INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 24 numbered problems. Note that Problem 4 occupies 3 pages, Problem 8 occupies 2 pages, Problem 10 occupies 2 pages, Problem 21 occupies 2 pages, Problem 24 occupies 2 pages. Including the cover sheet, you should have 66 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes the cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!

6. You must choose one of two exam options: CompE or EE. As shown in the table in the next page, each option consists of a core and electives. You must work all four core problems listed for your chosen option and four additional elective problems chosen from the list corresponding to your chosen option. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 16 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:
   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in EVERY sheet you are turning in.
   (c) In the table on the cover page, CIRCLE the problem numbers that you are turning in for grading.
   (d) Turn in the cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!
# CompE option

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# EE option

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Design a synchronous finite state machine (FSM) as follows:

- Use three positive edge-triggered D-type flip-flops (labeled A, B, C) to represent a 3-bit binary number, with flip-flop A as the most-significant bit (msb) and flip-flop C as the lsb.
- Provide an input control variable, X
- Obtain the desired behavior according to:

<table>
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<th>X</th>
<th>behavior</th>
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<td>0</td>
<td>count in the sequence 000, 001, 010, 011, 100, 101, 110, 111 (with wrap-around to 000)</td>
</tr>
<tr>
<td>1</td>
<td>count in the sequence 000, 010, 100, 110, 111, 001, 011, 101 (with wrap-around to 000)</td>
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**Part A** [3 points] – Draw a **state diagram** for a Moore-type finite state machine that describes this behavior.

**Part B** [7 points] – Use Karnaugh maps to find the **minimized sum-of-products** (SOP) solution for the three flip-flop input functions.
A

\[
\begin{array}{cccc}
A^* & B^* & C^* & \\
000 & 1 & 0 & \\
001 & 0 & 1 & \\
010 & 0 & 1 & \\
011 & 1 & 0 & \\
100 & 0 & 1 & \\
101 & 1 & 0 & \\
110 & 1 & 0 & \\
111 & 0 & 1 & \\
\end{array}
\]

\[
A^* (xABC) = \Sigma m(3, 4, 5, 6, 10, 11, 12, 13)
\]

\[
B^* (xABC) = \Sigma m(1, 2, 5, 9, 12, 13)
\]

\[
C^* (xABC) = \Sigma m(0, 2, 3, 4, 9, 11, 13)
\]
Code Number: 

\[ A^+ = \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} \]

\[ B^+ = \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} \]
\[ C^+ = \overline{x} C + \overline{XAC} + XAB \]
The block diagram above defines a system for a discrete-time filtering of continuous-time signals. Note: all parts of this question can be worked independently.

(a) Assume that \( h[n] = \delta[n] \). If \( x(t) = \cos\left(300\pi t - \frac{\pi}{4}\right) + \cos\left(800\pi t + \frac{\pi}{8}\right) \), find \( f_s \) so:

\[
y(t) = \cos\left(150\pi t + \frac{\pi}{4}\right) + \cos\left(350\pi t + \frac{\pi}{8}\right)
\]

(b) Find \( y(t) \) if \( x(t) = 2 + (-1)^n \) and \( H(z) = (1 - z^{-1})(1 + 0.9z^{-1}) \). (3 points)

(c) Let \( x(t) = A \cos(180\pi t) + B \cos(330\pi t) \) in the block diagram above. The N-point DFT of \( x[n] \) is represented by

\[
X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi k n/N}, k = 0, \ldots, N - 1
\]

Assume that \( N=32 \) and we determine the DFT has values of: \( X[6] = 64, X[11] = 160, X[21] = 160, \) and \( X[26] = 64 \) with all other values of \( k \) being zero (i.e., \( X[k] = 0 \), for \( k \neq 6, 11, 21, \) and \( 26 \)). (4 points)

Find \( f_s \).

(Extra Points) Only to make up for lost points on THIS problem. Total points on this problem will not exceed the maximum allowed for an individual problem on the prelim exam.

Find \( A \) and \( B \) in \( x(t) = A \cos(180\pi t) + B \cos(330\pi t) \) (3 points)
(a) \( x(t) = \cos\left(300\pi t - \frac{\pi}{2}\right) + \cos\left(800\pi t + \frac{\pi}{4}\right) \)

\( y(t) = \cos\left(150\pi t + \frac{\pi}{4}\right) + \cos\left(350\pi t + \frac{\pi}{8}\right) \)

The change in phase suggests that folding has occurred on the first sinusoid. The original frequency was 150Hz. The new frequency is 75 Hz. Therefore, \(150 - f_s = -75 \rightarrow f_s = 150 + 75 = 225 \text{ Hz} \)

(b) \( x(t) = 2 + (-1)^n = 2 + \cos(\pi n) \)

\( H(z) = (1 - z^{-1})(1 + 0.9z^{-1}) \rightarrow H(e^{j\omega}) = (1 - e^{-j\omega})(1 + 0.9e^{-j\omega}) \)

\( H(e^{j\omega}) = 0; H(e^{j\pi}) = (1 - e^{-j\pi})(1 + 0.9e^{-j\pi}) = 2 \times 0.1 = 0.2 \)

\( y(t) = 2 \times 0 + 0.2 \times \cos(\pi n) = 0.2\cos(\pi n) \)

(c) First let’s determine which normalized frequencies are present:

\( \tilde{\omega}_k = \frac{2\pi k}{N} = \frac{2\pi k}{32} = \frac{\pi k}{16} \)

for \( k = 6, 11, 21, 26 \rightarrow \tilde{\omega}_k = \frac{6\pi}{16}, \frac{3\pi}{8}, \frac{11\pi}{16}, \frac{21\pi}{16}, \frac{26\pi}{16}, \frac{11\pi}{16}, \frac{26\pi}{16}, \frac{6\pi}{8} \)

We know that there was a 90 Hz and 165 Hz signal in the signal. We can now find \( f_s \):

\( \tilde{\omega} = \frac{2\pi f}{f_s} \rightarrow 3\pi f = \frac{2\pi (90)}{f_s} \rightarrow f_s = 480 \)

(Extra Points)

Based on the concept of Discrete Fourier Series (DFS), we can tell that the size of the DFT is an integer multiple of a periodic signal because it is mostly zero except at the frequencies of interest. We also know that the height of the sinusoids in a DFS are scaled by \( N/2 \) (in this case 32/2 = 16). From the given information: \( X[6] = 64 = 16 \times A \rightarrow A = 4 \);

\( X[11] = 160 = 16 \times B \rightarrow B = 10 \)
Consider a doubly linked list that is implemented using the following struct definitions.

```c
typedef struct node_t {
    int data;
    struct node_t* prev;
    struct node_t* next;
}Node;

typedef struct dll_t {
    struct node_t* head;
    struct node_t* tail;
} DLList;
```

**Part A.** Assume a 64-bit system and consider the following function:

```c
DLList* create_dlinkedlist() {
    DLList* newList = (DLList*)malloc(sizeof(DLList));
    newList->head = NULL;
    newList->tail = NULL;
    return newList;
}
```

A.1 What integer is passed to `malloc` when this function executes? __________.

A.2 Which region of memory holds the variable `newList`? ________________.

A.3 How much space is allocated for `newList` in this region of memory? _______ bytes.

**Part B.** Write a C function `Reverse` that takes a pointer to a doubly linked list `dll` (which might be empty) and reverses the order of the nodes. Each node’s previous node becomes its next node and vice versa. Be sure to update the head and tail of the doubly linked list as well.

```c
void Reverse(DLList* dll){
```

```c
}
```
Consider a doubly linked list that is implemented using the following `struct` definitions.

```c
typedef struct node_t {
    int data;
    struct node_t* prev;
    struct node_t* next;
}Node;

typedef struct dll_t {
    struct node_t* head;
    struct node_t* tail;
} DLList;
```

**Part A.** Assume a 64-bit system and consider the following `create_dlinkedlist` function:

```c
DLList* create_dlinkedlist() {
    DLList* newList = (DLList*)malloc(sizeof(DLList));
    newList->head = NULL;
    newList->tail = NULL;
    return newList;
}
```

A.1 What integer is passed to `malloc` when this function executes? **16**.
A.2 Which region of memory holds the variable `newList`? **stack**.
A.3 How much space is allocated for `newList` in this region of memory? **8 bytes**.

**Part B.** Write a C function `Reverse` that takes a pointer to a doubly linked list `dll` (which might be empty) and reverses the order of the nodes. Each node’s previous node becomes its next node and vice versa. Be sure to update the head and tail of the doubly linked list as well.

```c
void Reverse(DLList *dll){
    Node *current = dll->head;
    Node *h = dll->head;
    Node *t = dll->tail;
    Node *n;
    Node *p;
    while(current){
        n = current->next;
        p = current->prev;
        current->prev = n;
        current->next = p;
        current = n;
    }
    dll->head = t;
    dll->tail = h;
    return;
}
```
Identify and correct the errors in the code segments below by adding new code, rewriting code, or eliminating code; or state that there is no error. The errors may be runtime or compile-time errors. The fact that some of the code segments do not appear to be very useful is not an error.

(a) Problem (1pt): 

Correct error below (1pt):

```cpp
#include <limits>
#include <cstdlib>
#include <random>
float x = (static_cast<float> rand()) / RAND_MAX;
switch (x) {
    case (x < 0.5):
        std::cout << "tails" << std::endl;
        break;
    case (x > 0.5):
        std::cout << "heads" << std::endl;
        break;
    default:
        std::cout << "landed on its side" << std::endl;
}
```
(b) Problem (1pt):
Correct error below (1pt):

```c
int sum( int x, int y)
{
    int result = 0;
    result = x + y;
}
```

(c) Problem (1pt):
Correct error below (1pt):

```c
int & sum( int a, int b)
{
    int s;
    s = a + b;
    return s;
}
```
(d) Problem (2pt): 

Correct error below (2pt):

```cpp
class MyArray {
    public:
        MyArray(unsigned int x = 1);

    private:
        int sz;
        int *Apotr;
    
};

MyArray::MyArray(unsigned int newsize)
{
    if (newsize < 1)
        newsize = 1;
    Aptr = new int[newsize];
    sz = newsize;
}
```
(a) Problem (1pt): **switch arguments cannot be floats**
Correct error below (1pt):

```cpp
#include <limits>
#include <cstdlib>
#include <random>
float x = (static_cast<float> rand()) / RAND_MAX;
switch (x)
{    
    case (x < 0.5):
        std::cout << "tails" << std::endl;
        break;
    case (x > 0.5):
        std::cout << "heads" << std::endl;
        break;
    default:
        std::cout << "landed on its side" << std::endl;
}
```

Change the switch to the following if structure

```cpp
if (x < 0.5)
    std::cout << "tails" << std::endl;
else if (x > 0.5)
    std::cout << "heads" << std::endl;
else
    std::cout << "landed on its side" << std::endl;
```
(b) Problem (1pt): **return statement missing**
Correct error below (1pt):

```c
int sum( int x, int y)
{
    int result = 0;

    result = x + y;
    return result;
}
```

(c) Problem (1pt): **returns a reference to a local variable**
Correct error below (1pt):

```c
int & sum( int a, int b)
{
    int s;
    s = a + b;

    return s;
}
```

change the first line above to remove the &

```c
int sum( int a, int b)
```
(d) Problem (2pt): There is no destructor to destroy the Aptr array
Correct error below (2pt):

```cpp
class MyArray {
    public:
        MyArray(unsigned int x = 1);
        ~MyArray();

    private:
        int sz;
        int *Aptr;
};

MyArray::MyArray(unsigned int newsize)
{
    if (newsize < 1)
        newsize = 1;
    Aptr = new int[newsize];
    sz = newsize;
}

MyArray::~MyArray()
{
    delete [] Aptr;
}
```
Compute the impedance matrix of the two-port network shown below. (Hint: You won’t be able to solve this problem by combining resistances, you will have to use mesh analysis instead. Note also that the network is symmetric.)
Problem XX (AREA) Code Number: SOLUTIONS

\[ I_1 = 1 \text{ A}. \text{ Mesh equations:} \]
\[ 2(I_3 - 1) + 4I_3 + 4(I_3 - I_4) = 0 \]
\[ 2(I_4 - 1) + 4(I_4 - I_3) + 4I_4 = 0 \]

Solution: \( I_3 = I_4 = \frac{1}{3} \text{ A}. \) Therefore:

\[ V_1 = 2 \left[ \left( 1 - \frac{1}{3} \right) + \left( 1 - \frac{1}{3} \right) \right] = \frac{8}{3} \text{ V} \]
\[ V_2 = 2I_3 + 2I_4 = \frac{4}{3} \text{ V} \]
\[ z_{11} = \frac{V_1}{I_1} = \frac{8}{3} \Omega \]
\[ z_{21} = \frac{V_2}{I_1} = \frac{4}{3} \Omega \]

Since the network is symmetric, \( z_{12} = z_{21} \) and \( z_{22} = z_{11}. \)
A. (3 points) A mathematical calculation is defined by this postfix expression:

\[ 6 \ 2 \ - \ 3 \ * \ 4 \ 11 \ - \ + \ 14 \ 5 \ 2 \ * \ - \ - \ 20 \ + \]

Assuming that all operators (addition, subtraction, and multiplication) are binary, draw the binary expression tree corresponding to this postfix expression.

B. (4 points) Write both the prefix and infix expressions corresponding to the expression tree from part A. For the infix expression, add parentheses where necessary to maintain the meaning of the mathematical expression.

Prefix:

Infix:

C. (3 points) A recursive function will be used to calculate the numerical result represented by the expression tree from part A. List, in order, all of the mathematical calculations made during the execution of the function on this tree. Show each calculation in a form similar to the following: \(10 + 5 = 15\).

1) 
2) 
3) 
4) 
5) 
6) 
7) 
8)
A. (3 points) A mathematical calculation is defined by this postfix expression:

\[ 6 2 - 3 * 4 11 - + 14 5 2 * - - 20 + \]

Assuming that all operators (addition, subtraction, and multiplication) are binary, draw the binary expression tree corresponding to this postfix expression.

![Binary Expression Tree]

B. (4 points) Write both the prefix and infix expressions corresponding to the expression tree from part A. For the infix expression, add parentheses where necessary to maintain the meaning of the mathematical expression.

Prefix: \[ + - + * - 6 2 3 - 4 11 - 14 * 5 2 20 \]

Infix: \[ (6 - 2) * 3 + (4 - 11) - (14 - 5 * 2) + 20 \]

C. (3 points) A recursive function will be used to calculate the numerical result represented by the expression tree from part A. List, in order, all of the mathematical calculations made during the execution of the function on this tree. Show each calculation in a form similar to the following: \(10 + 5 = 15\).

1) \(6 - 2 = 4\)
2) \(4 * 3 = 12\)
3) \(4 - 11 = -7\)
4) \(12 + -7 = 5\)
5) \(5 * 2 = 10\)
6) \(14 - 10 = 4\)
7) \(5 - 4 = 1\)
8) \(1 + 20 = 21\)
A very long transmission line is carrying a sinusoidal signal. The transmission line has impedance $Z_0$ and the load is a resistance $R_L$. You do not know $Z_0$ or $R_L$ so you measure the amplitude of the sinusoidal voltage on the transmission line at many locations along the transmission line. The below plot is the result, where the right edge, $x=0$, corresponds to the load point.

a. What is the voltage standing wave ratio?

b. What is the ratio $\frac{R_L}{Z_0}$? If you think there is more than one value, constrain it as much as possible.

c. If you had measured current instead of voltage, what would you have observed? Sketch the equivalent plot as above except for current instead of voltage. Assume for the purpose of this part that $Z_0 = 100 \, \Omega$
a. The VSWR is the ratio of $V_{\text{max}}$ to $V_{\text{min}}$, which can be directly read out of the plot as $\frac{1.5}{0.5} = 3$

b. Since the standing wave is established by the interference pattern between the outgoing and reflected wave, we can directly read out that the magnitude of the reflection coefficient is 0.5. And since the voltage is at a minimum near the load, that implies that the load has a smaller resistance than the line impedance, therefore the reflection coefficient is a negative number.

$$\text{A ratio } \frac{R_L}{Z_0} \text{ of } 1/3 \text{ satisfies the reflection coefficient } \Gamma = \frac{R_L - Z_0}{R_L + Z_0} = \frac{R_L - 1}{Z_0} = -0.5$$

c. The plot for current would like the same as voltage, except for the following changes: (1) The current would be maximized at the load, instead of minimized, and (2) the current would vary between 5 mA and 15 mA.
Part one (Power in a microprocessor): Consider a microprocessor logic block that can operate at a maximum clock frequency ($f$) of 2 GHz and 2.5 GHz when the power supply voltage ($V_{DD}$) is 1.2 V and 1.5 V, respectively.

Based on simplest possible assumptions, at which clock frequency will the block consume more power: 2 GHz or 2.5 GHz? Briefly explain your answer.
**Part two (MOSFET current-voltage characteristics):** The following figure shows how the drain current $I_D$ in a long channel MOSFET changes as a function of the drain voltage $V_D$ for different values of the gate voltage $V_G$.

**What is the threshold voltage $V_t$ of the MOSFET?**

**Calculate the saturation current at 3 V.**

![MOSFET $I_D - V_D$ characteristics.](figure1)

Figure 1: MOSFET $I_D - V_D$ characteristics.
Solution to Part one (MOS Capacitor):

Point B is outside of the depletion region. Hence, \( p = N_A = 10^{22} \text{ m}^{-3} \) and \( n = n_i^2 / N_A = 2.25 \times 10^{10} \).

Point A is at the oxide-semiconductor interface. As such, at \( V_G = V_t \) the electron and hole densities will switch @A. Hence, \( @A \) and \( V_G = V_t, n = N_A = 10^{22} \text{ m}^{-3} \) and \( p = n_i^2 / N_A = 2.25 \times 10^{10} \).

Part two (MOSFET current-voltage characteristics):

At \( V_G = 4 \text{ V} \), the drain saturation voltage \( V_{D, sat} = 3 \text{ V} = V_G - V_t \). Hence \( V_t = 1 \text{ V} \). (Or a student can do: At \( V_G = 2 \text{ V} \), the drain saturation voltage \( V_{D, sat} = 1 \text{ V} = V_G - V_t \). Hence \( V_t = 1 \text{ V} \).)

According the square law model (note that it is a long channel MOSFET), \( I_{D, sat} \propto (V_G - V_t)^2 \).

Hence,
\[
\frac{I_{D, sat}(V_G = 3\text{ V})}{I_{D, sat}(V_G = 4\text{ V})} = \frac{(3 - 1)^2}{(4 - 1)^2} \Rightarrow I_{D, sat}(V_G = 3\text{ V}) = \frac{4}{9} \times 1.43 \text{ A/m} = 0.64 \text{ A/m} \quad (1)
\]

(Or a student can do: Hence
\[
\frac{I_{D, sat}(V_G = 3\text{ V})}{I_{D, sat}(V_G = 2\text{ V})} = \frac{(3 - 1)^2}{(2 - 1)^2} \Rightarrow I_{D, sat}(V_G = 3\text{ V}) = 4 \times 0.16 \text{ A/m} = 0.64 \text{ A/m} \quad (2)
\] )
A non-ideal amplifier (pictured to the right using the operational amplifier schematic symbol) shown here has an open loop gain at low frequencies of 200 v/v, an input impedance of 30K ohms, an output impedance of 40 ohms and an open loop low pass bandwidth of 150 Hz. This amplifier is used in a non-inverting configuration to achieve a low frequency voltage gain of 10 v/v. If R2=181K ohms:

a) (4 points) Showing your work for full credit, what is the required value of R1?

b) (2 points) Showing your work for full credit, what is the new closed loop amplifier bandwidth?

c) (2 points) Showing your work for full credit, what is the new closed loop amplifier input impedance?

d) (2 points) Showing your work for full credit, what is the new closed loop amplifier output impedance?
A non-ideal amplifier (pictured to the right using the operational amplifier schematic symbol) shown here has an open loop gain at low frequencies of 200 v/v, an input impedance of 30K ohms, an output impedance of 40 ohms and an open loop low pass bandwidth of 150 Hz. This amplifier is used in a non-inverting configuration to achieve a low frequency voltage gain of 10 v/v. If R2=181K ohms:

a) (4 points) Showing your work for full credit, what is the required value of R1?

\[
A_{\text{v, closedloop}} = \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{A_{\text{openloop}}}{1 + \beta A_{\text{openloop}}}
\]

\[
10 = \frac{200}{1 + \beta 200} \rightarrow \beta = \frac{19}{200} = \frac{R_1}{R_{\text{out}} + R_2}
\]

\[
R_1 = \frac{19}{200} R_{\text{out}} = 19 K
\]

b) (2 points) Showing your work for full credit, what is the new closed loop amplifier bandwidth?

c) (2 points) Showing your work for full credit, what is the new closed loop amplifier input impedance?

d) (2 points) Showing your work for full credit, what is the new closed loop amplifier output impedance?

\[
f_{\text{closedloop}} = (1 + \beta A_{\text{openloop}}) f_{\text{openloop}} = (1 + (19/200)x200) 150 = 3000Hz
\]

\[
R_{\text{in, closedloop}} = (1 + \beta A_{\text{openloop}}) R_{\text{in, openloop}} = (1 + (19/200)x200) 30K = 600K\Omega
\]

\[
R_{\text{out, closedloop}} = \frac{R_{\text{out, openloop}}}{(1 + \beta A_{\text{openloop}})} = \frac{40}{(1 + (19/200)x200)} = 2\Omega
\]
Consider a single core processor with 4 GB of main memory, and a 256 Kbyte, 2-way set associative unified instruction and data cache with 64 byte lines. The instruction miss rate is 3% and the data miss rate is 4%. We find that 40% of all instructions generate a data memory reference.

1. **(3 pts)** Consider the following code segment. Assume the variables \(i\) and \(sum\) are in registers. Ignoring instruction references, what is the miss rate for accesses to \(Myarray\) in the execution of the loop. An int is 4 bytes.

   ```c
   int Myarray[1024];
   int i, sum;
   sum = 0;
   for (i = 0; i < 1024; i += 4){
       sum += Myarray[i];
   }
   ```

2. **(2 pts)** Can cache lines in two sets have the same value of tag? Justify your answer by showing the address breakdown.
3. (2 pts) If each cache directory entry has a dirty bit, a valid bit, and tag, what is the size of the cache directory in bits?

4. (3 pts) The average cycles per instruction with an ideal cache (e.g., no misses) is 1.2. Assuming a cache miss penalty of 20 cycles, what is the new cycles per instruction (CPI) due to the above miss rates (you can leave your answer in expression form).
1. (3 pts) Consider the following code segment. Assume the variables \( i \) and \( \text{sum} \) are in registers. Ignoring instruction references, what is the miss rate for accesses to \( \text{Myarray} \) in the execution of the loop.

```c
int Myarray[1024];
int i, sum;
sum = 0;
for (i = 0; i < 1024; i += 4)
    sum += Myarray[i];
```

There are 16, 32-bit words per cache line and the array is comprised of 64 lines. When accessing every 4\(^{th} \) word, each cache line will have 4 references. The first reference to a cache line will be a miss and the remaining 3 will be hits producing a miss rate of 1/4. This is repeated for each cache line. The full array fits in the cache so there will be no conflicts amongst array references.

2. (2 pts) Can cache lines in two sets have the same value of tag? Justify your answer by showing the address breakdown.

The address will have an 11-bit set index and a 15-bit tag. Two addresses can have the same value of tag but will map to two distinct sets and correspond to two distinct lines.

3. (2 pts) If each cache directory entry has a dirty bit, a valid bit, and tag, what is the size of the cache directory in bits?

There are \( \frac{2^{18}}{2^6} \) lines and with associativity of 2, there are \( 2^{12}/2 = 2^{11} \) sets. With 32-bit addresses, the size of the tag is 32-11-6 = 15 bits. Therefore, each cache directory entry has \( 1+1+15 = 17 \) bits. The total number of lines is \( 2^{12} \). Therefore, the total cache directory size is \( 2^{12} * 17 \) bits.

4. (3 pts) The average cycles per instruction with an ideal cache (e.g., no misses) is 1.2. Assuming a cache miss penalty of 20 cycles, what is the new cycles per instruction (CPI) due to the above miss rates (you can leave your answer in expression form).

\[ I = \text{total number of instructions} \]

- The number of cycles added due to instruction misses is \( I * 0.03 * \text{miss-penalty} = I * 0.60 \)
- The number of cycles added due to data miss cycles is \( I * 0.4 * 0.04 * \text{miss-penalty} = I * 0.32 \)

Note that 40\% of all instructions generate data references.

The per instruction increase in cycles is \( \frac{\text{Total Additional Cycles}}{\text{Total Instructions}} \)

\( = \frac{(0.6 * I) + (0.32 * I)}{I} = 0.92 \) New CPI = 1.2 +0.92 = 2.12
Consider a 3-phase, 230 V, 50 Hz, 100 kVA load operating at power factor 0.9. The load consumes 100 kVA of power for all three phases at 0.9 power factor lagging when the voltage across the load is 230 V (L-L). The load is connected to a source through a feeder with an impedance of 0.05 + j0.2 ohms in each phase wire. Neglect the no-load current. For this load condition:

(a) Determine the required supply voltage (L-L).
(b) Calculate the real and reactive power flowing from the supply.
Problem XX (Core: Energy-ECE3072)

\[ V_L = \frac{230 \angle 0^\circ}{\sqrt{3}} = 133.40 \]

\[ I_L = \frac{100}{3 \times 133} = 2.50 \angle \phi_L \]

\[ \phi_L = \cos^{-1}(0.9) = -25.8^\circ \]

\[ V_3 = V_L \angle 0^\circ + I_L \angle \phi_L = 133.40 + (2.50 \angle -25.8^\circ)(0.05 \angle 0^\circ) \]

\[ = 151.3 \angle 17.4^\circ \]

\[ S = \sqrt{3} V_3 I_L = \sqrt{3} (151.3 \angle 17.4^\circ)(250 \angle 25.8^\circ) \]

\[ = 4.775 \times 10^4 + j 4.984 \times 10^4 \]
3077 PROBLEM

Let X and Y be random variables that are (nonuniformly) distributed inside the unit circle according to the following joint probability density function:

\[ f_{X,Y}(x, y) = \begin{cases} 
  c \sqrt{1-x^2} & \text{for } x^2 + y^2 < 1, \\
  0 & \text{elsewhere},
\end{cases} \]

where c is an unspecified constant.

(a) Are X and Y are independent? [ ] Yes [ ] No

(b) The unspecified constant is \( c = [ ] \).

(c) Find the expected value \( E\left( \frac{1}{\sqrt{1-X^2}} \right) = [ ] \).

(d) The marginal pdf for \( X \) is \( f_X(x) = \begin{cases} 
  \text{Provide a careful sketch below:}\end{cases} \), for \(-1 < x < 1,\)

\( 0, \text{ elsewhere} \).

(e) Carefully sketch the conditional pdf \( f_{Y|X}(y| \frac{1}{\sqrt{2}}) \) for \( Y \), given that \( X = \frac{1}{\sqrt{2}} \):
Let $X$ and $Y$ be random variables that are (nonuniformly) distributed inside the unit circle according to the following joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} 
  c\sqrt{1-x^2}, & \text{for } x^2 + y^2 < 1, \\
  0, & \text{elsewhere,}
\end{cases}$$

where $c$ is an unspecified constant.

(a) Are $X$ and $Y$ independent? No.

Knowledge of $Y$ changes the probabilities for $X$. E.g., $P(X > 0.9) \neq 0$, while $P(X > 0.9 | Y > 0.9) = 0$.

(b) The unspecified constant is $c = \frac{3}{8}$.

The joint pdf must integrate to 1:

$$1 = \iint_{u.\ circle} f_{X,Y}(x, y)\, dx\, dy = c \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx\, dy = c \int_{-1}^{1} 2(1-x^2)\, dx = c \left(4 - \frac{4}{3}\right) \Rightarrow c = \frac{3}{8}.$$

(c) Find the expected value $E\left( \frac{1}{\sqrt{1-X^2}} \right) = \frac{3\pi}{8}$.

By the fundamental theorem of expectation:

$$E\left( \frac{1}{\sqrt{1-X^2}} \right) = \iint_{u.\ circle} \frac{1}{\sqrt{1-x^2}} f_{X,Y}(x, y)\, dx\, dy = c \iint_{u.\ circle} \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \, dx\, dy = c \iint_{u.\ circle} \, dx\, dy = c \text{(circle area)} = \frac{3\pi}{8}.$$

(d) The marginal pdf for $X$ is $f_X(x) = \begin{cases} 
  \frac{3}{4}(1-x^2), & \text{for } -1 < x < 1 \\
  0, & \text{elsewhere.}
\end{cases}$

Provide a careful sketch below:

$$f_X(x) = \int_{-1}^{1} f_{X,Y}(x, y)\, dy = c \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2}\, dy = 2c(1-x^2) = \frac{3}{4}(1-x^2) \text{ for } |x| < 1:
$$

(e) Carefully sketch the conditional pdf $f_{Y|X}(y|\frac{1}{\sqrt{2}})$ for $Y$, given that $X = \frac{1}{\sqrt{2}}$:

Observe that for all points inside the unit circle, the joint pdf $f(x, y)$ is independent of $y$. Therefore, slicing the joint pdf surface along the line $x = \frac{1}{\sqrt{2}}$ yields a rectangular crosssection. After normalizing so that it integrates to 1, we get:
Let $u(t)$ denote the unit step function, defined as $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$, and let $\delta(t)$ denote the Dirac delta function. Let $h(t) = u(t) - u(t - 2\pi)$ and $x(t) = \sin(t)u(t)$.

(a) (2 points) **Compute the second derivative** of $x(t)$ with respect to $t$. **Simplify** your answer as much as possible.

(b) (8 points) **Draw a labeled sketch** of $y(t)$, where $y(t)$ is the convolution of $h(t)$ with $x(t)$. 

Let $u(t)$ denote the unit step function, defined as $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$, and let $\delta(t)$ denote the Dirac delta function. Let $h(t) = u(t) - u(t - 2\pi)$ and $x(t) = \sin(t)u(t)$.\(^1\)

(a) (2 points) **Compute the second derivative** of $x(t)$ with respect to $t$. **Simplify** your answer as much as possible.

**Solution:** By the product rule for derivatives:

\[
\frac{d}{dt} \sin(t)u(t) = \sin(t)\delta(t) + \cos(t)u(t) = \sin(0)\delta(t) + \cos(t)u(t) = \cos(t)u(t). \\
\frac{d}{dt} \cos(t)u(t) = \cos(t)\delta(t) - \sin(t)u(t) = \cos(0)\delta(t) - \sin(t)u(t) = \delta(t) - \sin(t)u(t).
\]

(b) (8 points) **Draw a labeled sketch** of $y(t)$, where $y(t)$ is the convolution of $h(t)$ with $x(t)$.

**Solution:** The functions being convolved look like:

There are several approaches to this problem. The easiest is probably to “flip-and-shift” $h(t)$ while keeping $x(t)$ “fixed.” Consider three regions:

- For $t < 0$, there is no “overlap” between the support of $h(t - \tau)$ and $x(\tau)$, so $y(t) = 0$.
- For $0 < t < 2\pi$, there is “partial overlap,” and we can compute
  \[y(t) = \int_0^t \sin(\tau)d\tau = -\cos(\tau)|_{\tau=0}^{\tau=t} = -\cos(0) - [-\cos(t)] = \cos(t) - 1.\]
- For $t > 2\pi$, there is “total overlap,” and the convolution integral corresponds to the integration of a single cycle of a sinusoid, which gives $y(t) = 0$.

Combining these three cases, we may plot:

---

\(^1\)This is a variation of Problem 4 from MIT’s 6.003 OpenCourseWare, Fall 2009, Quiz 2.
A 480V, 60 Hz, 6-pole, three-phase, wye-connected induction motor has the following parameters:

\[ R_1 = 0.461 \, \Omega, \quad R_2 = 0.258 \, \Omega, \quad X_1 = 0.507 \, \Omega, \quad X_2 = 0.309 \, \Omega, \quad X_m = \infty \]

Where \( R_1 \) is the stator resistance, \( R_2 \) is the rotor resistance, \( X_1 \) is the stator leakage reactance, \( X_2 \) is the rotor leakage reactance, and \( X_m \) is the magnetizing reactance. All values are referred to the stator. Assume there is no iron loss for this machine.

Rotational losses are 2450W. The motor drives a mechanical load at a speed of 1170 rpm. Calculate the following information:

a) Slip  
b) Airgap power  
c) Output power delivered to the load
a) The slip is,
\[ s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1170}{1200} = 0.025 \]

b) The airgap power, \( P_g \), is the power in \( R_2/s \).
\[
P_g = 3 \frac{V_1^2}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2} \frac{R_2}{s} = 3 \frac{(480/\sqrt{3})^2}{0.258} \frac{0.258}{0.025}
\]
\[ = 20.34 \text{ kW} \]

c) The out is,
\[
P_{out} = (1 - s)P_g - P_{rot} = (1 - .025)(20340) - 2450 = 17.38 \text{ kW} \]
In Figure 1, assume the two op-amps are ideal with infinite voltage gain, infinite bandwidth, infinite input impedance, and zero output impedance. The op-amps are both in negative feedback loop and will not saturate. The circuit is driven by an ideal voltage source $V_s$. Assume that the $Z_1$, $Z_2$, $Z_3$, $Z_4$, and $Z_5$ are some physical impedances, made by resistors, inductors, and/or capacitors.

Please answer the following questions.

1. In Figure 1, solve $V_1$, $V_2$, $V_3$, $V_4$, $I_1$, $I_2$, $I_3$, $I_4$, $I_5$, and $I_s$.

2. In Figure 1, what is the input impedance of this circuit $Z_{in} = V_s/I_s$?

3. In Figure 1, assume $Z_2$ is a capacitor with a value of $C_2$, while $Z_1 = R_1$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = R_5$. What is the input impedance $Z_{in}$ in this special case?

   This impedance $Z_{in}$ equivalently behaves as a positive resistor, negative resistor, positive capacitor, negative capacitor, positive inductor, or negative inductor? (Only choose one.)

   What is the equivalent component value, i.e., equivalent resistor or capacitor or inductor value, of this impedance $Z_{in}$?

4. In Figure 1, assume $Z_1$ and $Z_5$ are capacitors with values of $C_1$ and $C_5$, respectively. Assume $Z_2 = R_2$, $Z_3 = R_3$, and $Z_4 = R_4$. What is the input impedance $Z_{in}$ in this special case?

   This impedance $Z_{in}$ equivalently behaves as a positive resistor, negative resistor, positive capacitor, negative capacitor, positive inductor, or negative inductor? (Only choose one.)

   What is the equivalent component value, i.e., equivalent resistor or capacitor or inductor value, of this impedance $Z_{in}$?

5. This circuit is often used to synthesize active inductors with large values that cannot be practically implemented using actual physical inductors.

   Please use the circuit in Figure 1 to synthesize a desired active inductor and achieve the target low-frequency 2nd order RLC band-pass filter shown in Figure 2. Note you can only use one capacitor ($C_x = 10\,\text{nF}$) and four resistors with equal resistance values ($R_x$).

   Recall in a 2nd order RLC band-pass filter, $\omega_0 = 1/\sqrt{L_0 C_0}$ and $Q = R_0/\sqrt{C_0 L_0}$. The center frequency of the RLC band-pass filter is $f_0 = 2\,\text{kHz}$, and its quality factor $Q = 25$. 

Figure 1

Figure 2
Solution 1. Both op-amps are ideal op-amps in negative feedback loops.

\[ V_2 = V_3 = V_4 \quad \Rightarrow I_5 = \frac{V_4}{Z_5} = -\frac{V_6}{Z_6} \]

Both op-amps are ideal op-amps with infinite input impedance.

\[ I_4 = I_5 = \frac{V_6}{Z_5} \quad \Rightarrow V_3 = (1 + \frac{Z_4}{Z_5}) V_4 = (1 + \frac{Z_4}{Z_5}) V_6 \]

\[ I_3 = \frac{V_4 - V_3}{Z_3} = \frac{V_3 - (1 + \frac{Z_4}{Z_5}) V_4}{Z_3} = -\frac{Z_6}{Z_5} V_6 \quad \text{and} \quad I_2 = I_3 \]

\[ V_1 = V_4 + Z_5 I_4 = V_4 + Z_5 I_2 = V_4 - \frac{Z_6}{Z_5} V_6 \]

\[ I_1 = \frac{V_4 - V_1}{Z_1} = \frac{V_3 - (V_4 - \frac{Z_6}{Z_5} V_6)}{Z_1} = \frac{Z_4}{Z_3} \frac{Z_4}{Z_5} \frac{1}{Z_1} V_6 \]

\[ I_5 = I_3 = \frac{V_6}{Z_5} = \frac{Z_6}{Z_5} V_6 \]

Theorem 1: \( V_1 = V_4 - \frac{Z_4}{Z_5} \frac{Z_6}{Z_5} V_6, \quad V_2 = V_4, \quad V_3 = (1 + \frac{Z_4}{Z_5}) V_4, \quad V_4 = V_6, \quad I_1 = I_3 = \frac{Z_4}{Z_3} \frac{Z_4}{Z_5} V_6, \quad I_5 = I_3 = \frac{V_6}{Z_5} \)

Question 2: \( Z_{in} = \frac{V_3}{I_3} = \frac{Z_3}{Z_5} \frac{Z_5}{Z_4} \)

Question 3: \( Z_{in} = R_1 R_4 R_3 \frac{1}{j \omega L_c R_4} = j \omega L \text{ left} \]

An effective positive inductor. Equivalent inductor value \( L_{\text{left}} = \frac{R_3 R_4}{R_4} C_s \)

Question 4: \( Z_{in} = \frac{R_3}{R_1 R_4} \frac{1}{j \omega L_c} = \frac{- \frac{R_3}{R_4}}{\omega L R_4 R_4 C_s} \)

An effective negative resistor. Equivalent resistor value \( R_{\text{left}} = -\frac{R_3}{\omega^2 R_4 R_4 C_s} \)
Question 5: \( L_0 = \frac{1}{\omega^2 C} = \frac{1}{\left(2\pi \times 2\times 10^3 \frac{rad}{s}\right)^2 \left(12 \times 10^{-9} F\right)} \)

\[ = 0.633 H \Rightarrow \text{This inductance value is too large to be implemented as a physical inductor.} \]

Using the circuit in Question 2, we can synthesize an active inductor with a value of \( L_{eq} = \frac{R_1 R_3 R_5 C_1}{R_4} = 0.633 H = R_{eq} C_{eq} \)

If \( C_2 = 10 mF = C_{eq} R_1 = R_3 = R_4 = R_5 = \sqrt{\frac{0.633 H}{10 mF}} = 7.96 k\Omega = R_{eq} \)
Consider an ideal silicon MOS capacitor, where the metal workfunction equals the semiconductor workfunction. The device is operated at room temperature (T = 300 K) under a bias voltage. Assume $n_i = 10^{10}/cm^3$, $kT = 0.026 \, eV$.

(a) Sketch the energy band diagram of the MOS capacitor assuming a Fermi potential $\phi_F = -0.3 \, V$ and a surface potential, $\phi_S = -0.7 \, V$ for the silicon. In your diagram, include all relevant energy levels (i.e., $E_F$ in the metal, $E_C$ and $E_V$ in the oxide, and $E_C$, $E_i$, $E_F$ and $E_V$ in the semiconductor.)

(b) Calculate carrier concentrations in the bulk silicon and also at the silicon-oxide interface. What is the biasing condition (Accumulation, Depletion, Inversion, Flat Band) and why?

(c) Assuming a potential of 0.2 V across the oxide layer, what is the gate voltage (i.e., the potential difference between the metal and the substrate)?
(a) Sketch the energy band diagram of the MOS capacitor assuming a Fermi potential \( \phi_F = -0.3 \) V and a surface potential, \( \phi_S = -0.7 \) V for the silicon. In your diagram, include all relevant energy levels (i.e., \( E_F \) in the metal, \( E_C \) and \( E_V \) in the oxide, and \( E_C, E_i, E_F \) and \( E_V \) in the semiconductor.)

(b) Calculate carrier concentrations in the bulk silicon and also at the silicon-oxide interface. What is the biasing condition (Accumulation, Depletion, Inversion, Flat Band) and why?

\[
\phi_F = \frac{1}{q}(E_{i,bulk} - E_F) = -0.3 \text{ V} \Rightarrow E_F - E_{i,bulk} = 0.3 \text{ eV}
\]

In the bulk Si: \( n = n_i e^{[E_F-E_{i,bulk}]/kT} = 10^{10} e^{0.3 eV/0.026 eV} \approx 1.026 \times 10^{15}/cm^3 \)

\[
p = \frac{n_i^2}{n} = \frac{10^{20}}{1.026 \times 10^{15}} \approx 9.75 \times 10^4/cm^3
\]

\[
\phi_S = \frac{1}{q}(E_{i,bulk} - E_{i,surf}) = -0.7 \text{ V} \Rightarrow E_{i,surf} - E_F = 0.4 \text{ eV}
\]

Si-Ox interface: \( n = n_i e^{[E_F-E_{i,surf}]/kT} = 10^{10} e^{-0.4 eV/0.026 eV} \approx 2.08 \times 10^3/cm^3 \)

\[
p = \frac{n_i^2}{n} = \frac{10^{20}}{2.08 \times 10^3} \approx 4.81 \times 10^{16}/cm^3
\]

MOS capacitor is biased in the inversion regime. \( n \)-type silicon substrate is inverted into \( p \)-type at the silicon-oxide interface under the applied bias.

(c) Assuming a potential of 0.2 V across the oxide layer, what is the gate voltage (i.e., the potential difference between the metal and the substrate)?

\[
V_G = \Delta \phi_{\text{silicon}} + \Delta \phi_{\text{oxide}} = -0.7 \text{ V} - 0.2 \text{ V} = -0.9 \text{ V}
\]
Consider the feedback system shown in the figure below. The plant has the transfer function \( G(s) = \frac{1}{(s+1)^2} \), and the controller has the transfer function \( G_c(s) = \frac{K_I}{s} + K_P \) for constants \( K_I > 0 \) and \( K_P > 0 \).

1. Let \( K_P = 3 \). Let the input \( r(t) \) be the ramp function, i.e., \( r(t) = t \) for every \( t \geq 0 \). Compute the range of \( K_I \) such that the steady-state error, defined by \( e_{ss} := \lim_{t \to \infty} e(t) \), satisfies the inequality \( e_{ss} < 0.1 \).

2. Same as part 1, except that \( K_P = 7 \).

Please note:

1. The closed-loop system must be stable.

2. The range you are requested to compute may be empty. In that case, explain why.
For stability, all zeros of the equation

\[ 1 + G_c(s) G(s) = 0 \] must be in the LHP

\[ 1 + \left( \frac{K_I}{s} + K_p \right) \left( \frac{1}{s+1} \right)^2 = 0 \]

\[ 1 + \frac{K_I + K_p s}{s(s+1)^2} = 0 \]

\[ s^3 + 2 s^2 + s + K_p s + K_I = 0 \]

\[ s^3 + 2 s^2 + (K_p + 1) s + K_I = 0 \]
Routh Test:

\[ \begin{align*}
1 & \quad k_p + 1 \\
2 & \quad k_I \\
\frac{2(k_p + 2 - k_I)}{2} & \quad k_I \\
\end{align*} \]

For stability, \( k_I < 2k_p + 2 \).

For tracking: The \( r-e \) transfer function is

\[ H_e(s) = \frac{1}{1 + CE(s)G(s)} = \frac{1}{1 + \frac{k_I + k_p s}{s(s+1)^2}} = \]
\[
\frac{S (S+1)^2}{S (S+1)^2 + K_I + K_P S}
\]

If \( r(t) = t \), \( R(s) = \frac{1}{s^2} \)

\[\Rightarrow \quad E(s) = H_c(s) \frac{1}{s^2} = \]

\[= \frac{1}{s} \frac{(S+1)^2}{S (S+1)^2 + K_I + K_P S} \]

\[\Rightarrow \text{If the closed-loop system is stable,} \]

\[E_{ss} = \frac{(S+1)^2}{S (S+1)^2 + K_I + K_P S} \bigg|_{S=0} = \frac{1}{K_I} \]

\[E_{ss} < 0.1 \quad \iff \quad \frac{1}{K_I} < 0.1 \quad \iff \quad K_I > 10.\]
Combining the two conditions,

\[ 10 < k_I < 2k_p + 2. \]

1. \( k_p = 3 \) \[\rightarrow\] \[ 10 < k_I < 8 \]
   
The range is empty.

2. \( k_p = 7 \) \[\rightarrow\] \[ 10 < k_I < 16 \]
(a) (6pts) Consider a network, where a node Z has three neighbors: W, X, and Y. The link cost from Z to W is 4, the link cost from Z to X is 3, and the link cost from Z to Y is 2. Suppose distance vector routing algorithm is used. Suppose W, X and Y have a part of the distance tables shown below, where p and q are two of the destinations. Each quantity in the table shows the minimum distance from the node to a destination via one of the neighbors. For example, “3” in the first row of Dw is the distance from W to p via neighbor s1.

\[
\begin{array}{c|ccc}
    & s_1 & s_2 & s_3 \\
\hline
    p & 3 & 12 & ?  \\
    q & 9 & ? & 7  \\
\end{array}
\quad
\begin{array}{c|ccc}
    & t_1 & t_2 & t_3 \\
\hline
    p & ? & 6 & 8  \\
    q & 12 & 4 & ?  \\
\end{array}
\quad
\begin{array}{c|ccc}
    & u_1 & u_2 & u_3 \\
\hline
    p & ? & 5 & 8  \\
    q & 10 & ? & 9  \\
\end{array}
\]

The following distance table in node Z is given after node Z receives the above three distance tables from its neighbors W, X and Y. Fill in a number for every “?” in W, X, and Y tables.

\[
\begin{array}{c|ccc}
    & W & X & Y \\
\hline
    p & 6 & 7 & 4  \\
    q & 7 & 5 & 5  \\
\end{array}
\]

(b) (1pts) Suppose a packet arrives at node Z with a destination p. What is the next node for which node Z forwards the packet to? Why?

(c) (3pts) Write an equation that you use to obtain the above results. Please define all variables you use in the equation clearly.

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SOLUTION

<table>
<thead>
<tr>
<th>D_w</th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>3</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>q</td>
<td>9</td>
<td>3</td>
<td>7</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>D_x</th>
<th>t1</th>
<th>t2</th>
<th>t3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>q</td>
<td>12</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D_y</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>q</td>
<td>10</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

(b) The next node is Y. The reason is that the packet is forwarded based on the shortest distance.

(c) Let \( u \) and \( w \) be a source and a destination respectively. Let \( C(u, v) \) be the link cost between node \( u \) and neighbor \( v \). Let \( d_a(b) \) be the shortest distance between nodes \( a \) and \( b \). Then the equation used is Bellman-ford equation

\[
d_u(w) = \min_v [C(u, v) + d_v(w)].
\]
Let \( X[k] \) be the 6-point DFT of the signal \( x[n] \). \( X[0] = 1 \), \( X[1] = 1 + j \), \( X[2] = j \), \( X[3] = a \), \( X[4] = b \), \( X[5] = c \). (Or, \( x=[1,1+j,j,a,b,c] \))

For all parts except (a) and (b), express your answer using the symbols \( a, b, c \). All parts are mutually independent.

(2 points) (a) If \( x[n] \) is real, find all possible values for \( a, b, \) and \( c \). You must explain your reasoning.

(2 points) (b) If \( x[n] = 0 \) for \( n \) odd, find all possible values for \( a, b, \) and \( c \). Do not assume \( x[n] \) is real. You must explain your reasoning.

(2 points) (c) Find \( x[0] \).

(2 points) (d) Let \( y[n] = x[n] \cos(\pi n) \). Find \( Y[k] \), the 6-point DFT of \( y[n] \).

(2 points) (e) Let \( w[n] \) be the length-3 signal where \( w[n] = x[n] + x[n+3] \) for \( n = 0, 1, 2 \). Find \( W[k] \), the 3-point DFT of \( w[n] \). (If you think about the problem, you should be able to solve it without any computation).

For all parts except (a) and (b), express your answer using the symbols $a, b, c$.

(2 points) (a) If $x[n]$ is real, find possible values for $a, b, c$. You must explain your reasoning.

For even $x[n]$, $X[k]$ must be conjugate symmetric modulo 6.

- $X_{0}=X_{3}$ real
- $X_{1}=-X_{5}^{*}$

$a$ must be real
$b = j^0 = -j = 1/\sqrt{2}$
$c = (1+j)^2 = 1-j$

(2 points) (b) If $x[n] = 0$ for $n$ odd, find possible values for $a, b, c$. Do not assume $x[n]$ is real. You must explain your reasoning.

METHOD 1: From the definition in time a algorithm, $X[k] = X[k+3]$

METHOD 2: $X[k] = \sum_{n=0}^{5} x[n] e^{-j2\pi kn/6} = \sum_{n=0}^{5} x[2n] e^{-j\pi kn}$

$\implies a = 1, b = 1+j, c = j$

(2 points) (c) Find $X[0]$.

$X[0] = \frac{1}{6} \sum_{k=0}^{5} X[k] = \frac{1}{6} (1 + 1+j + 1+a + b + c)$

$= \frac{1 + j + 2 - a - b + c}{6}$

(2 points) (d) Let $y[n] = x[n] \cos(\pi n)$. Find $Y[k]$, the 6-point DFT of $y[n]$.

$\cos(\pi n) = (-1)^n$

$X(k) \cos(\pi n) = \text{DFT} \left( \sum_{k=0}^{5} X[k] e^{-j\pi kn} \right) = \text{DFT} \left( \sum_{k=0}^{5} (X[k] e^{-j\pi kn}) \right)

\implies Y[k] = \begin{bmatrix} a & b & c & 1 & 1+j, j \end{bmatrix}$

(2 points) (e) Let $w[n]$ be the length-3 signal where $w[n] = x[n] + x[n+3]$ for $n = 0, 1, 2$. Find $W[k]$, the 3-point DFT of $w[n]$. (If you think about the problem, you should be able to solve it without any computation).

$W[k]$ is a time aliased version of $X[n]$ modulo 2

$-W[k]$ is $X[n]$ downsampled by 2.

$W[k] = \begin{bmatrix} 1 & 0 \end{bmatrix}$
Assume an ideal parallel-plate transmission line filled with air and having a plate separation of $a=15\text{mm}$. What is the dominant mode?
(a) Write an expression for the cutoff frequency and the modal characteristic impedance of the TEn modes.
(b) Repeat the same for the TMn modes.
(c) How many higher-order TE modes would propagate if the structure was excited by an incident wave with frequency 33 GHz?
a = 15mm = 15 \times 10^{-3} \text{m}, \text{ Air filled: } \mu_r = \varepsilon_r = 1, \text{ and dominant mode is TEM with cutoff frequency of 0 GHz.}

(a) \( f_{C,TE_n} = \frac{n}{2a\sqrt{\varepsilon \mu}} = \frac{n \times 3 \times 10^8}{2 \times 15 \times 10^{-3}} = 10 \cdot n \text{ GHz} \)

\[ Z_{TE_n} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{f_{C,TE_n}}{f}\right)^2} \]

(b) \( f_{C,TM_n} = f_{C,TE_n} = \frac{n}{2a\sqrt{\varepsilon \mu}} = \frac{n \times 3 \times 10^8}{2 \times 15 \times 10^{-3}} = 10 \cdot n \text{ GHz} \)

\[ Z_{TM_n} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{f_{C,TM_n}}{f}\right)^2} \]

(c) \( n =\frac{n}{2a\sqrt{\varepsilon \mu}} = f_{C,TM_n} = f_{C,TE_n} \leq 33\text{GHz} \)

10 \cdot n \leq 33 \text{ GHz}

n \leq 3.3 \text{ GHz}

\( \therefore \) 3 TE\(_n\) modes can propagate.
Arrayed Waveguide Grating Design

Arrayed waveguide gratings (AWGs) are used in fiber optic network telecommunications as multiplexers, demultiplexers, add-drop filters, and as $N \times N$ switches. The device, shown in the attached figure, consists of an array of channel waveguides with the optical path length of adjacent waveguides differing by a constant value. At the ends of the array of waveguides are input and output region slab waveguides. These slab waveguide regions, in turn, are connected to input and output channel waveguides that are linked to individual optical fibers. The arrayed waveguide grating can be conveniently implemented as silica waveguides on a silicon wafer.

The arrayed channel waveguides are spaced $d$ apart at the output slab waveguide. Each wavefront propagates a distance $L_f$ (focal length) across the output slab waveguide and converges to a output channel waveguide. The output channels are spaced $\Delta x$ apart. From the grating equation for the AWG, it can be shown that the angular dispersion for a wave of frequency $f$ in the output slab waveguide region is given by

$$\frac{d\theta}{df} = \frac{m \lambda^2 N_{g,ch}}{N_{sl} d c N_{ch}}$$

for small angles of deviation where $m$ is the diffraction order, $\lambda$ is the center wavelength (enters the center input channel waveguide and exits the center output channel waveguide), $N_{sl}$ is the effective index of the slab waveguide, $N_{ch}$ is the effective index of the channel waveguides, and $N_{g,ch}$ is the group velocity index of the channel waveguides given by $N_{g,ch} = N_{ch} - \lambda \frac{d N_{ch}}{d \lambda}$.

For a particular practical AWG, $d = 25 \mu m$, $\Delta x = 25 \mu m$, $m = 118$, $\lambda = 1553.81 \ nm$, $N_{sl} = 1.4529$, $N_{ch} = 1.4513$, and $N_{g,ch} = 1.4752$. Further, this AWG is to be designed to demultiplex communication channels that are separated by $100.0 \ GHz$. For this AWG, calculate in $deg/\ GHz$, showing all work, the magnitude of the angular dispersion in the output slab waveguide region for small angles of deviation. Calculate in microns the magnitude of the focal length $L_f$ needed for this AWG. Also, calculate in nanometers the freespace wavelength separation in $nm$ between adjacent output channel waveguides. Express your answers accurately to four significant figures. Put your final answers in the spaces provided below.

Angular dispersion = \(\frac{1}{100.0} \) deg/\ GHz.

Focal length, $L_f =$ \(25 \) $\mu m$.

Wavelength separation between channel waveguides = \(100.0\) nm.
Angular dispersion

\[
\frac{d\theta}{df} = -\frac{m \lambda^2 N_{g,ch}}{N_{sl} d c N_{ch}}
\]

\(m = 118\)
\(\lambda = 1553.81 \text{ nm}\)
\(N_{g,ch} = 1.4752\)
\(N_{sl} = 1.4529\)
\(d = 25 \mu m\)
\(N_{ch} = 1.4513\)
\(\Delta f = 100 \text{ GHz}\)
\(\Delta x = 25 \mu m\)

and so

\[
\frac{d\theta}{df} = -\frac{m \lambda^2 N_{g,ch}}{N_{sl} d c N_{ch}} = -2.6593 \times 10^{-5} \text{ rad/ GHz} = -1.5237 \times 10^{-3} \text{ deg/ GHz}
\]

Angle between adjacent output channel waveguides

\[
\text{Angle} = \frac{\Delta x}{L_f} = \frac{d\theta}{df} \Delta f
\]

Focal length

\[
L_f = \frac{\Delta x}{\Delta f} \left( \frac{d\theta}{df} \right)^{-1} = 9401 \mu m = 9.401 \text{ mm}
\]

Wavelength separation (using \(\lambda = c/f\))

\[
d\lambda = c \left( -\frac{1}{f^2} \right) df = -\frac{\lambda^2}{c} df = 0.80703 \text{ nm}
\]
An optical fiber communication link using binary “on-off keying” signal is assembled with an optical transmitter, a single mode fiber link of 50km with transmission loss of 0.22 dB/km, and a direct detection receiver. The optical transmitter consists of a cw (continuous wave) laser transmitter whose output is externally modulated by an optical modulator to produce the binary NRZ signal. At the other end, an optical receiver is used in which thermal noise is the dominant noise contributor (all other noise mechanisms are negligible). At room temperature (300 °K), the noise current is \( \sigma_I = 75.0 \, nA \). The average input optical power at the receiver is \( P_{rec} = 2.0 \, \mu W \), with incomplete extinction – that is, the modulator does not completely block the laser power. As a result, 20% of the optical power resides in the “zero” bit slots, with the rest in the “one” bits. Upon detection, the resulting current levels in the “one” and “zero” bits are \( I_1 = 0.8 \, \mu A \) and \( I_0 = 0.2 \, \mu A \) respectively.

a. Draw a system diagram of this optical fiber communication link with every element labelled; and estimate how much output power of the optical transmitter is required.

b. Evaluate \( Q = (I_1 - I_0)/(\sigma_1 + \sigma_0) \).

where \( \sigma_1 \) and \( \sigma_0 \) represent the total noise current at “one” and “zero”, respectively.

It is desired to operate the transmission link at a BER of \( 10^{-10} \), which means that \( Q \) must attain a value of 7. This is to be accomplished by doing two things simultaneously:

1) cool the receiver to \( T = 200 \, ^\circ K \) with a thermal electric cooler, and
2) increase the average input optical power at the receiver by reducing the length of optical fiber link.

c. What is the new required average optical power, and what is the fiber length reduction needed to accomplish this?

d. Suppose an ideal optical modulator is used, such that complete extinction is achieved (zero power in the “zero” bit slots), but with the output power of optical transmitter unchanged. With this accomplished, in addition to the aforementioned temperature reduction, re-answer part c.
a. Draw a system diagram of this optical fiber communication link with every element labelled; and estimate how much output power of the optical transmitter is required for $P_{\text{rec}}=2.0$ uW.

**Answer:** The fiber transmission loss = $50\text{ km} \times 0.22 \text{ dB/km} = 11 \text{ dB}$

The required transmitter output power $P_{\text{tx}}$ can be calculated from:

$$10\log \left( \frac{P_{\text{tx}}}{P_{\text{rec}}} \right) = 11$$

$$\therefore \quad P_{\text{tx}} = 2.0 \text{ uW} \times 10^{1.1} = 25.2 \text{ uW}$$

b. Evaluate $Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$.

**Answer:** with thermal noise dominant, $\sigma_1 = \sigma_0 = \sigma_T$

$$\therefore \quad Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{(0.8 - 0.2)}{(0.075 + 0.075)} \mu\text{A} = 4$$

It is desired to operate the transmission link at a BER of $10^{-10}$, which means that $Q$ must attain a value of 7. This is to be accomplished by doing two things simultaneously:

1) cool the receiver to $T = 200 \text{ K}$ with a thermal electric cooler, and
2) increase the average input optical power at the receiver by reducing the length of optical fiber link.

c. What is the new required average optical power, and what is the fiber length reduction needed to accomplish this?

Thermal noise current is in proportion to $T^{1/2}$, so the new noise current will be

$$\sigma_T = 200 \text{ K} = \sigma_T = 300 \text{ K} \times (200/300)^{1/2} = 75 \text{ nA} \times 0.817 = 61.24 \text{ nA}$$

With this change alone, Q value is now increased to $Q' = 4 \times (75\text{nA}/61.2\text{nA}) = 4.9$

To further increase Q to a value of 7, the currents must be increased by a factor of $7/4.9 = 1.43$. The received optical power must therefore be increased by this same factor. So the new received power will be $P_{\text{rec}} = 2.0 \times 1.43 = 2.86 \mu\text{W}$. The required loss reduction in dB from the fiber link is thus

$$P_{\text{reduction}}[\text{dB}] = 10\log (1.43) = 1.55 \text{dB}$$

The reduction in fiber length is $1.55 \text{dB}/0.22\text{dB} = 7 \text{ km}$
d. Suppose an ideal optical modulator is used, such that complete extinction is achieved (zero power in the “zero” bit slots), but with the output power of optical transmitter unchanged. With this accomplished, in addition to the aforementioned temperature reduction, re-answer part c.

Answer: With complete extinction, \( I_0 = 0 \), and \( Q \) with temperature reduction by a thermal electric cooler becomes:

\[
Q = \frac{(I_1 - I_0)}{(\sigma_1 + \sigma_0)} = \frac{(I_1 - 0)}{(\sigma_1 + \sigma_0)} = 0.8 \ \text{uA} / (0.061 + 0.061) \ \text{uA} = 6.67
\]

So the use of an ideal modulator plus temperature reduction has not obtained the desired \( Q \) value of 7. To further increase \( Q \) to a value of 7, the currents must be increased by a factor of \( 7/6.67 = 1.05 \). The received optical power must therefore be increased by this same factor.

So the new received power will be increased to \( P_{\text{rec}} = 2.0 \times 1.05 = 2.10 \mu\text{W} \). The required loss reduction in dB from the fiber link is thus

\[
P_{\text{reduction}}[\text{dB}] = 10 \times \log (1.05) = 0.21 \text{ dB}
\]

The reduction in fiber length for this case is: \( 0.21 \text{dB}/0.22 \text{dB} = 1 \text{ km} \)
The intracellular and extracellular concentrations and conductances for the axon of a newly discovered squid species at rest are given below.

A) Find the Nernst potential for Na\(^+\), K\(^+\), and Cl\(^-\).
B) Find the membrane resting potential.
C) What direction does each ion flow when the membrane is in equilibrium if \(V_m\) @ rest is -60 mV?

<table>
<thead>
<tr>
<th>Species</th>
<th>Intracellular (mM)</th>
<th>Extracellular (mM)</th>
<th>Conductances (mS/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>500</td>
<td>10</td>
<td>0.415</td>
</tr>
<tr>
<td>Na</td>
<td>70</td>
<td>350</td>
<td>0.010</td>
</tr>
<tr>
<td>Cl</td>
<td>24</td>
<td>350</td>
<td>0.582</td>
</tr>
</tbody>
</table>
A) \( E_{Na} = -\frac{kT}{q} \ln \left( \frac{70}{350} \right) = 40.2 \, \text{mV} \)

\( E_K = -\frac{kT}{q} \ln \left( \frac{500}{10} \right) = -97.8 \, \text{mV} \)

\( E_{Cl} = -\frac{kT}{q} \ln \left( \frac{24}{350} \right) = -67.0 \, \text{mV} \)

B) \( V_{rest} = \frac{g_K E_K + g_{Na} E_{Na} + g_{Cl} E_{Cl}}{g_K + g_{Na} + g_{Cl}} \)

\( = -78.6 \, \text{mV} \)

C) \( V_{rest} = -60 \, \text{mV} \)

\( I_K = g_K (V_{m} - E_K) = 0.415 \, ( -60 - -97.8 ) > 0 \Rightarrow K^+ \text{ flow outward} \)

\( I_{Na} = g_{Na} (V_{m} - E_{Na}) = 0.010 \, ( -60 - -40.2 ) < 0 \Rightarrow Na^+ \text{ flow inward} \)

\( I_{Cl} = g_{Cl} (V_{m} - E_{Cl}) = 0.582 \, ( -60 - -67 ) > 0 \Rightarrow Cl^- \text{ flow inward} \)
(10 pts) Movement Sensing. In assessing a person’s overall health / wellness, it is important to quantify a person’s movement. Typically, this would include multiple factors, such as the amount the person moves, the way the person moves, the symmetry between movement patterns on both sides of the body, etc. In two sentences or less, describe qualitatively what wearable sensor(s) you can use to quantify some of these parameters of movement as unobtrusively as possible, without getting in the way of the person’s activities.

Draw a block diagram depicting the processing steps for extracting the amount a person moves from this / these sensor(s). Be clear with your labeling, and specific with the blocks and their functions.
In **three sentences or less**, describe the steps you would use for an automated algorithm to quantify the **way** the person is moving from the sensor data. Be specific – statements such as, “I will input the sensor data into a machine learning algorithm” will not be accepted.

Why, from a physiological / health standpoint, do you think that the symmetry of the movement patterns of both sides of the body important? Limit your answer to **two sentences or less**.
(10 pts) *Movement Sensing.* In assessing a person’s overall health / wellness, it is important to quantify a person’s movement. Typically, this would include multiple factors, such as the amount the person moves, the way the person moves, the symmetry between movement patterns on both sides of the body, etc. In **two sentences or less**, describe qualitatively what wearable sensor(s) you can use to quantify some of these parameters of movement as unobtrusively as possible, without getting in the way of the person’s activities.

Multiple answers are possible for this question, but likely will involve the use of accelerometers or inertial measurement units. Qualitatively, the measurement of body acceleration at the limbs can be used to count the number of steps, estimate walking speed, assess gait, etc.

Draw a block diagram depicting the processing steps for extracting the amount a person moves from this / these sensor(s). Be clear with your labeling, and specific with the blocks and their functions.

An example block diagram that could work for this is shown below. This is just one example, as many solutions are possible. The important thing is to show the flow from signals (on the left) to some pre-processing and feature extraction, leading to signal parameters related to health (on the right). One or two sentences supporting what these parameters relate to (e.g., in this case the amount of movement (RMS power), frequency of steps taken (peak frequency), and some quantification of walking pattern (harmonic content)).
In **three sentences or less**, describe the steps you would use for an automated algorithm to quantify the **way** the person is moving from the sensor data. Be specific – statements such as, “I will input the sensor data into a machine learning algorithm” will not be accepted.

There are again many solutions. Good answers could include some discussion of using multiple accelerometers on the body rather than just one to quantify limb movements, joint angles, gait parameters, etc. It would also be good to include some information about what constitutes the “way” that a person moves. This could include for example the variability in the movements for a given task such as walking, etc.

Why, from a physiological / health standpoint, do you think that the symmetry of the movement patterns of both sides of the body important? Limit your answer to **two sentences or less**.

This again has many possible answers, but these could include say having an injury on one side leading to asymmetry because of discomfort / pain, arthritis in the joints affecting one side or the other, etc.