**INSTRUCTIONS**

1. Please check to ensure that you have a complete exam booklet. There are 22 numbered problems. Note that **Problem 2 occupies 2 pages, Problem 6 occupies 2 pages, Problem 11 occupies 2 pages, Problem 21 occupies 2 pages**. Including the cover sheet, you should have **57 pages**. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number **MUST APPEAR ON EVERY SHEET**. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. **DO NOT** write your name on any of these sheets. Use the preprinted numbers whenever possible, or **WRITE LEGIBLY!!!**

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. **DO NOT TURN IN ANY SHEETS FOR THE OTHER 14 PROBLEMS!!**

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:
   
   (a) **Separate the 8 problems to be graded as explained above.**
   
   (b) **Check to see that your Code Number is in EVERY sheet you are turning in.**
   
   (c) **On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.**
   
   (d) **Turn in this cover sheet (containing your code number) and the 8 problems to be graded.**
   
   (e) **All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!**

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**Problem 1 (Core: VLSI - ECE 2020) Solution**

**Problem**

**Part A** Implement the following expression using N and P type switches (NFETS and PFETS). Label all inputs and outputs.

\[ \text{OUT}_X = (A + B) \cdot C \cdot (D + E) \]

**Part B** Consider a memory system built using a 32 Mbit DRAM chip organized as 4 million addresses of 8-bit words. Given a memory system with 128 DRAM chips total and a 4-to-16 decoder, what are the following parameters?

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of banks</td>
<td></td>
</tr>
<tr>
<td>number of chips used in one bank</td>
<td></td>
</tr>
<tr>
<td>number of addresses</td>
<td></td>
</tr>
<tr>
<td>word size (in bits)</td>
<td></td>
</tr>
<tr>
<td>memory system capacity (in MBytes)</td>
<td></td>
</tr>
</tbody>
</table>

**Part C** Determine the appropriate expression for this mixed logic design. How many transistors are required?

\[ \text{OutY} = \text{__________________________} \]

Number of Transistors: ____________________________
Part A  Implement the following expression using N and P type switches (NFETS and PFETS). Label all inputs and outputs.

\[ \text{OUT}_X = (A + \bar{B}) \cdot \bar{C} \cdot (D + E) \]

Part B  Consider a memory system built using a 32 Mbit DRAM chip organized as 4 million addresses of 8-bit words. Given a memory system with 128 DRAM chips total and a 4-to-16 decoder, what are the following parameters?

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of banks</td>
<td>16 banks</td>
</tr>
<tr>
<td>number of chips used in one bank</td>
<td>8 chips/bank</td>
</tr>
<tr>
<td>number of addresses</td>
<td>(2^{26} = 64M)</td>
</tr>
<tr>
<td>word size (in bits)</td>
<td>64 bits/word</td>
</tr>
<tr>
<td>memory system capacity (in MBytes)</td>
<td>(2^{7}\times2^{26} \text{Mbit}/2^{5} \text{bits/byte} = 2^{9} \text{MB} = 512 \text{MBytes})</td>
</tr>
</tbody>
</table>

Part C  Determine the appropriate expression for this mixed logic design. How many transistors are required?

\[ \text{Out}_Y = (A + B + C) + D \cdot E \]

Number of Transistors: \(6T + 2 \times 4T + 2 \times 2T = 18T\)
The block diagram above defines a system for discrete-time filtering of continuous-time signals. 

Note: all parts of this question can be worked independently.

(a) Suppose that the discrete-time signal $x[n]$ is analyzed with a 100-point DFT, assuming that the C-to-D converter samples at a rate $f_s = 2000$ Hz and its input $x(t)$ is

$$
x(t) = A \cos(1960\pi t + 0.3\pi) + B \cos(2080\pi t - 0.2\pi)
$$

Since the signal $x[n]$ repeats with a period of 100, an arbitrary section of 100 points is analyzed with a 100-point FFT giving $X[k]$. If $X[k]$ has nonzero values at only 4 indices with the following magnitudes

$$
$$

determine the values of $A$ and $B$ in the definition of $x(t)$ above.

$$
A = \boxed{a}
$$

$$
B = \boxed{b}
$$

(b) Suppose that the system function of the LTI system is

$$
H(z) = \frac{6 - 8z^{-1}}{4 + 3z^{-1}}
$$

Determine the output $y[n]$ when the input signal is

$$
x[n] = 5 \cos(0.5\pi n - 0.15\pi) \quad -\infty < n < \infty
$$

Simplify your answer by writing $y[n]$ as a sinusoid, i.e., $y[n] = A \cos(\hat{\omega}n + \varphi)$

$$
y[n] = \boxed{c}
$$
(c) Suppose that the discrete-time LTI system is an IIR bandpass filter whose (causal) impulse response has the following form:

\[ h[n] = \begin{cases} 
0 & n < 0 \\
K_0 & n = 0 \\
K_1(0.93)^n \cos(\omega_1 n + \varphi_1) & n \geq 1 
\end{cases} \]

and whose frequency response (magnitude) is shown below.

Determine all the poles and zeros of the system. Be as accurate as possible.

Poles = 

Zeros = 

The block diagram above defines a system for discrete-time filtering of continuous-time signals. 

Note: all parts of this question can be worked independently.

(a) Suppose that the discrete-time signal \(x[n]\) is analyzed with a 100-point DFT, assuming that the C-to-D converter samples at a rate \(f_s = 2000\) Hz and its input \(x(t)\) is 

\[ x(t) = A \cos(1960\pi t + 0.3\pi) + B \cos(2080\pi t) \]

Since the signal \(x[n]\) repeats with a period of 100, an arbitrary section of 100 points is analyzed with a 100-point FFT giving \(X[k]\). If \(X[k]\) has nonzero values at only 4 indices with the following magnitudes

\[ |X[48]| = 700, \quad |X[49]| = 300, \quad |X[51]| = 300, \quad |X[52]| = 700 \]

determine the values of \(A\) and \(B\) in the definition of \(x(t)\) above.

\[ A = 6, \quad B = 14 \]

(b) Suppose that the system function of the LTI system is 

\[ H(z) = \frac{6 - 8z^{-1}}{4 + 3z^{-1}} \]

Determine the output \(y[n]\) when the input signal is 

\[ x[n] = 5 \cos(0.5\pi n - 0.15\pi) \quad -\infty < n < \infty \]

Simplify your answer by writing \(y[n]\) as a sinusoid, i.e., \(y[n] = A \cos(\omega_0 n + \phi)\)

\[ y[n] = 10 \cos(0.5\pi n + 0.35\pi) \quad H(e^{j\omega}) |_{\omega=0.5\pi} = j2 = 2e^{j0.5\pi} \]

(c) Suppose that the LTI system is an IIR bandpass filter whose impulse response has the following form

\[ h[n] = K_0 \delta[n] + K_1 (0.93)^n \cos(\omega_1 n + \varphi_1)u[n-1] \]

and whose frequency response (magnitude) is shown below.

Determine all the poles and zeros of the system. Be as accurate as possible.

Two poles at \(0.93\zeta \pm 0.6\pi\), and two zeros on unit circle at +1 and −1
Consider the following C code. Suppose the global variables are allocated starting at address 2000. Consider execution on a 32-bit byte addressed architecture with 32-bit addresses on which type int is one word (four bytes), all words must be word aligned, and chars are one byte.

```c
struct S{
    int x,y;
    char *c;
    char string[] = "IDK!"
    struct S *next;
} A[4];

struct S *p = &A[2];

void foo(){
    int i;
    struct S **handle = &p;
    for (i=0, i<4, i++) A[i].next = &A[(i+1)%4];
    //evaluate here
}
```

(a) What are the values of the following expressions (executed independently) evaluated just after the loop in foo?

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td></td>
</tr>
<tr>
<td>*handle</td>
<td></td>
</tr>
<tr>
<td>(**handle).next</td>
<td></td>
</tr>
<tr>
<td>(*handle)-&gt;next</td>
<td></td>
</tr>
<tr>
<td>(*handle)-&gt;next-&gt;next</td>
<td></td>
</tr>
<tr>
<td>(*handle)-&gt;next + 1</td>
<td></td>
</tr>
</tbody>
</table>

(b) Indicate in which segment of memory (data, text, stack, heap) the given variable resides.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Segment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A[]</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td></td>
</tr>
<tr>
<td>handle</td>
<td></td>
</tr>
</tbody>
</table>
Consider the following C code. Suppose the global variables are allocated starting at address 2000. Consider execution on a 32-bit byte addressed architecture with 32-bit addresses on which type int is one word (four bytes), all words must be word aligned, and chars are one byte.

```c
struct S{
    int x, y; // x, y, c, and next are 4 bytes
    char *c;  // and string is 5 bytes causing 3
    char string[] = "IDK!";  // bytes of slack to align next
} A[4];
struct S *p = &A[2];
void foo(){
    int i;
    struct S **handle = &p;
    for (i=0, i<4, i++) A[i].next = &A[(i+1)%4];
    // evaluate here
}
```

(a) What are the values of the following expressions (executed independently) evaluated just after the loop in foo?

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>2096</td>
</tr>
<tr>
<td>*handle</td>
<td>2048</td>
</tr>
<tr>
<td>(**handle)-&gt;next</td>
<td>2072</td>
</tr>
<tr>
<td>(*handle)-&gt;next</td>
<td>2072</td>
</tr>
<tr>
<td>(*handle)-&gt;next-&gt;next</td>
<td>2000</td>
</tr>
<tr>
<td>(*handle)-&gt;next + 1</td>
<td>2096</td>
</tr>
</tbody>
</table>

(b) Indicate in which segment of memory (data, text, stack, heap) the given variable resides.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Segment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A[]</td>
<td>data</td>
</tr>
<tr>
<td>i</td>
<td>stack</td>
</tr>
<tr>
<td>p</td>
<td>data</td>
</tr>
<tr>
<td>handle</td>
<td>stack</td>
</tr>
</tbody>
</table>
Consider the circuit below. If \( V_{ap} = 12 \text{V}, R_1=3\text{k}\Omega, R_2=6\text{k}\Omega, R_3=1\text{k}\Omega, R_4=6\text{k}\Omega, \) and \( R_5=3\text{k}\Omega), \) find the Norton equivalent circuit with respect to nodes A and B. Sketch the Norton equivalent circuit below, and provide expressions for both the Norton equivalent current and resistance, i.e. \( I_N \) and \( R_N \).

Sketch of Norton equivalent circuit:

\[
R_N = \rule{6cm}{0.25mm}
\]

\[
I_N = \rule{6cm}{0.25mm}
\]
Sketch the Norton equivalent circuit:

\[
R_N = \frac{R_3 \left( \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5} \right)}{R_3 + \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5}} = 800 \, \Omega
\]

Considering again the original circuit, find \( i_N \) by calculating the current through a short circuit between nodes A and B. As an intermediate step, find the voltage at node A:

\[
V_A = V_B = \frac{R_4 R_5}{R_4 + R_5} \frac{V_{ap}}{R_4 R_5 + R_1 R_2}
\]

Then by KCL at node A, we have:

\[
i_N = \frac{V_{ap} - V_A}{R_1} - \frac{V_A}{R_4} = 1 \, mA
\]
A set of useful electromagnetics equations are at the bottom of this page

As seen in the below diagram, the potential in a one-dimensional region is found to be
\[ V(x) = -2x^3 + 3x^2 \text{ Volts} \]

(a) In the region between \( x=0 \) and \( x=1 \), where is the electric field strongest?

(b) What is the electric field strength at that location, and which direction does it point?

(c) Does there exist negative charge, positive charge, both, or neither, within this region?

Potentially useful electromagnetics equations

<table>
<thead>
<tr>
<th>Electrostatics</th>
<th>Electrodynamics</th>
<th>Materials and Boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nabla \cdot \vec{D} = \rho_v )</td>
<td>( \nabla \cdot \vec{D} = \rho_v )</td>
<td>( \vec{D} = \varepsilon \vec{E} )</td>
</tr>
<tr>
<td>( \nabla \cdot \vec{B} = 0 )</td>
<td>( \nabla \cdot \vec{B} = 0 )</td>
<td>( \vec{B} = \mu \vec{H} )</td>
</tr>
<tr>
<td>( \nabla \times \vec{H} = \vec{J} )</td>
<td>( \nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt} )</td>
<td>( \hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0 )</td>
</tr>
<tr>
<td>( \nabla \times \vec{E} = 0 )</td>
<td>( \nabla \times \vec{E} = -\frac{d\vec{B}}{dt} )</td>
<td>( \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_S )</td>
</tr>
<tr>
<td>( \nabla^2 \vec{V} = -\frac{\rho_v}{\varepsilon} )</td>
<td>( \nabla^2 \vec{V} = -\frac{\rho_s}{\varepsilon} )</td>
<td>( \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s )</td>
</tr>
<tr>
<td>( Q = CV )</td>
<td></td>
<td>( \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 )</td>
</tr>
</tbody>
</table>
(a) The electric field can be found by taking the gradient of the voltage

\[ \vec{E} = -\nabla V \]

Which in one-dimensional space reduces to

\[ \vec{E} = - \frac{dV}{dx} \hat{x} \]

The electric field is strongest when the slope \( \frac{dV}{dx} \) is highest. This can be seen visually or calculated, and it occurs at \( x = 0.5 \).

(b) Taking the derivative of the voltage gives the electric field

\[ \vec{E} = (6x^2 - 6x) \hat{x} \]

Plugging in the value \( x = 0.5 \) gives \( E = -1.5\hat{x} \) V/m, so it is oriented in the negative (leftward) direction.

(c) The electric field can be found from Poisson's equation, or the second derivative of the voltage

\[ \nabla^2 V = -\frac{\rho}{\varepsilon_0} \]

Which in one-dimensional space reduces to

\[ \frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0} \]

Taking the second derivative of the voltage equation and solving for the charge density gives

\[ \rho = \varepsilon_0(12x - 6) \]

For \( x \) between 0 and 0.5, \( \rho \) is negative. For \( x \) between 0.5 and 1.0, \( \rho \) is positive. So the region contains some of both positive and negative charge. (actually an equal amount of both)
PART A [5 Points].

For the following MOSFET, draw the band diagrams across the x-axis (from left to right just below the surface) considering $V_G=0$, and $V_D=0$. Clearly include conduction band ($E_C$), valence band ($E_V$), intrinsic level ($E_i$), and Fermi level ($E_F$) in your diagram. For simplicity, you can assume, in the heavily n-type (n+) doped source/drain $E_F$ is at $E_C$. However, in the p-type (p) doped channel $E_F$ is in between $E_i$ and $E_V$. 

![MOSFET Band Diagram](image-url)
An engineer is measuring the drain current ($I_{DS}$) versus gate-to-source voltage ($V_{GS}$) characteristics of this MOSFET in the region: $0 < V_{GS} < V_{th}$, where $V_{th}$ is the threshold voltage of the MOSFET. All measurements are done at a constant $V_{DS}$. The two measured values are: (1) $I_{DS1}=1\text{nA}$ at $V_{GS1}=0.1\text{V}$, and (2) $I_{DS2}=100\text{nA}$ at $V_{GS2}=0.4\text{V}$.

**Compute the value of $V_{GS}$ at which $I_{DS}=100\text{nA}$.**

Remember subthreshold current of MOSFET can be given by:

$$I_{DS} = I_0 \exp\left(\frac{q(V_{GS}-V_{th})}{nKT}\right)(1 - \exp\left(\frac{qV_{DS}}{kT}\right)),$$

where $I_0$ is a constant, $n$ is the subthreshold swing factor, $k$ is the Boltzmann’s constant, $q$ is the electronic charge, and $T$ is the temperature. Also, remember $\log_{10}(x) = \ln(x)/2.3$, where $\ln$ represents the natural logarithm and $\log_{10}$ represents log-to-the-base-10.
PART A [5 Points].

For the following MOSFET, draw the band diagrams across the x-axis (from left to right just below the surface) considering $V_G=0$, and $V_D=0$. Clearly include conduction band ($E_C$), valence band ($E_V$), intrinsic level ($E_i$), and Fermi level ($E_F$) in your diagram. For simplicity, you can assume, in the heavily n-type (n+) doped source/drain $E_F$ is at $E_C$. However, in the p-type (p) doped channel $E_F$ is in between $E_i$ and $E_V$. 

![Band Diagram]
PART B [5 Points].

An engineer is measuring the drain current ($I_{DS}$) versus gate-to-source voltage ($V_{GS}$) characteristics of this MOSFET in the region: $0 < V_{GS} < V_{th}$, where $V_{th}$ is the threshold voltage of the MOSFET. All measurements are done at a constant $V_{DS}$. The two measured values are: (1) $I_{DS1}=1\text{nA}$ at $V_{GS1}=0.1\text{V}$, and (2) $I_{DS2}=1000\text{nA}$ at $V_{GS2}=0.4\text{V}$. **Compute the value of $V_{GS}$ at which $I_{DS}=100\text{nA}$.**

Remember subthreshold current of MOSFET can be given by:

$$I_{DS} = I_0 \exp\left(\frac{q(V_{GS} - V_{th})}{nKT}\right) \left(1 - \exp\left(\frac{qV_{DS}}{kT}\right)\right),$$

where $I_0$ is a constant, $n$ is the subthreshold swing factor, $k$ is the Boltzmann’s constant, $q$ is the electronic charge, and $T$ is the temperature. Also, remember $\log_{10}(x) = \ln(x)/2.3$, where $\ln$ represents the natural logarithm and $\log_{10}$ represents log-to-the-base-10.

$$\frac{I_{DS2}}{I_{DS1}} = \frac{1000}{1} = \frac{\exp\left(\frac{q(V_{GS2} - V_{TH})}{nKT}\right)}{\exp\left(\frac{q(V_{GS1} - V_{TH})}{nKT}\right)} = \exp\left(\frac{q(V_{GS2} - V_{GS1})}{nKT}\right),$$

$\Rightarrow$ $V_{GS2} - V_{GS1} = \frac{nKT}{q} \ln(1000) = 0.3$

$\Rightarrow$ similarly, $V_{GS3} - V_{GS1} = \frac{nKT}{q} \ln(100)$

$\Rightarrow$ **Hence,** $V_{GS3} = V_{GS1} + 0.3 \frac{\ln(100)}{\ln(1000)} = V_{GS1} + 0.3 \frac{\log_{10}(100)}{\log_{10}(1000)} = V_{GS1} + 0.2 = 0.3\text{V}$
Many Si-based npn bipolar junction transistors (BJTs) use a heavily n-type doped Si emitter region, a moderately doped p-type base region and a lightly doped n-type collector region. Draw and label the \textit{equilibrium energy-band diagram}, including \( E_G, E_O, E_F, E_i, E_C, \) and \( E_V \) where \( E_G \) is the energy gap, \( E_O \) is the vacuum energy level, \( E_F \) is the equilibrium Fermi energy, \( E_i \) is the intrinsic energy level, \( E_C \) is the conduction-band energy, and \( E_V \) is the valence-band energy for a such a BJT. Assume that the doping concentrations are: \( N_{DE} > N_{AB} > N_{DC} \) and that the dopant concentrations are constant. Using arrows to describe the above energy values on the diagram, draw and label the above energies, the semiconductor work function(s), \( \Phi_s \), and the semiconductor electron affinity (or affinities), \( \chi_s \), you use in each of the emitter, base, and collector regions. I have given you the Si conduction-band (\( E_C \)) and valence-band (\( E_V \)) energies and the vacuum level (\( E_O \)) in the emitter region of the Si BTJ. Label the equilibrium Fermi energy, \( E_F \). The vertical dashed lines are the metallurgical junctions for each region of the BJT.
Problem 7 (Core: MICROSYS - ECE 3040)

SOLUTION

ECE3040
Dupuis
Solution

(a) Homojunction BJT
The differential amplifier shown below has the following parameters:

\[ V_{CC} = 10 \text{ V}, \quad V_{EE} = 8.3 \text{ V}, \quad R_{EE} = 100 \text{ k}\Omega, \quad R_C = 10 \text{ k}\Omega \text{ and } \beta_F = 100. \]

(a) Calculate the Q-point \((I_C, V_{CE})\) for the transistors in the amplifier circuit.

(b) Draw the small-signal half-circuit for the differential mode.

(c) Calculate the differential mode gain assuming that \(r_\pi = 5000 \text{ \Omega}\).
The differential amplifier shown below has the following parameters: \( V_{CC} = 10 \) V, \( V_{EE} = 8.3 \) V, \( R_{EE} = 100 \, k\Omega \), \( R_C = 10 \, k\Omega \) and \( \beta_f = 100 \).

(a) Calculate the Q-point \((I_C, V_{CE})\) for the transistors in the amplifier circuit.

KVL in Loop 1 when \( v_1 = v_2 = 0 \):

\[-V_{BB} - 2I_{Ei}R_{EE} + V_{EE} = 0\]

\[I_{Ei} = (8.3 - 0.7)/200k\Omega = 38 \mu A\]

\[I_C = \beta I_{Ei}/(\beta + 1)\]

\[I_C = 38 \mu A\]

KVL in Loop 2:

\[V_{CC} - I_{C1}R_C - V_{CE} - 2I_{Ei}R_{EE} + V_{EE} = 0\]

\[V_{CE} = 10.3 \text{ V}\]
The differential amplifier shown below has the following parameters: $V_{CC} = 10\ V$, $V_{EE} = 8.3\ V$, $R_{EE} = 100\ k\Omega$, $R_C = 10\ k\Omega$ and $\beta_F = 100$.

(a) Calculate the $Q$-point ($I_C$, $V_{CE}$) for the transistors in the amplifier circuit.

KVL in Loop 1 when $v_i = v_o = 0$:

$$-V_{BE} - 2I_{E1}R_{EE} + V_{EE} = 0$$

$I_{E1} = (8.3 - 0.7)/200k\Omega = 38\mu A$

$I_{C1} = \beta I_{E1}/(\beta + 1)$

$I_{C1} = 38\ \mu A$

KVL in Loop 2:

$$V_{CC} - I_{C1}R_C - V_{CE} - 2I_{E1}R_{EE} + V_{EE} = 0$$

$V_{CE} = 10.3\ V$
(b) Draw the small-signal half-circuit for the differential mode.

(c) Calculate the differential mode gain assuming that $r_{ii} = 5000 \, \Omega$.

\[ V_{od/2} = -\beta i_b R_C = -\beta \frac{V_{id}}{2 \times r_{ii}} R_C = -200 \frac{V_{id}}{2} \]

\[ A_d = \frac{V_{od}}{V_{id}} = 200 \]
A 32-bit processor has 1 GByte of memory, 16 Kbyte direct mapped separate L1 instruction and data caches with 64 byte lines backed by a unified 8 Mbyte 8-way set associative L2 cache with 256 byte lines. Applications have a 4 GByte virtual address space and 16 Kbyte pages. A 32x32 integer matrix is stored in contiguous memory in row major form starting at physical address 0x10000000. Integers are 32-bits.

a. (3 pts) Show the breakdown of the physical addresses used to access each of the L1 and L2 caches. For example, identify which bits used for the tag, set, etc.

b. (2 pts) In what L2 set will you find the matrix element A[2,8]? The first element is A[0,0].

Set #: ____________

c. (3 pts) No matrix elements are initially in the caches. When sequentially accessing all elements along the main diagonal, how many misses will be encountered in the L1 data cache and the L2 cache?

L1 Cache: ______________

L2 Cache: ______________

d. (2 pts) If each page table entry has 1 valid bit, 1 dirty bit and the physical page number, what is the minimum size of the page table in bits?

Size: ___________________
A 32-bit processor has 1 GByte of memory, 16 Kbyte direct mapped separate L1 instruction and data caches with 64 byte lines backed by a unified 8 Mbyte 8-way set associative L2 cache with 256 byte lines. Applications have a 4 GByte virtual address space and 16 Kbyte pages. A 32x32 integer matrix is stored in contiguous memory in row major form starting at physical address 0x10000000. Integers are 32-bits.

a. **(3 pts)** Show the breakdown of the physical addresses used to access each of the L1 and L2 caches. For example, identify which bits used for the tag, set, etc.

<table>
<thead>
<tr>
<th>31</th>
<th>16</th>
<th>8</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>0</td>
<td>L1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>31</th>
<th>12</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>10</td>
<td>L2</td>
</tr>
</tbody>
</table>

b. **(2 pts)** In what L2 set will you find the matrix element A[2,8]? Note that the first element is A[0,0].

Set #: 0x001
This element is in the 73rd element of the matrix (32 + 32 + 9) or 292 bytes offset from 0x10000000 and the first matrix element is stored on a 256-byte boundary. Note that a L2 cache line will store two successive rows of the matrix. Therefore A[2,8] will be in the second cache line which maps to set 0x001 (the first set is 0x000).

c. **(3 pts)** No matrix elements are initially in the caches. When sequentially accessing all elements along the main diagonal, how many misses will be encountered in the L1 data cache and the L2 cache?

L1 Cache: ___32___

Each row is stored in 2 successive cache lines. Every access of an element on the diagonal will be to a new cache line. There are 32 elements on the diagonal.

L2 Cache: ____16____

There are two rows of the matrix in each cache line. Thus every miss will result in two diagonal elements being brought into the cache.
Problem 9 (Core: CSS - ECE 3056)  

Solution

d. (2 pts) If each page table entry has 1 valid bit, 1 dirty bit and the physical page number, what is the minimum size of the page table in bits?

Size: 

Each page table entry will have 18 bits for the physical page number and 1 bit each for the valid and dirty bits. There are \(2^{18}\) virtual pages. If you note that upper two bits of the physical page number are 00 (1 GByte physical address space) the physical page number could be stored with 16 bits rather than 18.
A balanced three phase source with a line-to-line voltage of 100 volts, is delivering power to a balanced Y-connected load with phase impedance of \((5 + j12)\) ohms (in each leg of the Y), and this Y-connected load is in parallel with a balanced delta connected load with a phase impedance of \((15 + j0)\) ohms (in each leg of the delta).

(a) Draw the circuit and find the magnitude of the total line current from the source in phase A.
(b) Calculate the complex power delivered by the source.
(c) Find the reactive power per phase in each leg of the delta.
(a) Draw the circuit and find the magnitude of the total line current from the source in phase A.

\[ I = \frac{100/\sqrt{3}}{4/\sqrt{3}} = 13.88 \text{A} \]

(b) Find the total complex power delivered by the source.

\[ S = 3(I^*) = 3 \left[ \frac{100}{\sqrt{3}} \right] \left[ 13.88 \angle 17.2^\circ \right] \]

\[ = 220 + j71.09 \]

\[ |S| = 240.3 \angle 17.2^\circ \]

(c) Find the reactive power per phase in one of the delta legs.

\[ \Delta \text{ has only } R, \text{ thus reactive power } Q \text{ in } \Delta \text{ is ZERO!} \]
Problem 11 (Core: DSP/TLCOM - ECE 3077) Solution

**Problem**

Suppose that you work at CNN and are assigned to report on a specific presidential candidate. You arrive in the morning at 8am and wait patiently at your desk until the candidate says something outrageous. Let $X$ denote the time of the first outrageous statement of the day (measured by the number of minutes after you arrive at work). Let $Y$ denote the time of the second outrageous statement. We can model $X$ and $Y$ as continuous random variables with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & 0 \leq x < y \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda$ is a positive constant.

In answering the following questions it may be useful to recall that if $Z$ is an exponential random variable with rate $\lambda > 0$, i.e., a random variable with pdf given by

$$f_Z(z) = \begin{cases} \lambda e^{-\lambda z} & z \geq 0, \\ 0 & z < 0, \end{cases}$$

then $\mathbb{E}[X] = \frac{1}{\lambda}$ and $\text{var}(X) = \frac{1}{\lambda^2}$.

1. (2 points) What are the marginal pdfs for $X$ and $Y$?

   $$f_X(x) = \quad f_Y(y) =$$

2. (2 points) Are $X$ and $Y$ independent? (Justify your answer.)

   Answer:
3. (2 points) What are the conditional probability density functions \( f_{X|Y}(x|y) \) and \( f_{Y|X}(y|x) \)?

\[
\begin{align*}
  f_{X|Y}(x|y) &= \\
  f_{Y|X}(y|x) &= 
\end{align*}
\]

4. (2 points) Suppose that \( \lambda = \frac{1}{60} \). What is the probability that the second outrageous statement of the day occurs before 9am? (Hint: You do not need to do integration by parts to compute the answer.)

Answer:

5. (2 points) Still supposing that \( \lambda = \frac{1}{60} \), what is the expectation and variance of the time of the second outrageous statement? (Hint: Write \( Y = X + W \) where \( W = Y - X \).)

Answer:
Solution

Suppose that you work at CNN and are assigned to report on a specific presidential candidate. You arrive in the morning at 8am and wait patiently at your desk until the candidate says something outrageous. Let $X$ denote the time of the first outrageous statement of the day (measured by the number of minutes after you arrive at work). Let $Y$ denote the time of the second outrageous statement. We can model $X$ and $Y$ as continuous random variables with joint pdf

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where $\lambda$ is a positive constant.

In answering the following questions it may be useful to recall that if $Z$ is an exponential random variable with rate $\lambda > 0$, i.e., a random variable with pdf given by

$$f_Z(z) = \begin{cases} \lambda e^{-\lambda z} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

then $E[X] = \frac{1}{\lambda}$ and $\text{var}(X) = \frac{1}{\lambda^2}$.

1. (2 points) What are the marginal pdfs for $X$ and $Y$?

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \lambda^2 y e^{-\lambda y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_X(x) = \int_0^x \lambda e^{-\lambda x} \, dy = \int_0^x \lambda e^{-\lambda x} \, dx = \lambda e^{-\lambda x}, \quad x \geq 0$$

$$f_Y(y) = \int_0^y \lambda e^{-\lambda y} \, dx = \int_0^y \lambda e^{-\lambda y} \, dx = \lambda^2 y e^{-\lambda y}, \quad y > 0$$

2. (2 points) Are $X$ and $Y$ independent? (Justify your answer.)

Answer: No

$$f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$$
3. (2 points) What are the conditional probability density functions $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$?

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

$$f_{Y|X}(y|x) = \begin{cases} e^{-\lambda(y-x)} & x < y \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \begin{cases} \frac{1}{y} & 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \begin{cases} e^{-\lambda(y-x)} & x < y \\ 0 & \text{otherwise} \end{cases}$$

4. (2 points) Suppose that $\lambda = \frac{1}{60}$. What is the probability that the second outrageous statement of the day occurs before 9am? (Hint: You do not need to do integration by parts to compute the answer.)

**Answer:**

\[
\begin{align*}
1 - \frac{2}{e} & \approx 0.26
\end{align*}
\]

\[
\begin{align*}
P(Y < 60) &= \int_{x=0}^{60} \int_{y=x}^{\infty} \frac{1}{y} e^{-\frac{y}{60}} dy \, dx \\
&= \int_{x=0}^{60} \left( e^{-\frac{x}{60}} - e^{-\frac{60}{60}} \right) dx \\
&= 1 - e^{-\frac{1}{60}} - 60 e^{-1} \\
&= \lambda^{-\frac{1}{60}} \Rightarrow 1 - 2 e^{-1}
\end{align*}
\]

5. (2 points) Still supposing that $\lambda = \frac{1}{60}$, what is the expectation and variance of the time of the second outrageous statement? (Hint: Write $Y = X + W$ where $W = Y - X$.)

**Answer:**

\[
\begin{align*}
E[Y] &= 120 \\
\text{var}(Y) &= 7200
\end{align*}
\]

\[
\begin{align*}
X \sim \text{Exp}\left(\frac{1}{60}\right), \quad W \sim \text{Exp}\left(\frac{1}{60}\right)
\end{align*}
\]

\[
\begin{align*}
E[Y] &= E[X+W] = E[X] + E[W] = 2 \cdot 60 = 120
\end{align*}
\]

Since $X$ and $W$ are independent:

\[
\begin{align*}
\text{var}(Y) &= \text{var}(X+W) = \text{var}(X) + \text{var}(W) = 2 \cdot 60^2 = 7200
\end{align*}
\]
Problem (5 points each) Suppose the impulse response, $h(t)$, of a continuous-time, linear time-invariant system is of the form

(a) Sketch $|H(j\omega)|$, the magnitude of the frequency response of this system, if $A = 5$ and $T = 10$. Label important features such as the location of the zero crossings and the height of the main lobe.

(b) Now suppose that $A$ and $T$ are unknown, and that we need to find them by inputting some test signals and looking at the output. When the system is subjected to the input

the output $y_1(t)$ is zero at $t = 5$, i.e. $y_1(5) = 0$. Also, when the input is $x_2(t) = tu(t)$ (where $u(t)$ is the unit step function), the output $y_2(t)$ is equal to 7 at $t = 9$, i.e. $y_2(9) = 7$. Determine $A$ and $T$. 
(a) In addition to making sure students know what features should be labeled so they don’t later complain about incomplete instructions, the request to label the “zero crossings” and “height of the main lobe” should help students recall that the Fourier transform of a “boxcar” function is a “sinc” function. The “DC” value of the frequency response is the integral of the impulse response, which in this case is

\[ \text{main lobe height} = A \times T = 5 \times 10 = 50. \]

Zero crossings occur at frequencies for which an integer number of cycles fit within the boxcar, namely \( \pm 1/T, \pm 2/T, \pm 3/T, \ldots \) Hertz, corresponding to \( \pm 2\pi/T, \pm 2(2\pi)/T, \pm 3(2\pi)/T, \ldots \) radians per second. With \( T = 10 \), the zero crossings occur at

\[ \text{zero crossing locations} = \pm \pi/5, \pm 2\pi/5, \pm 3\pi/5, \ldots \]

Alternatively, students may recall, or rederive, the details of various Fourier transform pairs and properties and proceed accordingly. They may reach a solution faster if they realize that because the problem is only asking for a magnitude plot, they need not worry about phase.

In any case, the final plot should look something like this:

(b) It is probably easiest to approach this problem by “flipping and shifting” the impulse response \( h(\cdot) \). Keeping \( x_1(\tau) \) “fixed” and comparing it with \( h(5 - \tau) \), we see that the right edge of the flipped-and-shifted boxcar corresponds to the right edge of the negative part of \( x_1(\tau) \). To get \( y_1(5) = 0 \), the flip-and-shifted boxcar must be just long enough to so that the included positive part of \( x_1(t) \), when integrated, cancels the negative part of \( x_1(t) \), when integrated. This realization yields

\[ T = 4. \]

To find \( A \), we continue to think of “flipping and shifting” \( h(\cdot) \); now \( h(9 - \tau) \) corresponds to a boxcar in \( \tau \) ranging from 5 to 9, yielding

\[ 7 = y(9) = \int_5^9 A t \, dt = \frac{A}{2} t^2 \Big|_{t=5}^{t=9} = \frac{A}{2} (81 - 25) = 28A, \]

\[ A = 1/4. \]

There may be other approaches to this problem; whatever works, works.
Consider the following circuit.

![Circuit Diagram]

a) (4pts) Draw the CMOS transistor-level schematic of F using the minimum number of transistors. Your solution should contain a single pair of pull-down and pull-up networks. Assume that complemented inputs are available.

b) (3pts) Assuming that the width of all transistors in part a) is 3w, compute the worst-case RC delay in terms of \( \tau \) when F is driving a load of 10\( \text{C}_{\text{inv}} \). \( w \) and \( R \) are the width and the resistance of the minimum-size nFET, respectively. \( \text{C}_{\text{inv}} \) is the input capacitance of the minimum-sized inverter, and \( \tau = R \cdot \text{C}_{\text{inv}} \). Assume that the hole mobility is 2x slower than that of electron.

c) (3pts) Determine the width of all transistors in part a) in terms of \( w \) so that the worst-case rise and fall time are matched, and the RC delay of driving a minimum-sized inverter becomes \( \tau \).
a) $F = \overline{a}b \cdot c + d = (\overline{a} + b)\overline{c}d$

b) pull-up: $3 \cdot 2R/3 \cdot 10C_{inv} = 20\tau$. 
pull-down: $2 \cdot R/3 \cdot 10C_{inv} = 20/3\tau$.

Thus, the worst-case delay is $20\tau$.

c) sizing is shown in the figure.
A 50 kVA 13,800/208-V Δ-Y distribution transformer has a resistance of 1% and a reactance of 7% per unit.

a) Determine the transformer’s phase impedance referred to the high-voltage side.

b) Calculate the transformer voltage regulation at full load and 0.8 PF lagging.
a) 

\[ V_{\text{Base, H}} = 13,800 \]

\[ S_{\text{Base}} = 50\text{kVA} \]

\[ Z_{\text{Base, H}} = \frac{3 \times (13,800V)^2}{50,000\text{VA}} = 11,426\Omega \]

\[ Z_{\text{eq, pu}} = 0.01 + j0.07 \]

\[ Z_{\text{eq}} = Z_{\text{eq, pu}} \times Z_{\text{Base, H}} = 114.2 = j800\Omega \]

b) In order to calculate the voltage regulation of a 3-phase transformer bank, we can determine the voltage regulation of any single transformer in the bank. The voltages on a single phase transformer are phase voltage. We have:

\[ VR = \frac{V_{\phi H} - aV_{\phi L}}{aV_{\phi L}} \]

\[ I_{\text{rated, \phi H}} = \frac{S}{3V_{\text{rated, \phi H}}} = \frac{50,000\text{VA}}{3 \times (13,800V)} = 1.208\text{A} \]

The rated phase voltage on the secondary of the transformer is: \( 280V / \sqrt{3} = 120V \).

Assume that the transformer secondary is operating at the rated voltage and current, and find the resulting primary phase voltage:

Thus the secondary voltage is \( 120V[0^\circ] \). For a full load and 0.8 PF lagging, the current is:

\[ I_\phi = 1.208 \times 36.87 \]

The voltage at the primary is equal to:

\[ V_{\phi H} = aV_{\phi L} + (R_{\text{eq}} + jX_{\text{eq}})I_\phi \]

\[ V_{\phi H} = 13,800[0^\circ] + (114.2 + j800\Omega)(1.208 \times 36.87) \]

\[ V_{\phi H} = 14,506[2.73^\circ] \]

Therefore:

\[ VR = \frac{V_{\phi H} - aV_{\phi L}}{aV_{\phi L}} = \frac{14,506 - 13,800}{13,800} \times 100\% = 5.1\% \]
Problem 15 (Breadth: EDA - ECE 3400) Solution

The small-signal input-output behavior of the cascode amplifier shown below on the left can be represented by the two-port model shown on the right. Compute the values of $G_m$ and $R_{out}$ for this amplifier. You may make the following simplifying assumptions:

- Neglect the effect of $\lambda$ in the DC analysis (i.e. assume $\lambda = 0$). However, you should include the MOSFET’s output resistance $r_o$ in your small-signal analyses.

- Neglect the body effect both in DC and in small-signal analysis.

Assume that all MOSFETs are in saturation. Recall that for a MOSFET in saturation $I_D = (K'/2)(W/L)(V_{GS} - V_T)^2$ and the small-signal parameters are:

$$g_m = \sqrt{2K'(W/L)I_D} \quad r_o = 1/(\lambda I_D)$$

Use the following MOSFET parameter values: n-channel: $K'_n = 25 \mu A/V^2$, $V_{TN} = 0.75 V$, $\lambda_n = 0.01 V^{-1}$; p-channel: $K'_p = 10 \mu A/V^2$, $V_{TP} = -0.75 V$, $\lambda_p = 0.02 V^{-1}$.
DC analysis
Node equation at the drain of M1:

\[ \frac{V_{GS1} + 5}{70 \, \text{k}\Omega} = \frac{10 \cdot 10^{-6} \cdot 200}{2} (V_{GS1} - V_{TP})^2 \]

Solve: \( V_{GS1} = -1.461 \, \text{V}, \quad I_{D1} = 50.56 \, \mu\text{A} = I_{D2} = I_{D3} = I_{D4} \).

Small-signal analysis
Small-signal parameters

\[
\begin{align*}
    r_{o2} &= \frac{1}{\lambda_p I_{D2}} = 989 \, \text{k}\Omega \\
    r_{o3} &= \frac{1}{\lambda_n I_{D3}} = 1.978 \, \text{M}\Omega = r_{o4} \\
    g_{m3} &= \sqrt{2K_n(W/L)D_{D3}} = 142.2 \, \mu\text{A/V} = g_{m4}
\end{align*}
\]

Computation of \( G_m \):

\[
G_m = \frac{i_o}{v_i} \text{ when } v_o = 0. \text{ Since } v_{gs3} = -v_{s3} \text{ the circuit at the top is equivalent to the circuit at the bottom. Applying current division:}
\]

\[
i_o = g_{m4}v_i \frac{g_{m3} + 1/r_{o3}}{g_{m3} + 1/r_{o3} + 1/r_{o4}}
\]
\[ G_m = \frac{g_{m4} g_{m3} + 1/r_{o3}}{g_{m3} + 1/r_{o3} + 1/r_{o4}} = 141.7 \mu A/V \]

Computation of \( R_{out} \)

\( R_{out} = v_o/i_o \) when \( v_i = 0 \). Therefore \( R_{out} \) is the parallel combination of \( r_{o2} \) with \( R_{d3} \), the equivalent resistance seen into the drain of \( M_3 \).

\[ R_{d3} = v_o/i_o \] in the circuit below.

\[ \begin{align*}
    v_{gs3} &= -v_{s3} = -r_{o4}i_o \\
    v_o &= r_{o3}(i_o - g_{m3}v_{gs3}) + r_{o4}i_o = r_{o3}(i_o + g_{m3}r_{o4}i_o) + r_{o4}i_o \\
    R_{d3} &= r_{o3} + r_{o4} + g_{m3}r_{o3}r_{o4} = 560.3 \text{ M}\Omega \\
    R_{out} &= r_{o2} || R_{d3} = 987.3 \text{ k}\Omega
\end{align*} \]
Consider the system represented in state space by the following equations.

\[
\begin{bmatrix}
-3 & 0 \\
-1 & -1
\end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 2 \end{bmatrix} + \begin{bmatrix}
x \\
u(t)
\end{bmatrix} \\
\begin{bmatrix}
1 \\
0
\end{bmatrix} y = \begin{bmatrix}
x \end{bmatrix}; \quad x[0] = \begin{bmatrix}
2 \\
0
\end{bmatrix}
\]

Solve for \(y(t)\). Show all steps.
\[ X(s) = (sI - A)^{-1}(x(0) + Bu(t)) = \begin{bmatrix} s + 3 & 0 \\ 1 & s + 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \]

\[ = \begin{bmatrix} \frac{1}{s + 3} & 0 \\ -\frac{1}{(s + 1)(s + 3)} & \frac{1}{s + 1} \end{bmatrix} \begin{bmatrix} 2s + 2 \\ s \end{bmatrix} = \begin{bmatrix} \frac{2(s + 1)}{s(s + 3)} \\ -\frac{s - 1}{s(s + 1)(s + 3)} \end{bmatrix} \]

\[ Y(s) = \begin{bmatrix} 1 & 0 \\ \frac{2(s + 1)}{s(s + 3)} & \frac{s - 1}{s(s + 1)(s + 3)} \end{bmatrix} = \begin{bmatrix} \frac{2(s + 1)}{s(s + 3)} \\ \frac{2(s + 1)}{s(s + 3)} = \frac{2}{3} \frac{1}{s + \frac{5}{3}} \end{bmatrix} \]

Obtaining the inverse Laplace transform

\[ y(t) = \frac{2}{3} + \frac{5}{3} e^{-3t} \]
What four items of information are needed by a host before it can operate normally on an IP network?

1. ________________________________
2. ________________________________
3. ________________________________
4. ________________________________

5. If these are not configured manually, what protocol might be used to get them over the network?
_______________________________

6. What is the primary advantage of a packet store-and-forward network?
_______________________________

7. What is the primary advantage of a circuit-switched network?
_______________________________

8. How do Internet routers build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular IP datagram?
_______________________________

9. How do Ethernet switches build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular Ethernet frame?
_______________________________

10. How does your PC determine the 32-bit or 128-bit IP address for “www.acmesales.com”?
_______________________________
What four items of information are needed by a host before it can operate normally on an IP network?

1. The host’s IP address
2. The gateway router’s IP address
3. The local DNS server’s IP address
4. The subnet mask (or CIDR size)

5. If these are not configured manually, what protocol might be used to get them over the network?

5. DHCP (Dynamic Host Configuration Protocol)

6. What is the primary advantage of a packet store-and-forward network?

   More efficient use of network resources by statistical multiplexing.

7. What is the primary advantage of a circuit-switched network?

   Guaranteed availability of connection and bit rate.

8. How do Internet routers build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular IP datagram?

   Use routing protocols like OSPF (exchange link information)

9. How do Ethernet switches build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular Ethernet frame?

   List the source addresses coming to each of the switch’s physical ports. (Self-learning)

10. How does your PC determine the 32-bit or 128-bit IP address for "www.acmesales.com"?

    It sends a DNS lookup request (to the local DNS Server).
Problem 18 (Breadth: EMAG - ECE 4350)

Problem

A y-polarized uniform plane wave \((E_i, H_i)\) at the frequency of 100 MHz propagates in air in the \(+x\) direction and impinges normally on a perfectly conducting plate at \(x = 0\). Assuming the amplitude of \(E_i\) to be 6 mV/m, write the phasor and instantaneous expressions for

(a) \(E_i\) and \(H_i\) of the incident wave;

(b) \(E_r\) and \(H_r\) of the reflected wave; and

(c) \(E_1\) and \(H_1\) of the total wave in air.

(d) Determine the location nearest to the conducting plane where \(E_1\) is zero.
1. A y-polarized uniform plane wave \((E_i, H_i)\) with a frequency of 100 MHz propagates in air in the +x direction and impinges normally on a perfectly conducting plate at \(x = 0\). Assuming the amplitude of \(E_i\) to be 6 mW/m, write the phasor and instantaneous expressions for (a) \(E_i\) and \(H_i\) of the incident wave; (b) \(E_r\) and \(H_r\) of the reflected wave; and (c) \(E_t\) and \(H_t\) of the total wave in air. (d) determine the location nearest to the conducting plane where \(E_1\) is zero.

**Solution**

At the given frequency 100 MHz,

\[
\omega = 2 \pi f = 2 \pi \times 10^8 \text{ (rad/s)}
\]

\[
\beta_i = k_0 = \frac{\omega}{c} = \frac{2 \pi \times 10^8}{3 \times 10^8} = \frac{2 \pi}{3} \text{ (rad/m)}
\]

\[
\eta_i = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120 \pi \Omega
\]

**a) For the incident wave (traveling wave):**

**i) Phasor form**

\[
E_i(x) = \hat{y} 6 \times 10^{-3} e^{-j2\pi x/3} \text{ (V/m)}
\]

\[
H_i(x) = \frac{1}{\eta_i} \hat{x} \times E_i(x) = \hat{z} \frac{10^{-4}}{2\pi} e^{-j2\pi x/3} \text{ (A/m)}
\]

**ii) Instantaneous form**

\[
E_i(x,t) = \text{Re}[E_i(x)e^{j\omega t}]
\]

\[
E_i(x,t) = \hat{y} 6 \times 10^{-3} \cos\left(2 \pi \times 10^8 t - \frac{2 \pi}{3} x\right) \text{ (V/m)}
\]

\[
H_i(x,t) = \hat{z} \frac{10^{-4}}{2\pi} \cos\left(2 \pi \times 10^8 t - \frac{2 \pi}{3} x\right) \text{ (A/m)}
\]
b) For the reflected wave (traveling wave):

i) Phasor form

\[ E_r(x) = -j 6 \times 10^{-3} e^{j/2\pi/3} \text{(V/m)} \]

\[ H_r(x) = \frac{1}{\eta_1} x E_r(x) = \frac{z}{2\pi} 10^{-4} e^{j/2\pi/3} \text{(A/m)} \]

ii) Instantaneous form

\[ E_r(x,t) = \text{Re} \left[ E_r(x) e^{j\omega t} \right] \]

\[ H_r(x,t) = \frac{z}{2\pi} 10^{-4} \cos \left( 2\pi \times 10^8 t + \frac{2\pi}{3} x \right) \text{(A/m)} \]

c) For the total wave (a standing wave):

i) Phasor form

\[ E_t(x) = E_r(x) + E_s(x) = -j 12 \times 10^{-3} \sin \left( \frac{2\pi}{3} \right) \text{(V/m)} \]

\[ H_t(x) = H_r(x) + H_s(x) = \frac{z}{\pi} 10^{-4} \cos \left( \frac{2\pi}{3} \right) \text{(A/m)} \]

ii) Instantaneous form

\[ E_t(x,t) = \text{Re} \left[ E_t(x) e^{j\omega t} \right] = j 12 \times 10^{-3} \sin \left( \frac{2\pi}{3} x \right) \sin \left( 2\pi \times 10^8 t \right) \text{(V/m)} \]

\[ H_t(x,t) = \frac{z}{\pi} 10^{-4} \cos \left( \frac{2\pi}{3} x \right) \cos \left( 2\pi \times 10^8 t \right) \text{(A/m)} \]

d) The electric field vanishes at the surface of the conductor plane at \( x=0 \). In air, the first null occurs at

\[ x = -\frac{\lambda_1}{2} = -\frac{\pi}{\beta_1} = -\frac{\lambda_1}{2} \text{ (m)} \]
The table below is extracted from the datasheet of a commercial helium-neon laser.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>632.8 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Power</td>
<td>0.8 mW</td>
</tr>
<tr>
<td>Polarization</td>
<td>random</td>
</tr>
<tr>
<td>Beam Diameter</td>
<td>0.48 mm</td>
</tr>
<tr>
<td>Beam Divergence</td>
<td>1.7 mrad</td>
</tr>
<tr>
<td>Mode Purity</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Longitudinal Mode Spacing</td>
<td>1090 MHz</td>
</tr>
<tr>
<td>Longitudinal Coherence Length</td>
<td>20 cm</td>
</tr>
<tr>
<td>Maximum Noise</td>
<td>1.0%</td>
</tr>
<tr>
<td>Maximum Drift</td>
<td>± 2.5%</td>
</tr>
<tr>
<td>Maximum Mode Sweeping Contribution</td>
<td>10%</td>
</tr>
<tr>
<td>Operating Voltage</td>
<td>1250 V</td>
</tr>
<tr>
<td>Operating Current</td>
<td>4 mA</td>
</tr>
<tr>
<td>Maximum Starting Voltage</td>
<td>10 kV DC</td>
</tr>
</tbody>
</table>

(a) Estimate the number of photons emitted from this laser in one second.

(b) Estimate the electric field amplitude (in V/m) of the light wave. You can assume that the light intensity is uniform across the beam profile.

(c) What is the major reason for the beam divergence of $\Delta \theta_{1/2} = 1.7$ mrad as indicated in the datasheet? Justify your reasoning.

(d) The helium-neon Laser consists of a Fabry-Pérot resonator, which gives rise to discrete longitudinal modes within the gain spectrum. Estimate the cavity length of the Fabry-Pérot resonator.

(e) The datasheet indicates that the He-Ne laser has a longitudinal coherence length of 20 cm. Estimate the linewidth ($\Delta \lambda$) of the laser output.

[Hint: The frequency linewidth $\Delta \nu$ is related to the longitudinal coherence length $L_c$ by $\Delta \nu \cdot L_c = \text{speed of light}$]
(a) 
\[ \lambda = 0.6328 \mu m \]
Photon energy:
\[ E = \frac{1.24}{\lambda} (eV) = \frac{1.24}{0.6328} (eV) = 1.96 \times 10^{-19} \text{J} = 3.14 \times 10^{-19} \text{J} \]
Output power: 0.8 mW = 0.8 \times 10^{-3} \text{J/sec}.
#photons/sec = \frac{0.8 \times 10^{-3} \text{J/sec}}{(3.14 \times 10^{-19} \text{J})} = 2.55 \times 10^{15} \text{sec}^{-1}

(b) 
Laser output power is \( P = 0.8 \text{ mW} = 0.8 \times 10^{-3} \text{ W} \); Spot size is \( A = \pi \left( \frac{0.48 \text{ mm}}{2} \right)^2 = 1.8 \times 10^{-7} \text{ m}^2 \).
So the beam intensity is: \( I = \frac{P}{A} = 4.4 \times 10^3 \text{ W/m}^2 \).

(c) 
The divergence results from the diffraction of light. This can be seen by estimating the divergence angle from the beam diameter at the output.
\[ D = 0.48 \text{ mm}, \Delta \theta_{1/2} = \frac{1.22 \lambda}{D} = \frac{1.22 \times 0.63 \mu m}{0.48 \text{ mm}} = 1.6 \text{ mrad} \]
This is very close to the value of 1.7 mrad shown in the datasheet.

(d) 
The resonance frequency is \( \nu_m = \frac{mc}{2d} \), so the longitudinal mode spacing is simply \( \delta \nu = \frac{c}{2d} \). Therefore the cavity length is \( d = \frac{c}{2 \delta \nu} = (3 \times 10^8 \text{ m/s}) / (2 \times 1.09 \times 10^9 \text{ Hz}) = 0.14 \text{ m} \).

(e) 
The frequency linewidth is \( \Delta \nu \approx \frac{1}{\tau_c} \approx \frac{c}{L_c} = \frac{3 \times 10^8}{2 \times 10^{-1}} = 1.5 \times 10^9 \text{ Hz} \)
The wavelength linewidth can be obtained by taking the differential of \( \lambda = \frac{c}{\nu} \)
\[ \Delta \lambda = \frac{c}{\nu^2} \Delta \nu = \frac{\lambda^2}{c} \Delta \nu = \frac{(633 \times 10^{-9})^2}{3 \times 10^8} (1.5 \times 10^9) = 2 \times 10^{-12} \text{ m} = 2 \text{ pm} \]
For optical fiber communications, the International Telecommunications Union (ITU) has specified a grid of standard frequencies for communication channels. One of these standard frequencies is 195.00 THz. A narrow optical pulse from a laser at this carrier frequency that has a bandwidth of 10 nm propagates through a 100 km length of fused silica optical fiber. The pulse requires a time of 487.8 µsec to propagate through the 100 km optical fiber. The dispersion in the fiber is known to be \( D = +21.0985 \text{ ps/km/nm} \) at 195.00 THz. The pulse spreading due to dispersion limits the maximum bit rate by limiting the minimum time between pulses. That is, two adjacent pulses cannot be closer together than the amount of this spreading in order not to overlap at the output end of the fiber.

Calculate, showing all work, the freespace wavelength in microns, the group velocity refractive index, and the maximum bit rate in bits/sec. Express all answers accurately to four significant figures. Put your final answers in the spaces provided.

Freespace wavelength = \underbrace{\phantom{0000}} \mu m

Group velocity refractive index = \underbrace{\phantom{0000}}

Maximum bit rate = \underbrace{\phantom{0000}} \text{ bits/sec}
**Problem 20 (Specialized: OPTICS - ECE 4502) Solution**

\[ f = 195.00 \text{ THz} \]
\[ \Delta \lambda = 10 \text{ nm} \]
\[ L = 100 \text{ km} \]
\[ \Delta T = 487.8 \mu\text{sec} \]
\[ D = 21.0985 \text{ ps/km/nm} \]

Freespace wavelength \( \lambda = \frac{c}{f} = \)

\[ = \frac{3 \times 10^8 \text{ m/sec}}{195 \times 10^{12} \text{ sec}^{-1}} = 1.5385 \mu\text{m} \]

\[ v_g = \frac{c}{N_g} = \frac{L}{\Delta T} \]

\[ \rightarrow N_g = \frac{c \Delta T}{L} = 1.4634 \]

Maximum bit rate \( \frac{1}{\Delta t} = \frac{1}{D L \Delta \lambda} = \frac{1}{21098.5 \text{ ps}} = 47.3967 \times 10^6 \text{ bits/sec} \)
For this question assume that all operational amplifiers are ideal.

(a) Determine an expression for the gain $G$ where $v_o = G(v_b - v_a)$.
(b) Select $R$ and $R_G$ such that the amplifier provides a gain of approximately 60 dB.
(c) What is this amplifier design called?
(d) Why is it often preferable to make biopotential measurements using this configuration vs. a standard single-input, single-op-amp circuit to amplify a signal?
Sketch a general action potential for a nerve cell. Label the schematic with critical features, regions, and estimated values. Be sure to indicate time and voltage scales and amplitudes. Explain these features, regions and values to convey your understanding of membrane potentials.
For this question assume that all operational amplifiers are ideal.

(a) Determine an expression for the gain $G$ where $v_o = G(v_b - v_a)$.

$$G = 1 + 2\frac{R}{R_G}$$

(b) Select $R$ and $R_G$ such that the amplifier provides a gain of approximately 60 dB.

$$G = 20\log(1+2(500 \text{ k-ohms}/1 \text{ k-ohms})) = 20\log(1+1000) = 60.01 \text{ dB}$$

$$R = 500 \text{ k-ohm}, \quad R_G = 1 \text{ k-ohms}$$

(c) What is this amplifier design called?

*Instrumentation amplifier (R\text{}\text{}G is the gain setting resistor)*

(d) Why is it often preferable to make biopotential measurements using this configuration vs. a standard single-input, single-op-amp circuit to amplify a signal?

*Biopotential signals are often on the order of ten of microVolts and therefore require substantial amplification. However, undesired signals are also amplified. One approach is to amplify differentially thereby rejecting signals common to both, such as 60 Hz noise. Ideally this can be accomplished with a single, high-gain op amp with a differential input. However, the addition of two buffering stages increases the input resistance.*
Sketch a general action potential for a nerve cell. Label the schematic with critical features, regions, and estimated values. Be sure to indicate time and voltage scales and amplitudes. Explain these features, regions and values to convey your understanding of membrane potentials.

A: Resting Potential. The resting potential is about -70mV. During the rest phase the membrane is in steady-state and exhibits some permeability to K+ ions, i.e. some K+ channels are open. Sodium ions move in and out of the cell, there is no net flow of any ion. Furthermore, the membrane potential retains a constant value.

B: Initiation of Action Potential. The initiation of an action potential occurs when the membrane potential sufficiently depolarizes such that it crosses a threshold potential. A local depolarizing event causes some local voltage-gated Na+ channels to open allowing Na+ ions to enter the membrane. These ions further depolarize the membrane enabling more Na+ ions to enter the membrane. Eventually, the membrane is depolarized sufficiently such that it crosses threshold. Many more Na+ channels open further depolarizing the membrane.

C: Highest Conductivity to Na+. After threshold is crossed the number of Na+ channels open increases considerably. Potassium channels begin to open also enabling K+ to flow out. As a result the membrane becomes further depolarized. Technically, the Na+ conductance is greatest before the peak of action potential.

D: Highest Conductivity to K+. The opening of potassium channels enables K+ to flow out and causes an eventual hyperpolarization of the membrane. This occurs during the falling edge of the action potential. Technically, the K+ conductance is greatest before the minimum of action potential. The cell returns to rest by the action of the Na+/K+ pumps.
Problem 22 (Specialized: BIO - ECE 4782) Solution

PROBLEM

(4 pts) Cardiovascular Anatomy / Physiology. Fill in the blanks with the best possible answer, and provide a short answer response.

The cardiac cycle has two main phases: ____________ and ______________. During both of the phases, there are periods of time in which the ventricle either builds up pressure, or reduces its pressure, while the volume of blood within the ventricle does not change. What are the key physiological parameters that describe the health of the heart during these two phases?

(4 pts) Draw a pressure-volume loop for the left ventricle, labeling each segment and each corner point. Show which segments are associated with filling and which are associated with ejection. Then, using a dotted line, show how the loop would change with increased preload.

(2 pts) Draw an electrocardiogram trace with three normal heartbeats, then one premature ventricular contraction (PVC), then two normal heartbeats. Clearly label two or three (at most) particular aspects of the shape and timing of the PVC that are unique to that particular arrhythmia.
(4 pts) **Cardiovascular Anatomy / Physiology.** Fill in the blanks with the best possible answer, and provide a short answer response.

The cardiac cycle has two main phases: __systole__ and __diastole__. During both of the phases, there are periods of time in which the ventricle either builds up pressure, or reduces its pressure, while the volume of blood within the ventricle does not change. What are the key physiological parameters that describe the health of the heart during these two phases?

Students should discuss ventricular pressure, stroke volume, time intervals, and / or electrical conduction as examples of key parameters.

(4 pts) Draw a pressure-volume loop for the left ventricle, labeling each segment and each corner point. Show which segments are associated with filling and which are associated with ejection. Then, using a dotted line, show how the loop would change with increased preload.

Students should show the classic PV loop, labeling isovolumetric contraction, ejection, isovolumetric relaxation, and filling. IVC and ejection are systole, IVR and filling are diastole.

With a dotted line, students should indicate that the right side of the PV loop moves further to the right (increased EDV).

(2 pts) Draw an electrocardiogram trace with three normal heartbeats, then one premature ventricular contraction (PVC), then two normal heartbeats. Clearly label two or three (at most) particular aspects of the shape and timing of the PVC that are unique to that particular arrhythmia.

Students should show several ECG beats which have P, QRS, and T components clearly shown. The PVC should be (1) premature, (2) wider in the QRS complex, and (3) lacking a P-wave.