1}{3} e^{-z/3} + e^{-z} - \frac{4}{3} e^{-4z/3}, & z \geq 0 \\ 0, & \text{otherwise} \end{cases} \]
   Recognizing that each term in the sum above looks like the probability density function of a simple exponential random variable, we have that
   \[ E[Z] = 3 + 1 - \frac{3}{4} = 3.25. \]

4. Define the event \( C = \{2 \leq B \leq 4\} \). Using the CDF of an exponential random variable we can write \( P(C) = F_B(4) - F_B(2) = (1 - e^{-4}) - (1 - e^{-2}) = 0.117 \).
   The conditional PDF for this problem is therefore given by:
   \[ f_{B|C}(b|c) = \frac{f_B(b)}{P(C)} = \frac{e^{-b}}{0.117} \quad \text{for } 2 \leq b \leq 4 \]
   Using this, the probability of interest is given by:
   \[ P(B \geq 3|C) = \int_{3}^{\infty} f_{B|C}(b|c) \, db = \int_{3}^{4} \frac{e^{-b}}{0.117} \, db = \frac{-(e^{-4} - e^{-3})}{0.117} = 0.269 \]
Problem 1: Fourier Transform and Fourier Series (10 points)

(a) Find the Fourier transform of \( x(t) \). (5 points)
(b) Now consider the periodic signal \( y(t) \) with a period of \( T_0 = 6 \) seconds. Find the Fourier series coefficients \( a_k \) of \( y(t) \). \( (5 \text{ points}) \)
Problem 12 (Core: CONTROLS - ECE 3084) Solution

(a) Find the Fourier transform of \( x(t) \). (5 points)

Brutal force approach

\[
X(j\omega) = \int_{-2}^{-1} 2te^{-j\omega t} dt + \int_{-1}^{1} 2e^{-j\omega t} dt + \int_{1}^{2} (4-2t)e^{-j\omega t} dt = 2\left[\int_{-2}^{-1} te^{-j\omega t} dt + \int_{-1}^{1} e^{-j\omega t} dt + \int_{1}^{2} (2-t)e^{-j\omega t} dt\right] = \ldots = 6\text{sinc}\left(\frac{\omega}{2}\right)\text{sinc}\left(\frac{3\omega}{2}\right)
\]

The easier solution is to recognize that signal \( x(t) \) is convolution of two signals:

\[
x(t) = 2(x_1(t) * x_2(t)) = 2\left([u(t+1.5) - u(t-1.5)] * [u(t+0.5) - u(t-0.5)]\right)
\]

\[
X(j\omega) = 2(F(x_1(t)) \cdot F(x_2(t))) = 2\left(\frac{\sin\left(\frac{\omega}{2}\right)}{\omega/2} \cdot \frac{\sin\left(\frac{3\omega}{2}\right)}{3\omega/2}\right) = \ldots = 6\text{sinc}\left(\frac{\omega}{2}\right)\text{sinc}\left(\frac{3\omega}{2}\right) = 8\frac{\sin\left(\frac{\omega}{2}\right)\sin\left(\frac{3\omega}{2}\right)}{\omega^2} = \frac{4}{\omega^2}(\cos(\omega) - \cos(2\omega))
\]
Problem 12 (Core: CONTROLS - ECE 3084) Solution

(b) Now consider the periodic signal \( y(t) \) with a period of \( T_0 = 6 \) seconds. Find the Fourier series coefficients \( a_k \) of \( y(t) \). (5 points)

\[
T_0 = 6 \quad \text{and} \quad \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}
\]

\[
a_k = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} y(t) e^{-j k \omega_0 t} \, dt
\]

**Brutal force**

\[
a_0 = \frac{1}{6} \int_{-3}^{3} y(t) \, dt = 1 \quad \text{This is DC component of the signal}
\]

\[
a_k = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} y(t) e^{-j k \omega_0 t} \, dt = \frac{2}{T_0} \left[ \int_{-2}^{-1} t e^{-j k \omega_0 t} \, dt + \int_{-1}^{1} e^{-j k \omega_0 t} \, dt + \int_{1}^{2} (2 - t) e^{-j k \omega_0 t} \, dt \right] = \ldots
\]

\[
= \frac{6}{T_0} \frac{\sin \left( \frac{k \omega_0}{2} \right) \sin \left( \frac{3k \omega_0}{2} \right)}{\sin \left( \frac{\pi}{6} \right) \sin \left( \frac{k \pi}{2} \right)} = \sin \left( \frac{k \pi}{6} \right) \sin \left( \frac{k \pi}{2} \right)
\]

**More elegant approach:**

\[
a_k = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} y(t) e^{-j k \omega_0 t} \, dt = \frac{1}{T_0} \int_{-2}^{2} x(t) e^{-j k \omega_0 t} \, dt \quad \text{Using results from part (a), we can write}
\]

\[
a_k = \sin \left( \frac{k \pi}{6} \right) \sin \left( \frac{k \pi}{2} \right) = \frac{8}{6} \frac{\sin \left( \frac{k \pi}{6} \right) \sin \left( \frac{k \pi}{2} \right)}{\left( \frac{k \pi}{3} \right)^2} = \begin{cases} 0 \text{ for } k \neq 0 \text{ even} \\ \frac{8}{6} \frac{\sin \left( \frac{k \pi}{6} \right) \sin \left( \frac{k \pi}{2} \right)}{\left( \frac{k \pi}{3} \right)^2} \end{cases}
\]

\[
a_0 = \frac{1}{6} \int_{-3}^{3} y(t) \, dt = 1
\]
Consider the design of a full adder cell for a ripple carry adder in standard static CMOS style. All gates should be sized for equal worst case rise and fall times. Delay should be calculated in units of $\tau = R_n C_{inv}$, where $R_n$ is the equivalent resistance of a minimum sized nFet ($W = W_{\text{min}}$ and $L = L_{\text{min}}$), and $C_{inv} = 3C_{\text{fet}}$ is the input capacitance of a minimum sized inverter with equal rise and fall times. Note that we are assuming that the transconductance ratio of identically sized nFet to pFet $\beta_n / \beta_p = 2$. Also, assume that the parasitic delay of a minimum sized inverter $p_{\text{inv}} = \tau$. You may wish to use the method of logical effort to solve this problem.

(a) (2) Write the equations for the full adder sum $s_i$ and carry out $c_{i+1}$ in terms of inputs $a_i, b_i$ and $c_i$.

(b) (2) Design and draw two alternative implementations of $c_{i+1}$ using
(i) 3 NAND2 gates and 1 NAND3 gate, and (ii) 2 NAND2 gates and 1 OAI21 gate. (Hint: factor a term).

(c) (6) Size each circuit in (b) for minimum delay in a ripple carry adder. In each case size the first stage of logic as small as feasible given the design rules, and minimize the load from any logic that is not on the critical path. Calculate the per bit delay on the critical path for each circuit. Note that you may leave sizes in terms of input capacitance.
Consider the design of a full adder cell for a ripple carry adder in standard static CMOS style. All gates should be sized for equal worst case rise and fall times. Delay should be calculated in units of \( \tau = R_n C_{inv} \), where \( R_n \) is the equivalent resistance of a minimum sized nFET \( (W = W_{min} \ and \ L = L_{min}) \), and \( C_{inv} = 3C_{fet} \) is the input capacitance of a minimum sized inverter with equal rise and fall times. Note that we are assuming that the transconductance ratio of identically sized nFET to pFET \( \beta_n/\beta_p = 2 \). Also, assume that the parasitic delay of a minimum sized inverter \( p_{inv} = \tau \). You may wish to use the method of logical effort to solve this problem.

(a) (2) Write the equations for the full adder sum \( s_i \) and carry out \( c_{i+1} \) in terms of inputs \( a_i, b_i \) and \( c_i \):

\[
\begin{align*}
  s_i &= a_i \oplus b_i \oplus c_i = a_i \overline{b_i} \overline{c_i} + \overline{a_i} b_i c_i + \overline{a_i} \overline{b_i} c_i + a_i \overline{b_i} c_i \\
  c_{i+1} &= a_i b_i + a_i c_i + b_i c_i = a_i b_i + (a_i + b_i) c_i
\end{align*}
\]

(b) (2) Design and draw two alternative implementations of \( c_{i+1} \) using

(i) 3 NAND2 gates and 1 NAND3 gate, and (ii) 2 NAND2 gates and 1 OA\|21 gate. (Hint: factor a term).

\[\text{Diagram}(\text{i})\]

\[\text{Diagram}(\text{ii})\]
(c) (6) Size each circuit in (b) for minimum delay in a ripple carry adder. In each case size the first stage of logic as small as feasible given the design rules, and minimize the load from any logic that is not on the critical path. Calculate the per bit delay on the critical path for each circuit. Note that you may leave sizes in terms of input capacitance.

\[ F = G \cdot B \cdot H = \left( \frac{5}{3} \cdot \frac{5}{3} \right) \cdot \left( \frac{1}{3} \cdot \frac{3}{2} \right) = 7.8 \quad \hat{F} = F^{1/2} = 2.8 \tau \]

Per bit delay \( D = 2 \cdot \hat{F} + P = 2 \cdot 2.8 + 5 = 10.6 \tau \)

Sizing of NAND3:

\[ x = \frac{C_{out}}{C_{in}} = \frac{\frac{9}{3} \cdot \frac{7}{2}}{2.8} = 1.4 \text{C}_{in} \]

(ii) 

\[ F = G \cdot B \cdot H = \left( \frac{5}{3} \right)^2 - 1 \left( \frac{9}{3} \cdot \frac{3}{2} \right) = 4.4 \quad \hat{F} = F^{1/2} = 2.1 \tau \]

\[ P = \left( \frac{9}{3} + 2 \right) \tau \quad D = 2 \cdot 2.1 + 4 \cdot \tau = 8.9 \tau \]

Size of NAND2:

\[ x = \frac{4/3 \cdot 5/3}{2.1} = 1.1 \text{C}_{in} \]
PROBLEM

A 6-pole, 230-V (L-L), 60 Hz, Y-connected stator, three phase induction motor has the following parameters on a per phase basis, all referred to the stator side:

Stator resistance = 0.5 ohm
Stator leakage reactance = 0.75 ohm
Rotor resistance referred to stator side = 0.25 ohm
Rotor leakage reactance referred to stator side = 0.5 ohm
Core loss equivalent resistance referred to the stator side = 500 ohm
Magnetizing current is small enough to be neglected.
Neglect mechanical losses.

Use the approximate equivalent circuit for the motor, which has the core loss resistance across the input terminals to the circuit. Draw the circuit, insert your symbols, and then determine the efficiency of the motor at its rated slip of 2.5%.
A 6-pole, 230-V (L-L), 60 Hz, Y-connected stator, three phase induction motor has the following parameters on a per phase basis, all referred to the stator side:

Stator resistance = 0.5 Ω
Stator leakage reactance = 0.75 Ω
Rotor resistance referred to stator side = 0.25 Ω
Rotor leakage reactance referred to stator side = 0.5 Ω
Core loss equivalent resistance referred to the stator side = 500 Ω
Magnetizing current is small enough to be neglected.
Neglect mechanical losses.

Use the approximate equivalent circuit for the motor, which has the core loss resistance across the input terminals to the circuit. Draw the circuit, insert your symbols, and then determine the efficiency of the motor at its rated slip of 2.5%.

**Solution**

\[ I_c = \frac{230}{1.732} \div 500 = 0.266 \text{ ohm} \]

Equivalent impedance of the series circuit is \( Z_e = R_1 + R_2/s + j(X_1 + X_2) = 10.5 + j1.25 \)

Hence equivalent rotor current is \( I_2 = \frac{(230/1.732)/(10.5 + j1.25)}{12.558^\circ \text{-6.79}^\circ} \)

Stator and rotor copper losses are \( 3I_2^2 (0.5 + 0.25) = (3)12.558^2 (0.75) = 344.84 \text{ W} \)

Core loss = \( 3(230/1.732)^2 / 500 = 106.13 \text{ W} \)

Power output = \( 3I_2^2 R_2/s = (3)12.558^2 (0.25/0.025) = 4462.82 \text{ W} \)

Efficiency = \( 4462.82 / (4462.82 + 344.84 + 106.13) = 87.96\% \)
Analyze the following feedback circuit. Assume the op-amp is ideal with infinite voltage gain, infinite bandwidth, infinite input impedance, and zero output impedance. The op-amp is powered by ±25V. The diode D1 is an ideal diode with turn-on voltage of 0.7V. The NMOS MOSFET M1 has its device parameters as $K' = 25\mu A/V^2$, $W/L = 40\mu m/1\mu m$, and $V_{TN} = 0.5V$.

Recall that for a MOSFET in saturation $I_D = \left(\frac{K'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2 = \left(\frac{K'}{2}\right)\left(\frac{W}{L}\right)\left(V_{ad}\right)^2$, while for a MOSFET in triode $I_D = K'\left(\frac{W}{L}\right)\left[V_{ad} \times V_{DS} - \frac{(V_{DS})^2}{2}\right]$, where $V_{ad} = V_{GS} - V_{TN}$.

Neglect the channel-length-modulation and body-effect of the MOSFET transistor. Use the constant voltage drop model for the diode. Please answer the following questions.

1. Label the op-amp Plus “+” and Minus “-” inputs to ensure a negative feedback.

2. Assume the input voltage $0<V_{in}<5V$. Is diode D1 ON or OFF? Is NMOS MOSFET M1 in its Saturation, Triode, or Off region?

3. Assume the input voltage $0<V_{in}<5V$. Label the actual directions of all the branch currents ($I_{R1}$, $I_{R2}$, $I_{R3}$, $I_{R4}$, $I_{D1}$, and $I_{M1}$).

4. Assume the input voltage $0<V_{in}<5V$. Solve all the node voltages ($V_1$, $V_2$, $V_3$, $V_4$ and $V_{out}$) and branch currents ($I_{R1}$, $I_{R2}$, $I_{R3}$, $I_{R4}$, $I_{D1}$, and $I_{M1}$) as functions of $V_{in}$ and the given device parameters.

5. Assume the input voltage $-5V<V_{in}<0$. Is diode D1 ON or OFF? Is NMOS MOSFET M1 in its Saturation, Triode, or Off region?

6. Assume the input voltage $-5V<V_{in}<0$. Label the actual directions of all the branch currents ($I_{R1}$, $I_{R2}$, $I_{R3}$, $I_{R4}$, $I_{D1}$, and $I_{M1}$).

7. Assume the input voltage $-5V<V_{in}<0$. Solve all the node voltages ($V_1$, $V_2$, $V_3$, $V_4$ and $V_{out}$) and branch currents ($I_{R1}$, $I_{R2}$, $I_{R3}$, $I_{R4}$, $I_{D1}$, and $I_{M1}$) as functions of $V_{in}$ and the given device parameters.
Question 1:

![Diagram](image1)

Question 3:

![Diagram](image2)

Question 6:

![Diagram](image3)
Problem 15 (Breadth: EDA - ECE 3400) Solution

Question 2: Diode $D_1$ is ON. NMOS MOSFET $M_1$ is OFF.

Question 4: $0 \leq \text{Vin} < 5 \text{V}$. $D_1$ is ON. The negative feedback loop is closed by $D_1$ and $R_1$.

$V_1 = 0 \text{V}$, $V_2 = -0.7 \text{V}$, NMOS MOSFET $M_1$ is OFF.

$V_3 = -0.7 \text{V} - \text{Vin}$, $V_4 = -0.7 \text{V}$, $V_{out} = 0 \text{V}$

$I_{R_1} = \frac{-\text{Vin}}{R_1}$, $I_{D_1} = I_{R_3}$, $I_{R_1} = I_{R_4} = I_{R_2} = 0$

Question 5: Diode $D_1$ is OFF. NMOS MOSFET $M_1$ is in its Saturation Mode.

Note that $M_1$ is a "diode-connected" transistor.

Question 7: $-5 \text{V} < \text{Vin} < 0 \text{V}$. $D_1$ is OFF. NMOS $M_1$ is in Saturation Mode.

The negative feedback loop is closed by $R_x, M_1$ and $R_3$.

$V_1 = 0 \text{V}$, $V_{out} = -\frac{R_3}{R_1} \cdot \text{Vin} = -2 \text{Vin} = 2 |\text{Vin}|$ since $-5 \text{V} < \text{Vin} < 0 \text{V}$

$I_{R_1} = I_{R_3} = I_{M_1} = I_{R_4} = |\text{Vin}|/R_1$, $I_{D_1} = I_{R_3} = 0$

Since NMOS $M_1$ is in Saturation Mode, its current is given as

$I_{R_1} = \frac{1}{2} (k') \frac{V_1}{W} (V_{th})^2 \Rightarrow V_{th} = \frac{-2 \cdot I_{M_1}}{k'} \cdot \left( \frac{1}{W} \right) = V_{th} - V_{th}$

$\Rightarrow V_2 = V_4 = V_{out} + \sqrt{2 \cdot I_{M_1} \left( \frac{L}{W} \right)} = 2 |\text{Vin}| + V_{th} + \sqrt{2 \cdot |\text{Vin}| / R_1 \cdot \frac{1}{k'} \cdot \left( \frac{1}{W} \right)}$

$\Rightarrow V_3 = V_2 + I_{R_3} \cdot R_3 = V_4 + \frac{|\text{Vin}|}{R_1} \cdot x R_3$

Note $R_3 = R_1 = 10 \text{k}\Omega$

$V_3 = V_2 + |\text{Vin}| = 3 |\text{Vin}| + V_{th} + \sqrt{2 \cdot |\text{Vin}| / R_1 \cdot \frac{1}{k'} \cdot \left( \frac{1}{W} \right)}$
Problem 16 (Breadth: CONTROLS - ECE 3550) Solution

Suppose you have a big ugly unknown electromechanical system, Σ, in front of you, that is full of nonlinear elements inside, but is otherwise such that its overall behavior from input to output is linear and time invariant.

You have a (perfect) sinusoidal signal generator in the lab, that produces signals $A \cos \omega t$, with programmable amplitude $A$, and frequency $\omega$ ranging from zero to the “very large”.

You also have an (ideal) spectral analyzer that has two input lines and two outputs, and is such that if $u_1(t) = \cos \omega t$ and $u_2(t) = B \cos \omega (t + \theta)$ (same frequency!) it produces the outputs $y_1 = A$ and $y_2 = \theta$.

In addition you have an actuator mapping a voltage input to whatever form the input of Σ may take, with (for simplicity) transfer function $G_a(s) = 1$, and a sensor that takes the output of Σ and maps it to a voltage, also with transfer function $G_s(s) = 1$.

i) As you have no idea what the system Σ is doing, you start to experiment and apply some pure sinusoids of various frequencies at the input $\omega_1, \ldots, \omega_N$. In each case the output grows unboundedly and you have to shut the system off for safety. Could you actually have concluded that if the system goes unstable at one frequency, it must do so at all frequencies?

ii) Now you remember you learned something about feedback. Indeed, when you feed the output of Σ back to the input in the usual feedback scheme, you nd that unity feedback makes the output go to zero asymptotically when arbitrary initial conditions are set up in Σ, but otherwise no external input is applied. Would this imply that for any sinusoidal input, the output of the feedback system is bounded?

iii) You also remember that the Nyquist criterion may help in deciding for which feedback gains, $k$ around Σ, a closed loop system would be stable. Would it be possible to determine using experimental and analytic methods, all values of the proportional feedback gain, $k$ for which the unknown system would remain asymptotically stable in closed loop. If so, describe how? If not, explain why not.
i) It is given that $\Sigma$ is LTI, and hence is characterizable by a transfer function $H(s)$. Since the Laplace transform of a $A \cos \omega t$ is $\frac{As}{s^2 + \omega^2}$, the Laplace transform of the outputs are $Y_i(s) = H(s) \frac{As}{s^2 + \omega^2}$. Since the corresponding time responses are all unstable, it either happens that $H(s)$ must have roots in the right half plane, or if that’s not the case, it must have had roots precisely at $\pm j\omega_i$ for $i = 1, \ldots, N$ (which is probably unlikely). Indeed, even if the roots $\pm j\omega_i$ have multiplicity one, it follows that $Y_i(s)$ has roots of at least multiplicity two on the imaginary axis, and thus $y_i(t)$ grows unboundedly.

If you performed only one such experiment, in the first case, it would necessarily follow that the responses to all sinusoidal inputs would grow unboundedly. In the second case, factor $H(s) = \frac{1}{s^2 + \omega_i^2} H_r(s)$, for some positive integer $\alpha$ and where $H_r(s)$ is irreducible and has no other factors $s^2 + \omega_i^2$ in its denominator. Clearly the response to $\cos \omega_1 t$ will be unbounded, but if $H_r(s)$ is asymptotically stable, then the response to sinusoidal inputs at frequencies differing from $\omega_1$ remains bounded.

Hence the answer is thus: NO!

ii) Using unity feedback in standard configuration, the closed loop has transfer function (See Fig 1)

$$H_{cl}(s) = \frac{H(s)}{1 + H(s)}.$$  

The property that is described is the asymptotic stability property with respect to the initial conditions, and a necessary and sufficient condition for this is that all poles of $H_{cl}(s)$ are in the open left half plane. But then it follows in turn that for a sinusoidal input $\cos \omega t$

$$Y(s) = H_{cl}(s)U(s) = H_{cl}(s) \frac{s}{s^2 + \omega^2},$$

and since the denominator of $Y(s)$ only has roots in the closed left half plane and the ones on the imaginary axis have multiplicity one, $y(t)$ must remain bounded.

Thus the answer is: YES!

![Figure 1: $H_{cl}(s)$](image)

iii) Since it is established in (ii) that the frequency function $H_{cl}(j\omega)$ is well defined, hook up your laboratory equipment to the feedback system (both output of the signal generator and feedback system output need to be hooked up to the analyzer so that correct amplitude gain
and phase can be detected) and take a frequency sweep of the output $Y(j\omega)$. The output of your spectral analyzer gives components related to the Bode diagrams (if you transform to logarithmic form), or better, by plotting polar plot, you obtain directly the Nyquist plot for the closed loop system.

For the sake of the argument, suppose this Nyquist plot looks like the one in Figure 2, if you sweep $\omega$ from zero to a very large frequency (let’s call it in nity). Note that it has to be entirely bounded from what we know. Recall that the Nyquist plot is the conformal mapping of a "D-shaped" contour in the right half plane. Since $H_d(s)$ is asymptotically stable, no indentations have to be made on the $j\omega$ axis, and this contour does not encircle any poles of $H_d(s)$. Hence $P$, the number of encircled "open loop poles” (which is here $H_d$) is zero. The Nyquist criterion says that the number of closed loop (treating $H_d(s)$ as the “open loop” with additional feedback with gain $k_0$) poles inside the same D-shaped contour is equal to $P = 0$ plus the number of encirclements, $n$, of the critical point $-\frac{1}{k_0}$ in counter clockwise (CCW) direction (since the $D$ is run in CCW direction). Since there also cannot be poles on D-shaped contour either, it follows that $n = 0$ corresponds to asymptotic stability of the closed loop. For the given gure we nd

$$
\begin{align*}
    n &= 0 & -\frac{1}{k_0} < c_1 \\
    n &= 2 & c_1 < -\frac{1}{k_0} < c_2 \\
    n &= 1 & c_2 < -\frac{1}{k_0} < c_3 \\
    n &= 0 & c_3 < -\frac{1}{k_0}
\end{align*}
$$

In general you should nd the intervals on the real axis where $n = 0$ and determine the corresponding intervals for $k_0$, which is readily done by graphing $k_0$ as function of $-\frac{1}{k_0}$, (a hyperbola with the real and imaginary axes as asymptotes, and which lies in II-nd and IV-th quadrant).

Finally, by Figure 3, it follows that the requisite gain for $\Sigma$ is $k = k_0 + 1$.

Conclusion: Yes! It can be done!.
Figure 3: Equivalence of the feedback systems
Problem 17 (Breadth: TELECOM - ECE 3600) Solution

PROBLEM

Given that the end-end delay between Host A and Host B is 10 milliseconds, and a bottleneck link has bandwidth 120 Mbps. Assume that the bottleneck link dominates the end-end delay which is half of the round trip delay (RTT). Assume maximum segment size (MSS) is 1500 bytes, and all segments are MSS.

(a) (3pts) Assume TCP now uses a window size \( n \) to sent segments. What should \( n \) be to fill the “bit pipe” at the bottleneck link in one RTT? (In other words, what is the bandwidth-delay product when the delay is considered as RTT?)

(b) (3pts) Consider that TCP starts from slow-start. Assume that there is no loss. How many RTTs are needed for the TCP congestion window size to grow to be at least \( n \) as in (a)? (Provide your solutions in terms of \( n \) first, then plug in numbers.)

(c) (3pts) Now consider that TCP starts from window size of one MSS, and increases the window size through additive increase. Assume that there is no loss. How many RTTs are needed for the TCP congestion window size to grow to be at least \( n \) as in (a)? (Provide your solutions in terms of \( n \) first, then plug in numbers.)

(d) (1pt) Using the results you have obtained from (b) and (c), provide an explanation why the practical TCP (i.e., the one we use now) adopts slow-start and additive-increase at different (i.e., the beginning and later) stages of a TCP session.
SOLUTION

Given that the end-end delay between Host A and Host B is 10 milliseconds, and a bottleneck link has bandwidth 120 Mbps. Assume that the bottleneck link dominates the end-end delay which is half of the round trip delay (RTT). Assume maximum segment size (MSS) is 1500 bytes, and all segments are MSS.

(a) (3pts) Assume TCP now uses a window size \( n \) to sent segments. What should \( n \) be to fill the “bit pipe” at the bottleneck link in one RTT? (In other words, what is the bandwidth-delay product when the delay is considered as RTT?)

\[ A: n = \frac{2 \times 120 \times 0.001 \times 1,000,000}{1500 \times 8} = 200 \]

(b) (3pts) Consider that TCP starts from slow-start. Assume that there is no loss. How many RTTs are needed for the TCP congestion window size to grow to be at least \( n \) as in (a)? (Provide your solutions in terms of \( n \) first, then plug in numbers.)

\[ A: \text{#RTT} = (\log n) + 1 = (\log 200) + 1 = 8 \]

(c) (3pts) Now consider that TCP starts from window size of one MSS, and increases the window size through additive increase. Assume that there is no loss. How many RTTs are needed for the TCP congestion window size to grow to be at least \( n \) as in (a)? (Provide your solutions in terms of \( n \) first, then plug in numbers.)

A: Window size increases from 1, 2, 3, ..., \( n \). So \( n-1 = 199 \) RTTs are needed.

(d) (1pt) Using the results you have obtained from (b) and (c), provide an explanation why the practical TCP (i.e., the one we use now) adopts slow-start and additive-increase at different (i.e., the beginning and later) stages of a TCP session.

A: The beginning of a TCP session needs to exploit available bandwidth, and thus uses slow-start. It would take a long time for additive-increase to grow to a large enough rate. Later, when the bandwidth is utilized and congestion may occur. Additive-increase is then appropriate to grow the rate cautiously for congestion avoidance.
Consider the symmetric 2-port network made from three resistors as shown, for use in a 50 Ω system:

The incident and reflected voltage waves defined at the indicated ports are related by

\[
\begin{bmatrix}
V_1^- \\
V_2^-
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
V_1^+ \\
V_2^+
\end{bmatrix}
\]

where the 2 by 2 matrix is known as the scattering matrix or \( S \)-matrix.

(a) Find the \( S \)-matrix for the above system.

(b) If port 2 is (improperly) terminated in a load resistor with \( R = 25 \) Ω, what is the reflection coefficient looking into port #1?
Consider the symmetric 2-port network made from three resistors as shown, for use in a 50 Ω system:

The incident and reflected voltage waves defined at the indicated ports are related by

\[
\begin{bmatrix}
V_1^- \\
V_2^-
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
V_1^+ \\
V_2^+
\end{bmatrix}
\]

where the 2 by 2 matrix is known as the scattering matrix or S-matrix.

(a) Find the S-matrix for the above system.

*S₁₁ and S₂₂ can be determined from:*

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{8.56} + \frac{1}{141.8} = 0.0114 + 0.0071 = 0.0185
\]

\[
R = \frac{1}{0.0185} = 54.0
\]

\[
\begin{bmatrix}
V_1^- \\
V_2^-
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]
The figure below shows five situations in which light of free-space wavelength $\lambda_0 = 600$ nm is incident normally on a very thin film (the middle layer in each case). The indicated refractive indices are $n_1 = 1.5$ and $n_2 = 2.2$.

a) In each case, consider what happens to the reflected light in the limit where the thickness of the thin layer approaches zero. Please circle “enhanced” or “suppressed”. No need to explain your answers.

Case A: Enhanced Suppressed
Case B: Enhanced Suppressed
Case C: Enhanced Suppressed
Case D: Enhanced Suppressed
Case E: Enhanced Suppressed

b) What fraction of the light power is reflected in case A and case C, respectively, when the thickness of the thin layer goes to zero?

c) Between case C and case E, which case may lead to a better elimination of the reflection? What is the optimal thickness of the thin film in that case?

d) For the best anti-reflective case you considered in part (c), where the reflection of light at $\lambda_0 = 600$ nm is efficiently suppressed under normal incidence, what wavelength (within the visible spectrum) is minimally reflected when the light is incident at 30°?
a) Case A: Reflection Suppressed;
Case B: Reflection Suppressed;
Case C: Reflection Enhanced;
Case D: Reflection Enhanced;
Case E: Reflection Suppressed.

We use $n_0$, $n_f$, $n_s$ to represent the refractive index of the top, the film, and the substrate, respective. Note that (1) when $n_0 < n_f < n_s$ or $n_0 > n_f > n_s$, there is no addition phase difference between the two reflected beams. Constructive interference between the two reflected beams occurs when $2n_0 t \cos \theta_t = m \lambda_0$; (2) When the value of $n_f$ is larger (or smaller) than both $n_0$ and $n_s$, there is a half-cycle relative phase shift between the two reflected beams. Destructive interference in reflection occurs when $2n_f t \cos \theta_t = m \lambda_0$;

b) When the intermediate layer disappears, the reflection between two semi-infinite media is evaluated by the Fresnel’s equation at the normal incidence.

Case A: $R = \frac{|n_1 - n_2|}{|n_1 + n_2|}^2 = 3.6\%$; Case C: $R = \frac{|1-n_2|}{1+n_2}^2 = 14.1\%$.

c) In order to obtain effective suppression of the reflection, we need the magnitudes of the two reflection coefficients to be balanced.

$\rho_{top} = \frac{(n_{air} - n_f)}{(n_{air} + n_f)}$; $\rho_{bottom} = \frac{(n_f - n_s)}{(n_f + n_s)}$. So $\rho_{top} = \rho_{bottom}$ can be achieved when $n_{air} < n_f < n_s$, and ideally, $n_f = \sqrt{n_{air}n_s}$. Case C satisfies this requirement, as $n_1 \approx \sqrt{n_2}$. The optimal thickness meets the condition $\Delta p = 2n_1 t = \lambda_0/2$, i.e., $t = \lambda_0/(4n_1)$ (quarter wavelength coating). So the optimal thickness is $(600 \text{ nm})/(4 \times 1.5) = 100 \text{ nm}$.

d) In case C with $t = 100 \text{ nm}$, the destructive interference in reflection occurs when

$2n_f t \cos \theta_t = (m + 1/2) \lambda_0$. At normal incidence ($\theta_t = 0$), this is fulfilled for $m = 0$ and $\lambda_0 = 600 \text{ nm}$, as discussed in part (c).

For $\theta_t = 30^\circ$, we first find $\theta_t$ (i.e., the angle in the film) using Snell’s law.

$\sin \theta_t = \frac{\sin \theta_i}{n_f} = 1/3$, so $\cos \theta_t = \frac{\sin \theta_i}{n_f} = 0.9428$. So in this case, destructive interference in reflection occurs for the wavelength

$\lambda_0 = \frac{2n_f \cos \theta_t}{m + 0.5} = \frac{2 \times 1.5 \times (100 \text{ nm}) \times 0.9428}{m + 0.5}$. Let $m = 0$, we find $\lambda_0 = 566 \text{ nm}$.
Problem 20 (Specialized: OPTICS - ECE 4502) Solution

PROBLEM

A step-index optical fiber consists of a cylindrical core surrounded by a cladding as shown in the figure below. The refractive index of the core $n_{\text{core}}$ is greater than the index of the cladding $n_{\text{clad}}$.

\begin{center}
\begin{tabular}{c|c}
& $n_{\text{clad}}$ \\
\hline
Cladding & \\
\hline
Core & $n_{\text{core}}$ \\
\hline
Cladding & $n_{\text{clad}}$
\end{tabular}
\end{center}

For this optical fiber, draw the dispersion curve ($\omega$ vs. $\beta$) for the fundamental mode of this fiber on the axes provided below. For purposes of simplification assume that the core and cladding materials are dispersionless (their refractive indices do not vary with frequency). On your dispersion diagram include the “light lines” for the core and the cladding. Also, on your diagram, label the “zero dispersion frequency.”
The dispersion diagram for a step-index optical fiber appears as shown below.
A. Draw a standard electrocardiogram for a healthy human being. Label the intervals and describe the relevant heart function for each interval.

B. The resting (or filling) phase of the heart cycle is called the ________________.

C. The contractile (or pumping) phase of the heart cycle is called the ________________.
A. Draw a standard electrocardiogram for a healthy human being. Label the intervals and describe the relevant heart function for each interval.

B. The resting (or filling) phase of the heart cycle is called the ___diastole____.

C. The contractile (or pumping) phase of the heart cycle is called the ___systole____.

PR interval - measure of the AV conduction time
QRS complex – activation of the ventricles
TP segment – establishes a baseline for the measurement
T wave – recovery of the ventricular cardiac cells
QT interval – total duration of the ventricular systole
(10 pts) **Body Posture / Position Sensing.** In assessing a person’s overall health / wellness, it is important to determine the person’s body posture or position in real-time. Such information can facilitate the classification of daily living activities that the person is involved in, and also provide a context for other sensed parameters (such as heart rate). In two sentences or less, describe qualitatively what sensor(s) you can use to determine a person’s body position as unobtrusively as possible, without getting in the way of the person’s activities.

Draw three positions for the person (standing upright, seated, and supine), and the resultant signals measured using your sensing hardware to the right of each position. Be clear with your labeling.
In three sentences or less, describe the steps you would use for an automated algorithm to determine the person’s position from the sensed data shown in the figures you have drawn. Be specific – statements such as, “I will input the sensor data into a machine learning algorithm” will not be accepted.

What are two confounding factors that can potentially cause errors in your detection of body position from your sensed signals? Your answer should only provide the two factors, and one sentence about each describing why it can reduce your accuracy.
(10 pts) Body Posture / Position Sensing. In assessing a person’s overall health / wellness, it is important to determine the person’s body posture or position in real-time. Such information can facilitate the classification of daily living activities that the person is involved in, and also provide a context for other sensed parameters (such as heart rate). In two sentences or less, describe qualitatively what sensor(s) you can use to determine a person’s body position as unobtrusively as possible, without getting in the way of the person’s activities.

A pair of tri-axial accelerometers can be used, with the first one placed on the person’s trunk, and the second placed on the lower limbs. It would also be possible to use a single accelerometer (tri-axial) on the trunk only, but the number of positions that can be detected would be limited.

Draw three positions for the person (standing upright, seated, and supine), and the resultant signals measured using your sensing hardware to the right of each position. Be clear with your labeling.

The key to the solution would be that the effect of gravity on the different sensor data is shown with the "signals" to the right of the positions. There are many possible solutions to the problem depending on where the sensor is placed, and what orientation with respect to the body. The important thing is that the three axes of data are shown for each position, and that the one facing downward (toward the ground) has a +1 g acceleration depicted.
In three sentences or less, describe the steps you would use for an automated algorithm to determine the person’s position from the sensed data shown in the figures you have drawn. Be specific – statements such as, “I will input the sensor data into a machine learning algorithm” will not be accepted.

Several answers are possible. Some pre-processing to reduce high frequency noise and also focus only on the “DC” component of the accelerations is expected. Some type of threshold based decision rules should be described. Again, a combination of the three axes is required, and there are multiple ways to go about that.

What are two confounding factors that can potentially cause errors in your detection of body position from your sensed signals? Your answer should only provide the two factors, and one sentence about each describing why it can reduce your accuracy.

1. Motion artifacts: these can cause large disturbances in the acceleration signals that would potentially result in errors in position detection.
2. Positioning / calibration: the user will initially place the device at some orientation / position on the body, and this position must be known for the algorithms to be successful in determining the positions following this initial state.

Again, many other answers are possible here, but these are the two most obvious ones.