INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 22 numbered problems. Note that Problem 2 occupies 2 pages, Problem 3 occupies 2 pages, Problem 12 occupies 2 pages, Problem 20 occupies 2 pages, Problem 22 occupies 3 pages. Including the cover sheet, you should have 29 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!!

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 14 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:

   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in EVERY sheet you are turning in.
   (c) On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.
   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

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Problem 1 (Core: VLSI - ECE 2020)  

PROBLEM

Answer all the parts of the question.

a. (2 pts each) Use algebraic techniques to simplify:
   i. \( F = X(A + B + CDE + FG + W + X + YZ) \)
   ii. \( F = AB + \overline{AC} + BCD \)

b. (3 pts) Consider the following Boolean Function:
   \[
   F_{(A,B,C,D,E)} = \overline{(A + B)(C + D)} + \overline{(C + D)} + E
   \]
   Use mixed logic design style to implement the function using OR gates and INVERTERs only.

c. (3 pts) Consider a latch with an input signal (In) and a enable signal (Enable). The latch is transparent when the enable signal (Enable) is high and is blocking when the enable signal (Enable) is low. Draw the timing diagram corresponding to the input pattern (In) and the Enable signal (Enable) as shown below. Consider zero delay between the input and the output.
When answering the five parts on the next page, consult the following resource:

Each of the magnitude only frequency response plots (A–F) in the figure below corresponds to one of the systems $S_1 - S_7$, specified by either a system function $H(z)$ or a difference equation.

$S_1 : \quad y[n] = 0.6y[n - 1] + 4x[n] - 4x[n - 1]$

$S_2 : \quad y[n] = 0.8y[n - 1] + 0.5x[n] + 0.5x[n - 1]$

$S_3 : \quad H(z) = \frac{0.5(1 - z^{-1})}{1 + 0.8z^{-1}}$

$S_4 : \quad y[n] = \frac{1}{2}x[n] + \frac{5}{4}x[n - 1] + \frac{3}{2}x[n - 2] + \frac{5}{4}x[n - 3] + \frac{1}{2}x[n - 4]$

$S_5 : \quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}$

$S_6 : \quad y[n] = \sum_{k=0}^{3} x[n - k]$

(a) For Frequency Response (A), determine the output when the input is $x[n] = 3$ for $-\infty < n < \infty$.

(b) For Frequency Response (B), determine the impulse response of the system.

(c) Determine which system has the frequency response (C).

(d) For Frequency Response (E), make a pole-zero plot for the corresponding system.

(e) The frequency response (magnitude) of system $S_4$ is the plot shown in (F). The impulse response of $S_4$ is $h_4[n]$ and the length of $h_4[n]$ is finite. If $h_4[n]$ is zero-padded to a length of 100, and a 100-point DFT is computed, the result is $H_{100}[k]$. Explain how to use the plot in (F) to obtain an estimate of the magnitude of the DFT at $k = 90$ without computing the DFT.

$|H_{100}[90]| \approx \boxed{4}$
Problem 3 (Core: CSS - ECE 2036)  

What is printed by the program below?

```cpp
#include <iostream>
using namespace std;

class Base {
   // Define a base class
   public:
      virtual void Sub1() = 0;
      void Sub2();
      virtual void Sub3();
      virtual void Sub4();
   };

class A : public Base {
   // Class A derives from Base
   public:
      void Sub1();
      void Sub2();
      void Sub4();
   };

class B : public Base {
   // Class B derives from Base
   public:
      void Sub1();
      void Sub2();
      void Sub3();
   };

   // Base Class Methods
   void Base::Sub2() { cout << "Hello from Base::Sub2()" << endl; }
   void Base::Sub3() {
      cout << "Hello from Base::Sub3()" << endl;
      Sub1(); // DON'T MISS THIS CALL IN YOUR ANSWER
      Sub4(); // DON'T MISS THIS CALL IN YOUR ANSWER
   }
   void Base::Sub4() { cout << "Hello from Base::Sub4()" << endl; }

   // Class A Methods
   void A::Sub1() { cout << "Hello from A::Sub1()" << endl; }
   void A::Sub2() { cout << "Hello from A::Sub2()" << endl; }
   void A::Sub4() { cout << "Hello from A::Sub4()" << endl; }

   // Class B Methods
   void B::Sub1() { cout << "Hello from B::Sub1()" << endl; }
   void B::Sub2() { cout << "Hello from B::Sub2()" << endl; }
   void B::Sub3() { cout << "Hello from B::Sub3()" << endl; }

   // A Helper Subroutine
   void SubP(Base* x) {
      x->Sub1();
      x->Sub3();
      x->Sub4();
   } // CONTINUED ON NEXT PAGE
```

Program q5.cc
// Another helper
void SubR(Base& x)
{
    x.Sub2();
    x.Sub3();
    x.Sub4();
}

// Another Helper
void SubV(B b0)
{
    b0.Sub1();
    b0.Sub2();
}

int main()
{
    A a;
    B b;
    SubP(&a);
    SubR(b);
    SubV(b);
}

Program q5.cc (continued)
The circuit shown below is at steady state before the switch closes. Determine the capacitor voltage $v(t)$ for $t > 0$. 

![Circuit Diagram](image-url)
Consider the problem of turning an ordinary array of numbers into a heap.

a) Describe in words an efficient method to perform this conversion. Your method should be faster than simply inserting all elements from the array into a heap. Analyze the running time of your solution for an \( n \)-element heap and compare it to the running time of the method that inserts all of the elements one-by-one into a heap.

b) Execute your method on the below array. Show several intermediate stages of your method on the array (using any representation you choose) but draw the final heap using the array implementation of a heap.

\[
\begin{array}{cccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
5 & 13 & 17 & 2 & 9 & 8 & 26 & 19 & 31 & 8 & 4 & 16 & _ & _ & _
\end{array}
\]
Problem 6 (Core: EMAG - ECE 3025)  

**Problem**

Consider the below circuit. A long transmission line with impedance 50 Ω carries an AC steady state sinusoidal voltage from a high power generator to a RADAR antenna. The antenna is represented as a 200 Ω resistor. To eliminate reflections that could return to the generator and damage it, a stub is inserted in between the line and load that is exactly one quarter of a wavelength long, with impedance $Z_0$. This technique is called quarter-wave matching.

![Circuit Diagram]

(a) What value of $Z_0$ ensures perfect matching, i.e., none of the power from the generator returns back to it?

(b) What is the voltage standing wave ratio (VSWR) in the 50 Ω line, and in the quarter-wave stub?

(c) Using the principles of transmission line theory, explain qualitatively why this technique is successful in eliminating reflections. How does it work?
You are given a system with a clock rate of 1 GHz. Registers have a setup and hold time of 5 ps. The settling time constant for the registers is $\tau = 10\, ps$. What is the probability of failure if the signal is given one clock period to resolve?
A non-ideal operational amplifier shown here has an open loop gain at low frequencies of 100 v/v, an input impedance of 20K ohms, an output impedance of 50 ohms and an open loop low pass bandwidth of 200 Hz. This amplifier is used in a non-inverting configuration to achieve a low frequency voltage gain of 10 v/v. If R2=91K ohms:

a) (4 points) Showing your work for full credit, what is the required value of R1?

b) (2 points) Showing your work for full credit, what is the new closed loop amplifier bandwidth?

c) (2 points) Showing your work for full credit, what is the new closed loop amplifier input impedance?

d) (2 points) Showing your work for full credit, what is the new closed loop amplifier output impedance?
Consider the 3-phase, 230 V, 50 kVA, load operating at power factor 0.8 (lagging), which as shown in the following figure, is supplied by three 20 kVA, 1330/230 V, 50 Hz transformers connected in Y-Δ by means of a common 3-phase feeder whose impedance is 0.002 + j0.01 Ohms per phase. The transformers are supplied from a 3-phase source through a 3-phase feeder whose impedance is 0.5 + j5 Ohms per phase. The equivalent impedance of one transformer referred to the low-voltage side is 0.1 + j0.25 Ohms. Determine the required supply voltage (phase-to-phase) if the load voltage is 230 V.
Problem 10 (Core: DSP/TLCOM - ECE 3077) Code Number: _________

PROBLEM

Light bulbs manufactured by a certain company are known to last 200 hours on the average. It is assumed that the lifetime $T$ of a particular light bulb has an exponential distribution, i.e., its pdf is of the form

$$f_T(t) = \alpha e^{-\alpha t}, \quad t \geq 0.$$ 

a) What fraction of the bulbs manufactured will last longer than 250 hours?

b) Bulbs are tested one after the other to see if they last longer than 250 hours. Let $X$ be the number of trials to obtain the tenth bulb lasting longer than 250 hours. What is the probability that $X = 20$?

c) Bulbs cost $1.25 to manufacture and are sold for $2.50. If a bulb burns out before 100 hours the customer is entitled to a full refund. What is the expected profit per bulb?
The integral shown below, which is a function of $t$, will not yield to the usual techniques of freshman calculus. Solve and simplify it using your knowledge of Fourier transform theory. Explain your reasoning.

\[
\frac{1}{\pi^2} \int_{-\infty}^{\infty} \frac{\sin(800\pi a) \sin(400\pi (t - a))}{a(t - a)} da =
\]
Problem 12 (Breadth: VLSI - ECE 3150)  
Code Number: _________

**Problem**

Consider the Boolean function \( Z = a \oplus b \oplus c \oplus d \) (XOR4) which is also called the odd parity function (true if an odd number of inputs are true). Please assume an equal worst-case rise/fall time \((\gamma = 2)\). Please use \( C_{\text{inv}} = 3C_{\text{fet}} \) as the input capacitance of a minimum size inverter, and express all delay in units of \( \tau = R_{n}C_{\text{inv}} \) where \( R_{n} \) is the equivalent resistance of a minimum size nfet (with gate width and length \( L = L_{\text{min}} \) and \( W = W_{\text{min}} \) respectively).

a. (3pts) Consider a symmetric mirror form complex gate for this function, where each **product** term in the switching function of the pullup corresponds to a 1 of \( Z \) (a minterm), and each **product** term in the switching function of the pulldown represents a 0 of \( Z \) (a maxterm). Assume that each true and complemented literal is available. Give the per input logical effort \( g \) for this gate. You do not need to draw the transistor schematic but you should show how you arrived at your answer. (Note that there are 8 inputs for this gate).

b. (2pts) Consider a minimum size version of this gate. Give the delay of this gate driving a minimum size inverter (include both effort and parasitic components of delay).
c. (5pts) Consider an implementation of Z (not necessarily the gate in a.) designed to minimize delay driving a very large load of $10,000C_{inv}$. Assume each necessary input is the output of a minimum size inverter. Estimate the worst case delay of this implementation from input inverter to output load. You need not design the entire implementation, but you should sketch the approach.
Problem 13 (Breadth: POWER - ECE 3300)  

The illustrated system comprises three synchronous machines. SM1 operates as a generator while machines SM2 and SM3 are a motor generator pair (common shaft, speed of machines SM2 and SM3 are identical). In addition assume that the generated voltage in SM2 and SM3 are equal and the power factor of SM2 is unity. Further neglect losses in the motor/generator set so that the real power input to the motor equals the real power output of the generator. The illustrated circuit is the per-phase equivalent model of the system.

The system operates in steady state conditions. The constant power load absorbs 1.2 MW (one phase) with a power factor of 0.95 (current lagging) while the terminal voltage is 2,400 V, 50 Hz (phase to neutral).

Computed the generated real and reactive power of synchronous machine SM1.

Use a simple synchronous machine model, i.e. constant voltage behind a synchronous impedance. Each of the three synchronous machines has a per phase inductance of 0.0015 Henries.

Hint: Compute the frequency of the currents and voltages in SM1 and SM2.
In the following circuit, the amplifier output resistance is 500 $\Omega$, and $A(s)$ is given by:

$$A(s) = \frac{10^7}{s+50}$$

(a) Find an expression for the loop gain.

(b) What is the maximum load capacitance, $C_L$, which can be connected to the amplifier if the phase margin is to be at least 50°?

(c) How do you change the amplifier to be able to increase the load capacitance by a factor of 10 while maintaining the 50° phase margin?
PART I: (5 pts)
Consider an electron moving in a hypothetical semiconductor crystal with a lattice constant = 0.730 nm. You may assume that the electron scattering limited velocity is $1 \times 10^8$ cm/s. Give a compelling argument (using numbers) that the proof of the existence of band structure requires solution of the Schrödinger equation in this periodic system. Take $h = 6.634 \times 10^{-34}$ J-s, and $m^* = 9.11 \times 10^{-31}$ kg. For credit you must carefully show how you got your answer.

PART II: (5 pts)
Carefully consider two hypothetical diamond lattice semiconductors (Technium and Georgium) and their respective band structures, and then answer the questions below. For credit you must carefully demonstrate how you got your answers.

![Band Structures](image)

i) Which semiconductor has the largest electron mobility?

ii) Which semiconductor has the smallest lattice constant?
Consider the feedback system shown in Figure 1, where \( G(s) \) represents the plant, and the compensation is a proportional controller with gain \( K > 0 \). The root locus of the system is shown in Figure 2, where only one of the poles is specified. Suppose that the DC gain of the plant is \( G(0) = 1 \), and the closed-loop system stability range of the gain \( K \) is \( K < \frac{22}{3} \). Find \( G(s) \), and compute the points where the root locus crosses the imaginary axis.

**Problem**

![Figure 1: Closed-Loop System](image1.png)

![Figure 2: Root Locus](image2.png)
Problem 17 (Breadth: TELECOM - ECE 3600) Code Number: __________

**Problem**

TCP

How does a reliable transport protocol (e.g. TCP) overcome the following problems:

1. Lost packet?

2. Bit error in packet?

3. Packets that arrive in wrong order?

4. Packets arrive too fast for receiver to handle.

Internet Delay

Between hosts A in Atlanta and B in Las Dallas there are 2 routers (X,Y). The link between routers (---) is 10 Mbps. The access links (LANs, ===) are 1000 Mbps. The total distance from A to B is 2000 km. The signal travels about 2E8 m/s. A starts to send a large file using TCP, sending 1200 byte packets to B. B acks with 100 byte packets. There is no other traffic on this network.

A ===X---Y===B

5. What is the time required to transmit a 1200 Byte datagram at 10 Mbit/s? __________ ms

6. What is the propagation delay for a one-way trip in milliseconds (ms)? __________ ms

7. If the router buffers are empty, what is the total round trip transmission delay (neglecting processing delay and the transmission delay on the X-Y link) Assume the ACK packet is also 1200 bytes? __________ ms

8. Other traffic builds up the average level in X's X-to-Y output buffer to 500 kBytes. What does this queuing delay add to the RTT? __________ ms

9. If a Web Browser (or any network program) wants to find the IP address from a name (e.g., "www.gatech.edu"), to what type of server does it send a query? __________

10. If that server does not have information in its cache, it does a recursive lookup. What type of server does it send its first query to? __________
A lossless transmission line uses a dielectric insulating material with \( \varepsilon_r = 4 \). If the line capacitance is \( C' = 10 \, \text{pF/m} \), find:

(a) The phase velocity \( u_p \)
(b) The line inductance \( L' \), and
(c) The characteristic impedance \( Z_0 \)
Problem 19 (Specialized: OPTICS - ECE 4500) Code Number: ___________

**Problem**

(a) An imaging system consists of two thin lenses. The focal length of the first lens is 4 cm, and the focal length of the second lens is 2.5 cm. The second lens is placed 10 cm behind the first lens. An object is placed 5 cm in front of the first lens. Find the location of the image formed by the system, relative to the position of the second lens. You may use the paraxial approximation.

(b) A coating is applied to the front surface of the first lens to minimize reflections. The first lens has a refractive index of 1.5, and the coating has a refractive index of 1.23. Calculate the coating thickness that will minimize the reflected power for normally incident light with a free-space wavelength of 500 nm.
Consider a 1x4 fiber optic splitter shown here, in which input power through the single mode fiber at the left, $P_{in}$, is (ideally) split equally into the four fibers at the right. The device works in a similar manner in reverse, in which power that is input in any of the four fibers at the right leaves through the left fiber, but with a power reduction of one-fourth (6dB). The loss in decibels (dB) is defined as $10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$. A power quantity in decibels relative to a milliwatt is defined as $P_{dBm} = 10 \log_{10} \left( \frac{P_{mW}}{1mW} \right)$.

(a. Suppose an input power, $P_{in} = 10$ dBm is used, and the device behaves ideally as described above. What dBm value will each of the output ports carry, and thus how many mW of power is found at each port?

(b. In a real device, the sum of all four output powers is likely to be less than the input power. The power that is lost is the insertion loss of the device, expressed in decibels through

$$L_{ins} = 10 \log_{10} \left( \frac{\sum_{i=1}^{4} P_i}{P_{in}} \right)$$

Suppose that the insertion loss for a given 1x4 splitter is quoted as 4dB, and the output powers are otherwise all equal. What is the power loss in dB between $P_{in}$ and $P_2$, for example? Be careful here, and clearly show your reasoning for full credit.

(c. It is possible to construct a 4x4 star coupler, using two 1x4 splitters arranged front-to-front as shown below.

Briefly describe what this device does when power is input to any of the four left ports.

d. Assuming a 4-dB insertion loss in each of the two devices, what is the net dB loss in transmitting, for example, between input port 1 and output port 2?

(continued next page)
There are better ways of implementing a star coupler. One involves using four 3dB directional couplers (one of which is shown below). In each coupler, light input in either of the two input ports at left is equally divided between the two output ports at the right. Make a sketch of how four of these can be connected to make a 4x4 star. Also, calculate the net loss in dB for light that transmits through your arrangement, assuming zero insertion loss for the couplers.
PROBLEM

(4 pts) Cardiovascular Anatomy / Physiology. Fill in the blanks with the best possible answer.

The two large veins which bring deoxygenated blood to the right atrium from most of the body are the __________________ and __________________.

The volumetric flow rate of blood from the heart is called ________________ and is the produce of heart rate and ________________.

(4 pts) Draw an electrocardiogram (ECG) waveform for one heartbeat, and label the P and T waves, the QRS complex, and the approximate amplitude of the QRS complex.

(2 pts) If you are designing an algorithm for classifying premature atrial contractions (an arrhythmia related to hypoxic foci in the atria), provide a mathematical expression relating the R-to-R interval that precedes (RR\textsubscript{i-1}) and follows (RR\textsubscript{i+1}) the arrhythmia to the average interval for the entire recording (\overline{RR}), assuming the rest of the recording is composed of only normal heartbeats.
(3 points) **Electrochemical Balance.** For this question, we are concerned with a cell (T=20°C) with the initial ionic concentrations listed below. A\(^-\) represents arbitrary anions. The membrane is passive and only permeable to K\(^+\) and Cl\(^-\) ions.

<table>
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<th>Ion</th>
<th>Intracellular (mM)</th>
<th>Extracellular (mM)</th>
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<tr>
<td>K(^+)</td>
<td>145</td>
<td>145</td>
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<tr>
<td>Na(^+)</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Cl(^-)</td>
<td>55</td>
<td>245</td>
</tr>
<tr>
<td>A(^-)</td>
<td>100</td>
<td>0</td>
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Calculate 1) the final concentration of extracellular K\(^+\) and 2) the resulting membrane potential at equilibrium. Show all work and specify units.
(4 points) **Current-voltage relationships.**

The figure below illustrates 4 different steady-state current-voltage (I-V) plots. Each I-V plot represents a combination of one or more membrane ion channels: $\text{Na}^+$, $\text{Cl}^-$, and/or $\text{K}^+$. Each ion channel may be linear, or have a voltage-activated or voltage-inactivated conductance. The Nernst potential for the 3 ionic species are $E_K = -90$ mV, $E_{Cl} = -55$ mV, and $E_{Na} = +50$ mV.

Determine a combination of ionic current(s) sufficient to generate each I-V curve. For each current in your answer, specify both the ionic species and whether the conductance is constant, voltage-activated, or voltage-inactivated. Each answer should have an explanation justifying your answer with equations or reasoning. You are also free to draw on the graphs. Note: it is possible that an I-V plot may have more than one correct answer, and credit will be given for any correct answer.
(3 points) **Nerve Stimulation.** A strength duration curve for a nerve is below. Determine parameters (amplitude, pulse duration) for a periodic stimulus protocol that will maximize battery life for an implantable stimulator. The maximum safe charge injection (total charge to be injected over a single pulse) is 0.15 $\mu$C. The stimulator circuitry cannot source a current larger than 10 mA. Assume a monophasic pulse for your analysis (this is not realistic, but simplifies assumptions). Justify your answer.