INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 22 numbered problems. Note that Problem 7 occupies 2 pages, Problem 12 occupies 2 pages, Problem 17 occupies 3 pages. Including the cover sheet, you should have 56 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!!

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 14 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:

   (a) Separate the 8 problems to be graded as explained above.

   (b) Check to see that your Code Number is in EVERY sheet you are turning in.

   (c) On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.

   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.

   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!
Problem 1 (Core: VLSI-ECE2020)  Solution

PROBLEM

A 3-bit synchronous finite state machine (FSM) with one data input (X) behaves as follows after receiving a clock pulse:

When X=0 the FSM increments the state code by 2, with “wrap-around” (e.g. state S7 transitions to S1).

When X=1 the FSM decrements the state code by 5, with “wrap-around” (e.g. state S3 transitions to S6).

The input X may change value only once per clock cycle.

Three D-type flip flops (Q2, Q1, Q0) will be used for the state register.

(a) [2 points] Complete the symbolic state transition diagram.

(b) [2 points] Complete the encoded state transition table according to the format below:

<table>
<thead>
<tr>
<th>Present State (X)</th>
<th>Next State (Q2Q1Q0)</th>
<th>Present State (X)</th>
<th>Next State (Q2Q1Q0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>000</td>
<td>000</td>
<td>000</td>
</tr>
<tr>
<td>001</td>
<td>001</td>
<td>010</td>
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<tr>
<td>011</td>
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<td>100</td>
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<tr>
<td>101</td>
<td>101</td>
<td>110</td>
<td>110</td>
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<tr>
<td>111</td>
<td>111</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) [3 points] Write expressions for the flip flop input functions, using sum-of-minterms format [i.e. \( \sum m \)]

\[ D_2(X,Q2,Q1,Q0) = \sum \]
\[ D_1(X,Q2,Q1,Q0) = \sum \]
\[ D_0(X,Q2,Q1,Q0) = \sum \]

(d) [3 points] Use Karnaugh maps to find the minimized sum-of-products (SOP) expressions for D2, D1, and D0.
A 3-bit synchronous finite state machine (FSM) with one data input (X) behaves as follows after receiving a clock pulse:

- When X=0 the FSM increments the state code by 2, with “wrap-around” (e.g. state S7 transitions to S1).
- When X=1 the FSM decrements the state code by 5, with “wrap-around” (e.g. state S3 transitions to S6).

The input X may change value only once per clock cycle.

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(a) [2 points] Complete the symbolic state transition diagram.

(b) [2 points] Complete the encoded state transition table according to the format below:

<table>
<thead>
<tr>
<th>Present State (X)</th>
<th>Next State (Q2+Q1+Q0+)</th>
<th>Present State (X)</th>
<th>Next State (Q2+Q1+Q0+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>010</td>
<td>100</td>
<td>001</td>
</tr>
<tr>
<td>001</td>
<td>011</td>
<td>010</td>
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<td>000</td>
<td>100</td>
</tr>
<tr>
<td>011</td>
<td>111</td>
<td>011</td>
<td>000</td>
</tr>
</tbody>
</table>

(c) [3 points] Write expressions for the flip flop input functions, using sum-of-minterms format [i.e. ∑m]

\[
D2(X,Q2,Q1,Q0) = \sum m(2,3,4,5,9,10,11,12) \\
D1(X,Q2,Q1,Q0) = \sum m(0,1,4,5,8,11,12,15) \\
D0(X,Q2,Q1,Q0) = \sum m(1,3,5,7,10,12,14) 
\]

(d) [3 points] Use Karnaugh maps to find the minimized sum-of-products (SOP) expressions for D2, D1, and D0.
Consider a chip that is dissipating 1 W of power as heat, which is represented by $q_1$ in the equivalent thermal circuit below. Assume that the thermal resistance for the package is 10 K/W, for the heat sink is 0.1 K/W, and for the equivalent thermal resistance for convection is 20 K/W. Assume the ambient temperature ($T_{amb}$) is 25°C.

(a) Assume that the thermal paste has a thermal conductivity of $k = 1.1 \ [W/m*K]$. Assume that the thickness of the paste is on average 100 µm and the package on which it is applied is 5 cm² in area. What is the steady-state temperature of the devices on the chip (i.e. $T_{dev}$)?

(b) What would happen if you forgot the thermal paste? Assume that the average spacing between the package and the heat sink is now reduced to 50 µm and the effective thermal conductivity of this interface region dominated by air gaps is now 0.02 [W/m*K]. What is the new temperature of your devices on the chip (i.e. $T_{dev}$)?
Consider a chip that is dissipating 1W of power as heat, which is represented by \( q_1 \) in the equivalent thermal circuit below. Assume that the thermal resistance for the package is 10 K/W, for the heat sink is 0.1 K/W, and for the equivalent thermal resistance for convection is 20 K/W. Assume the ambient temperature \( T_{\text{amb}} \) is 25°C.

(a) Assume that the thermal paste has a thermal conductivity of \( k = 1.1 \) [W/m*K]. Assume that the thickness of the paste is on average 100 µm and the package on which it is applied is 5 cm² in area. What is the steady-state temperature of the devices on the chip (i.e. \( T_{\text{dev}} \))?

\[
T_{\text{dev}} = T_{\text{amb}} + q_1 \left( R_{\text{rh,package}} + R_{\text{rh,paste}} + R_{\text{rh,heatsink}} + R_{\text{rh,convection}} \right)
\]

\[
R_{\text{th,paste}} = \frac{\Delta x}{k \cdot A} = \frac{100 \cdot 10^{-6} \text{m}}{1.1 \text{W/mK} \cdot (0.0005) \text{m}^2} = 0.18 \frac{K}{W}
\]

\[
T_{\text{dev}} = 298K + 1W \left( 10 \frac{K}{W} + 0.18 \frac{K}{W} + 0.1 \frac{K}{W} + 20 \frac{K}{W} \right) = 298K + 30.28K = 328.28K = 55.28°C
\]

(b) What would happen if you forgot the thermal paste? Assume that the average spacing between the package and the heat sink is now reduced to 50 µm and the effective thermal conductivity of this interface region dominated by air gaps is now 0.02 [W/m*K]. What is the new temperature of your devices on the chip (i.e. \( T_{\text{dev}} \))?

\[
R_{\text{th,airgap}} = \frac{\Delta x}{k \cdot A} = \frac{50 \cdot 10^{-6} \text{m}}{0.02 \text{W/mK} \cdot (0.0005) \text{m}^2} = 5 \frac{K}{W}
\]

\[
T_{\text{dev}} = 298K + 1W \left( 10 \frac{K}{W} + 5 \frac{K}{W} + 0.1 \frac{K}{W} + 20 \frac{K}{W} \right) = 298K + 35.1K = 333.1K = 60.1°C
\]
Consider the following logic gate:

(a) Re-design the circuit shown above with minimum number of transistors. Draw the schematic of the new circuit. Your design should have as few as possible devices connected to the output node [6 point].

(b) In the new schematic, size the NMOS and PMOS devices so that the maximum possible resistances of the pull-down and pull-up paths are same as that of an inverter with an NMOS of $W/L = 1$ and PMOS of $W/L = 2$. (You can put the sizing numbers on the transistor diagram). [If you cannot solve part (a), you can solve part (b) considering the original schematic, but you will not receive credit for the part a] [3 point]

(c) Considering your sizing solution from part b, compute the RC delay for the following transitions. Assume the resistance of NMOS of size W is R. Neglect junction capacitances. Assume the output load capacitance is $C_{LOAD}$. [If you did not solve part (a), you can solve part (c) considering the original schematic and corresponding sizing in part (b)] [1 points]

<table>
<thead>
<tr>
<th>Transition</th>
<th>RC Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD: (0111) → (0011)</td>
<td></td>
</tr>
</tbody>
</table>
Part (a)

\[ F = \bar{C} \cdot D \cdot (A + B) + A \cdot B \cdot C + C = \bar{C} \cdot D \cdot (A + B) + C = D \cdot (A + B) + C \]

Part (b)

See the sizes in the above figure.

Part (c)

There is no output transition for the specific input transition. So, the delay is zero (0).
A coaxial cable is constructed from two concentric conducting cylinders of radii $a$ and $c$, respectively. The inner cylinder is coated with a magnetic material ($\mu_r=100$) out to radius $b$ (see figure). For an assumed current $I$ flowing out of the page in the center conductor and returning on the outer conductor, the magnetic field between the conductors is determined to be:

$$H(\rho) = \hat{\phi} \frac{I}{2\pi\rho} \quad a < \rho < c$$

(a) Find an expression for the inductance per unit length of the cable. Leave your answer in terms of $a$, $b$, $\mu_r$, etc.

(b) Can you relate your result in part (a) to the series or parallel combination of inductances?

Circle one: Series Parallel Neither

(c) Find a numerical value for the energy per unit length (energy per meter) stored in the cable if $I = 100$ mA, $a = 3$ mm, $b = 3.5$ mm, $c = 4$ mm
Solution

(a) 

\[ L = \frac{1}{I} \int_a^c B_\phi d\rho \]

\[ = \mu_0 \int_a^b \frac{\mu_r}{2\pi \rho} d\rho + \mu_0 \int_b^c \frac{1}{2\pi \rho} d\rho \]

\[ = \frac{\mu_0 \mu_r}{2\pi} \ln(b / a) + \frac{\mu_0}{2\pi} \ln(c / b) \]

(b) the arrangement is analogous to the SERIES combination of two inductors

(c) 

\[ W_m = \int_{\phi=0}^{2\pi} \int_{\rho=a}^{c} \frac{\mu}{2} \cdot \vec{H} \cdot \vec{H} \rho d\rho d\phi \]

\[ = \frac{\pi \mu_0 I^2}{4\pi^2} \left\{ \int_a^b \frac{\mu_r}{\rho} d\rho + \int_b^c \frac{1}{\rho} d\rho \right\} \]

\[ = \frac{\mu_0 I^2}{4\pi} \left\{ \mu_0 \ln(b / a) + \ln(c / b) \right\} \]

for the values provided,

\[ W_m = 1.55 \times 10^{-8} \text{ (J/m)} \]
Problem 5 (Core: E&M-ECE4350)

Problem

The amplitude of a 2GHz linearly polarized (x-directed) incident wave is 10 V/m at the surface of a soil medium. The soil medium is non-magnetic, has relative permittivity \( \varepsilon_r = 16 \) and conductivity \( \sigma = 5 \times 10^{-4} \) (S/m).

a) Calculate the loss tangent of the soil medium.
b) At what depth would the wave have an amplitude of 1 mV/m?
c) Calculate the average power density of the wave at the depth calculated in part b).
Problem 5 (Core: E&M-ECE4350) Solution

**SOLUTION**

a) 
\[
\tan d = \frac{\sigma}{\omega\epsilon} = \frac{5 \times 10^{-4}}{2\pi \times 2 \times 10^9 \times 16 \times 8.854 \times 10^{-12}} = 0.000281
\]

b) 
\[
\vec{E}_s = 10 e^{-j\beta z} a_x \quad V/m
\]
\[
\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[ 1 + \left( \frac{\epsilon''}{\epsilon'} \right)^2 - 1 \right]^{1/2}
\]
\[
\alpha = 2\pi \times 2 \times 10^9 \sqrt{\frac{4\pi \times 10^{-7} \times 16 \times 8.854 \times 10^{-12}}{2}} \left[ \sqrt{1 + (0.000281)^2} - 1 \right]^{1/2} = 0.023 Np/m
\]
\[
|\vec{E}_s| = 10^{-3} = 10 e^{-0.023z} \Rightarrow z = 393.6 m
\]

c) 
\[
\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{16 \times 8.854 \times 10^{-12} \sqrt{1 - j0.000281}}} \equiv 94.25 \Omega
\]
\[
\vec{H} = \frac{1}{\eta} a_z \times \vec{E} = \frac{10}{94.25} e^{-0.023z} e^{-j\beta z} a_y
\]
\[
P_{avg}(z) = \frac{1}{2} \text{Re} \left[ 10 e^{-0.023z} e^{-j\beta z} a_x \times \frac{10}{94.25} e^{-0.023z} e^{j\beta z} \right] = 7.27 nW/m^2
In the following figure, (a) find $R_L$ such that maximum power is dissipated in $R_L$ and (b) calculate the value of maximum power.
Problem 6 (Core: EDA-ECE2040)

Solution

\[ R_L = \frac{30}{11} \Omega = 2.73 \Omega \]

\[ P_{\text{max}} = \frac{V_s^2}{4R_L} = \frac{\left( \frac{78}{11} \right)^2}{4 \cdot \frac{30}{11}} = 4.61 \text{ W} \]
Problem 7 (Core: EDA-ECE3400)

Problem

If you do not write your answer in the line provided, you will not get credit for your answer. Only answers given on the lines below will be graded in a binary way whether correct or not.

Answers to Questions Here (pt value) Answers to Questions Here (pt value)
1. (2pt) 2. (1pt)
3. (1pt) 4. (1pt)
5. (2pt) 6. (2pt)
7. (1pt)
Assume we have an idealized device A (in Fig. 1a) such that we get no current from the $V_2$ node, and the current between the $V_1$ node and the $V_3$ node, $I$, follows

$$I = I_s \left( \exp \left( \frac{0.5V_2 - V_1 + 0.01V_3}{2U_T} \right) - \exp \left( \frac{0.5V_2 - V_3 + 0.01V_1}{2U_T} \right) \right), \quad (1)$$

and be approximated for significant difference between $V_1$ and $V_3$ ($V_1 - V_3$)

$$I = I_s \exp \left( \frac{V_2 - V_1 + 0.01V_3}{2U_T} \right) \quad (2)$$

$I_s$ is a constant equal to 1pA. $U_T$ is the thermal voltage, $kT/q$, and for this problem should be assumed to equal 25mV. We will assume (1) happens for Case I, and we will assume (2) happens for Case II.

For the following two statements, identify if they are True or False:

1. What is the minimum voltage (within 10 percent) between $V_1$ and $V_3$ for (2) to be within 2 percent of (1).

2. Where device A models a MOSFET device, we call Case II saturation. 3. Where device A models a BJT device, we call Case II saturation.

Next assume the following circuit in Fig 1b. $C = 1pF$. Assume $V_{dd} = 2.5V$.

4. What is the gain of this circuit?

5. For $V_\tau = 0.3V$, what is the resulting -3dB corner frequency of this circuit?

For the following circuit in Fig. 1c, assume that the current source acts like an ideal current source that can source a voltage below $V_{dd}$. Still assume $V_{dd} = 2.5V$.

What is the large-scale gain of the resulting device?

What is the gain of the circuit if we flip the terminals ($V_1$ and $V_3$) in this circuit?
<table>
<thead>
<tr>
<th>Answers to Questions Here (pt value)</th>
<th>Answers to Questions Here (pt value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 100mV (2pt)</td>
<td>2. True (1pt)</td>
</tr>
<tr>
<td>3. False (1pt)</td>
<td>4. 1/2 (1pt)</td>
</tr>
<tr>
<td>5. 2.5kHz (2pt)</td>
<td>6. 50 (2pt)</td>
</tr>
<tr>
<td>7. 50 (1pt)</td>
<td></td>
</tr>
</tbody>
</table>
Consider the simplified three-phase electric power system of Figure P1 consisting of a balanced generator, a symmetric three-phase transmission line and a symmetric three-phase electric load. The source is wye connected and the electric load is delta connected as illustrated in the figure. The phase A voltage of the source is 6.92 kV with a phase angle of zero degrees. Each leg of the three phase load has an impedance of 36+j15 ohms. Each phase of the symmetric line has a self-impedance of j9 ohms. The mutual impedance between any two phases is j4 ohms. Other pertinent information is indicated in the figure.

(a) Compute the real power absorbed by the three-phase electric load.
(b) Compute the reactive power absorbed by the three phase electric load.
(c) Assume that the three legs of the three-phase electric load are connected in a wye configuration without changing the values of R and X. (1) compute the real power of the wye connected load, and (2) compute the reactive power of the wye connected load.

![Figure P1](image-url)

**Figure P1**

\[ E_a = 6.92 \ e^{j0} \text{ kV} \]
Parts (a) and (b): Convert the delta connected load into an equivalent wye connected load and then perform per phase analysis. Results:

\[ 6.92 \, \text{e}^{j \theta} \, \text{kV} = j9 \, \tilde{I}_a + j4 \, \tilde{I}_b + j \, \tilde{I}_c + (12 + j5) \, \tilde{I}_a \]
\[ \Rightarrow \]
\[ 6.92 \, \text{e}^{j \theta} \, \text{kV} = (j9 - j4) \, \tilde{I}_a + (12 + j5) \, \tilde{I}_a \]
\[ \Rightarrow \]
\[ \tilde{I}_a = \frac{6.92 \, \text{e}^{j \theta}}{(12 + j10)} \]
\[ = 0.443 \, \text{e}^{j31.80^\circ} \, \text{kA} \]

\[ S = (3) (12 + j5) \, \tilde{I}_a \cdot \tilde{I}_a^* \]
\[ = 7.065 \, \text{MW} + j2.9437 \, \text{MVAR} \]
\[ \Rightarrow \]
(a) \quad P = 7.065 \, \text{MW}
(b) \quad Q = 2.9437 \, \text{MVAR}

Part (c): Perform per phase analysis assuming that the leg in the wye connected load has an impedance of 36+j15 ohms.

\[ 6.92 \, \text{e}^{j \theta} \, \text{kV} = (j9 - j4) \, \tilde{I}_a + (36 + j15) \, \tilde{I}_a \]
\[ \Rightarrow \]
\[ \tilde{I}_a = \frac{6.92 \, \text{e}^{j \theta}}{(36 + j20)} \]
\[ = 0.168 \, \text{e}^{j29.05^\circ} \, \text{kA} \]
\[ S = (3) (36 + j15) \, \tilde{I}_a \cdot \tilde{I}_a^* \]
\[ = 3.0428 \, \text{MW} + j1.27 \, \text{MVAR} \]
\[ \Rightarrow \]
(1) \quad P = 3.0428 \, \text{MW}
(2) \quad Q = 1.27 \, \text{MVAR}
A 3-phase, 5 MVA, 11 kV, 60 Hz synchronous machine has a synchronous reactance of 10 Ohms per phase and has negligible stator resistance. The machine is connected to the 11 kV, 60 Hz bus and is operated as a synchronous condenser (producing zero active power and being used for reactive power only). Neglect rotational losses. (a) For normal excitation, find the stator current. Draw the phasor diagram. (b) If the excitation (field current) is increased to 150 percent of the normal value at synchronization, find the stator current and power factor. Draw the phasor diagram. (c) If the excitation is decreased to 50 percent of the normal value at synchronization, find the stator current and power factor. Draw the phasor diagram.
Problem 9 (Core: Energy-ECE3072) Solution

**SOLUTION**

a) \( P = 3V_4I_4 \cos \theta = 0 \Rightarrow I_4 = 0 \)

Also, \( \delta = 0 \), HENCE \( |V_4| = |E_f| = \frac{11,000}{\sqrt{3}} V = 6.35 \text{ kV} \)

\[ E_f \]

\[ V_4 \]

b) \( \delta = 0 \)

\( I_4 = \frac{V_4/L - E_f/L_0^*}{jX_s} = \frac{6.351/10^* - 1.5 \cdot 6.351/10^*}{j10} = 317.55 / 90^* \text{ A} \)

\( P_f = \cos 90^* = 0 \Rightarrow \text{LEADING} \)

\[ I_4 \]

\[ V_4 \]

\[ E_f \]

\[ jX_s I_4 \]

c) \( \delta = 0 \)

\( I_4 = \frac{6.351/10^* - 0.5 \cdot 6.351/10^*}{j10} = 319.55 / -90^* \)

\( P_f = \cos 90^* = 0 \Rightarrow \text{LAGGING} \)

\[ I_4 \]

\[ V_4 \]

\[ E_f \]

\[ jX_s I_4 \]
Light from two identical light sources is absorbed on both sides of a silicon wafer of thickness 520 µm. The wafer is p-type, and is uniformly doped with $10^{17}$ cm$^{-3}$ acceptors. The wafer has been optically illuminated sufficiently long for steady-state conditions to be reached. For purposes of this problem, you should assume that the incident light is absorbed uniformly within a depth of 10 µm from the surface (i.e. no light penetrates past 10 microns into the wafer). The diffusion coefficient for electrons is 2.5 cm$^2$/sec. If the photogeneration rate for holes is $10^{17}$ cm$^{-3}$/sec and the minority carrier lifetime is 1 millisecond, what is the excess electron concentration for all positions in the wafer? Hint: break the problem up into three regions (two of which have identical solutions) and use the x axis pictured above. Your answer should be a numeric expression with x being the only independent variable.
Because the Si wafer is p-type, the steady-state minority electron diffusion equation
\[ D_n \frac{\partial^2 \delta n_p}{\partial x^2} - \frac{\delta n_p}{\tau_n} + G_L = 0 \]
governs the spatial distribution of excess electrons in the wafer.

Its general solution is of the following form:
\[ \delta n_p = Ae^{-x/L_n} + Be^{x/L_n} + G_L \tau_n \]
and we may therefore write

For -10µm < x < 0 µm:
\[ \delta n_p = Ae^{-x/L_n} + Be^{x/L_n} + G_L \tau_n \]

For 0µm < x < 500 µm:
\[ \delta n_p = Ce^{-x/L_n} + De^{(x-500\cdot10^{-6})/L_n} \]

For 500µm < x < 510 µm:
\[ \delta n_p = Ee^{-(x-500\cdot10^{-6})/L_n} + Fe^{(x-500\cdot10^{-6})/L_n} + G_L \tau_n \]
Note that by symmetry, C = D, E = B and F = A. Simplifying, we have

For \(-10\mu m < x < 0 \mu m:\)
\[
\delta n_p = Ae^{-x/L_n} + Be^{x/L_n} + G_L \tau_n
\]

For \(0\mu m < x < 500 \mu m:\)
\[
\delta n_p = Ce^{-x/L_n} + Ce^{(x-500 \cdot 10^{-6})/L_n}
\]

For \(500 \mu m < x < 510 \mu m:\)
\[
\delta n_p = Be^{-(x-500 \cdot 10^{-6})/L_n} + Ae^{(x-500 \cdot 10^{-6})/L_n} + G_L \tau_n
\]

Next, boundary conditions must be applied. The first boundary condition is that the minority electron current density is zero at the top and bottom surfaces of the wafer, since no electrons can enter or leave the wafer across its surface. The minority electron diffusion current density is 
\[
j_n = qD_n dn/dx,
\]
and we therefore have
\[
\frac{A}{L_n} e^{10 \cdot 10^{-6}/L_n} = \frac{B}{L_n} e^{-10 \cdot 10^{-6}/L_n}
\]
or equivalently
\[
B = Ae^{20 \cdot 10^{-6}/L_n}
\]

Note that \(L_n = (D_n \tau_n)^{1/2} = (.00025 \times .001)^{1/2} = 500\) microns.

Now we can write:

For \(-10\mu m < x < 0 \mu m:\)
\[
\delta n_p = A \left( e^{-x/L_n} + e^{(x+20 \cdot 10^{-6})/L_n} \right) + G_L \tau_n
\]

For \(0\mu m < x < 500 \mu m:\)
\[
\delta n_p = Ce^{-x/L_n} + Ce^{(x-500 \cdot 10^{-6})/L_n}
\]

For \(500 \mu m < x < 510 \mu m:\)
\[
\delta n_p = A \left( e^{-(x-500 \cdot 10^{-6})/L_n} + e^{(x-500 \cdot 10^{-6}/L_n} \right) + G_L \tau_n
\]

The next boundary condition is continuity of minority electron density at \(x=0\). This implies
\[
A \cdot (1 + e^{20/500}) + G_L \tau_n = C \cdot (1 + e^{-1})
\]
The final boundary condition is continuity of minority electron current density at \(x=0\). This in turn implies
\[
A \cdot (1 - e^{20/500}) = C \cdot (1 - e^{-1})
\]

These last two equations may be solved simultaneously to yield the final unknown coefficients, A and C. Note that \(G_L \tau_n = 10^{14} \text{ cm}^{-3}\).
Problem 10 (Core: Microsystems-ECE3040) Solution

The solutions are \( A = -4.6968 \cdot 10^{13} \text{ cm}^{-3} \) and \( B = 3.0323 \cdot 10^{12} \text{ cm}^{-3} \). With “x” in units of meters,

\[
\delta n_p = \begin{cases} 
10^{14} - 4.6968 \cdot 10^{13} \left( e^{-x/500 \cdot 10^{-6}} + e^{(x+20 \cdot 10^{-6})/500 \cdot 10^{-6}} \right) \text{ cm}^{-3} & -10 \mu m < x < 0 \\
3.0323 \cdot 10^{12} \left( e^{-x/500 \cdot 10^{-6}} + e^{(x/500 \cdot 10^{-6})^{-1}} \right) \text{ cm}^{-3} & 0 < x < 500 \mu m \\
10^{14} - 4.6968 \cdot 10^{13} \left( e^{-(x-20 \cdot 10^{-6})/500 \cdot 10^{-6}} + e^{(x-500 \cdot 10^{-6})/500 \cdot 10^{-6}} \right) \text{ cm}^{-3} & 500 \mu m < x < 510 \mu m 
\end{cases}
\]
Consider an ideal **silicon pnp bipolar junction transistor (BJT)**. The three regions of the transistor (i.e., the emitter, the collector and the base) are uniformly doped. Use a list of the parameters below, wherever applicable, to find an express for the base transport factor for the BJT biased at the forward-active operation mode.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_E$</td>
<td>The emitter doping concentration</td>
<td>$D_E$</td>
<td>The diffusion coefficient for the minority carriers in the emitter</td>
</tr>
<tr>
<td>$N_B$</td>
<td>The base doping concentration</td>
<td>$D_B$</td>
<td>The diffusion coefficient for the minority carriers in the base</td>
</tr>
<tr>
<td>$N_C$</td>
<td>The collector doping concentration</td>
<td>$D_C$</td>
<td>The diffusion coefficient for the minority carriers in the collector</td>
</tr>
<tr>
<td>$\tau_E$</td>
<td>The minority carrier lifetime in the emitter</td>
<td>$V_{CB}$</td>
<td>The collector-base voltage</td>
</tr>
<tr>
<td>$\tau_B$</td>
<td>The minority carrier lifetime in the base</td>
<td>$V_{EB}$</td>
<td>The emitter-base voltage</td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>The minority carrier lifetime in the collector</td>
<td>$W_B$</td>
<td>The quasi-neutral base width</td>
</tr>
<tr>
<td>$T$</td>
<td>The temperature in Kevin</td>
<td>$n_i$</td>
<td>The intrinsic carrier concentration</td>
</tr>
<tr>
<td>$k$</td>
<td>The Boltzmann’s constant</td>
<td>$q$</td>
<td>The electric charge</td>
</tr>
</tbody>
</table>

To do so,

a. State the definition of the base transport factor first. (1 points)

b. Use the proper equations and boundary conditions (2 points) to find the minority carrier distribution (2 points) and the corresponding minority carrier current (2 points) in the base.

c. Use proper assumptions. Derive and simplify the final answer to the question. (3 points)
Problem 11 (Core: Microsystems-ECE3450) Solution

a. The base transport factor \( \alpha_T = \frac{I_C}{I_P} \) for PNPBJT.

b. The general solution to the excess hole concentration in the base is expressed as:
\[
\Delta p_B(x) = Ae^{-x/L_B} + Be^{x/L_B}; \quad \text{where } L_B = \sqrt{\tau_B D_B}
\]
Assume that the emitter-base voltage is \( V_{EB} \) and the collector-base voltage is \( V_{CB} \).
The boundary conditions are:
\[
\Delta p_B\big|_{\text{emitter-side of the base}} = p_{B0}(e^{qV_{EB}/kT} - 1) \quad \text{and} \quad \Delta p_B\big|_{\text{collector-side of the base}} = p_{B0}(e^{qV_{CB}/kT} - 1)
\]
, where \( p_{B0} = \frac{n_i^2}{N_B} \)

Let the emitter-side of the base be \( x = 0 \). The collector-side of the base is \( x = W_B \).
In the forward-active biasing mode, \( \Delta p_B(W_B) = p_{B0}(e^{qV_{CB}/kT} - 1) \equiv -p_{B0} \)

Solve for \( A \) and \( B \). One will have:
\[
\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1) \frac{\sinh[(W_B - x)/L_B]}{\sinh(W_B / L_B)} - p_{B0} \frac{\sinh[x/L_B]}{\sinh(W_B / L_B)}
\]
The emitter hole current density \( J_{EP} \) is:
\[
J_{EP} = -qD_B \frac{\partial}{\partial x} \left[ \Delta p_B(x) \right]_{x=0} = \frac{qD_B p_{B0}}{L_B} \left[ (e^{qV_{EB}/kT} - 1) \frac{\cosh(W_B / L_B)}{\sinh(W_B / L_B)} + \frac{1}{\sinh(W_B / L_B)} \right]
\]
The collector hole current density \( J_{CP} \) is:
\[
J_{CP} = -qD_B \frac{\partial}{\partial x} \left[ \Delta p_B(x) \right]_{x=W_B} = \frac{qD_B p_{B0}}{L_B} \left[ (e^{qV_{EB}/kT} - 1) \frac{1}{\sinh(W_B / L_B)} + \frac{\cosh(W_B / L_B)}{\sinh(W_B / L_B)} \right]
\]
c. Assume that the junction area is \( A \). One will have:
\[
\alpha_T = \frac{I_{CP}}{I_{EP}} = \frac{J_{CP}}{J_{EP}} \cdot A = \frac{(e^{qV_{EB}/kT} - 1) + \cosh(W_B / L_B)}{(e^{qV_{EB}/kT} - 1) \cosh(W_B / L_B) + 1}
\]
or,
\[
\alpha_T = \frac{(e^{qV_{EB}/kT} - 1) + \cosh(W_B / \sqrt{D_B \tau_B})}{(e^{qV_{EB}/kT} - 1) \cosh(W_B / \sqrt{D_B \tau_B}) + 1}
\]

Typically, \( W_B < < L_B \) and \( e^{qV_{EB}/kT} \gg 1 \). The base transport factor can be further simplified as:
\[
\alpha_T = \frac{1}{\cosh(W_B / \sqrt{D_B \tau_B})}
\]
PROBLEM

Consider the system below where $h[n]$ describes an arbitrary LTI system.

$x(t) = A \cos(1000\pi t) + B \cos(300\pi t) \cos(800\pi t)$

(NOTE: Each of these problems can be worked independently)

(a) Determine the Nyquist rate to sample $x(t)$ (i.e., to avoid aliasing)

Nyquist Rate = 

(b) Assume now that $f_s = 2000$ Hz (regardless of your answer for (a)) and the impulse response $h[n]$ and its frequency response $H(e^{j\hat{\omega}})$ have the form:

\[ h[n] = \begin{cases} 1, & n = 0, \ldots, L - 1 \\ 0, & \text{otherwise} \end{cases} \]

\[ H(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{\hat{\omega}/2} e^{-j\hat{\omega}(L-1)/2} \]

find the smallest value of $L$ such that the output is $y(t) = 0$

$L = $
(c) Assume now that the LTI system is defined as follows:

\[ y[n] = 0.5y[n - 1] + x[n] + x[n - 2] \]

Find the system function \( H(z) \) and find all of the poles and zeros (include any poles and/or zeros at \( z = 0 \) and/or \( z = \infty \)).

\[ H(z) = \]

<table>
<thead>
<tr>
<th>Poles:</th>
<th>Zeros:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consider the system below where \( h[n] \) describes an arbitrary LTI system.

\[
x(t) \xrightarrow{\text{IDEAL C-to-D CONVERTER}} x[n] \xrightarrow{h[n]} y[n] \xrightarrow{\text{IDEAL D-to-C CONVERTER}} y(t)
\]

\[ x(t) = A \cos(1000\pi t) + B \cos(300\pi t) \cos(800\pi t) \]

(NOTE: Each of these problems can be worked independently)

(a) Determine the Nyquist rate to sample \( x(t) \) (i.e., to avoid aliasing)
\( x(t) \) contains the frequencies: 250, 500, and 550 Hz. Nyquist rate is twice the highest frequency so it is \( 2 \times 550 = 1100 \) Hz

Nyquist Rate = 1100 Hz

(b) Assume now that \( f_s = 2000 \) Hz (regardless of your answer for (a)) and the impulse response \( h[n] \) and its frequency response \( H(e^{j\omega}) \) have the form:
\[
h[n] = \begin{cases} 1, & n = 0, \ldots, L - 1 \\ 0, & \text{otherwise} \end{cases} \quad H(e^{j\omega}) = \frac{\sin(\omega L/2)}{\omega/2} e^{-j\omega(L-1)/2}
\]

Find the smallest value of \( L \) such that the output is \( y(t) = 0 \)

Recall: \( \omega = \frac{2\pi f_s}{f_s} \). If the sampling frequency is \( f_s = 2000 \) Hz with frequencies 250, 500, and 550 Hz from \( x(t) \), then \( X(e^{j\omega}) \) contains the discrete-time frequencies:

\[
\omega_1 = \frac{2\pi 250}{2000} = 0.25\pi, \quad \omega_2 = \frac{2\pi 500}{2000} = 0.5\pi, \quad \omega_3 = \frac{2\pi 550}{2000} = 0.55\pi
\]

\( H(e^{j\omega}) \) contains zeros at integer multiples of \( 2\pi/L \) (i.e., the zeros are spaced by \( \Delta\omega = 2\pi/L \) on the discrete frequency axis). The smallest value of \( L \) will place zeros that are spaced by the distance between 0.5\( \pi \) and 0.55\( \pi \) (a larger spacing would not place zeros in the appropriate place and a smaller spacing would increase the value of \( L \)).

Therefore,

\[
\Delta\omega = \frac{2\pi}{L} = (0.55\pi - 0.5\pi) = 0.05\pi
\]

\[
L = \frac{2\pi}{0.05\pi} = 40
\]

\( L = 40 \)
(c) Assume now that the LTI system is defined as follows:

\[ y[n] = 0.5y[n - 1] + x[n] + x[n - 2] \]

Find the system function \( H(z) \) and find all of the poles and zeros (include any poles and/or zeros at \( z = 0 \) and/or \( z = \infty \)).

To find \( H(z) \), rearrange the difference equation using z-transforms.

\[
\begin{align*}
Y(z) &= 0.5Y(z)z^{-1} + X(z) + X(z)z^{-2} \\
Y(z) - 0.5Y(z)z^{-1} &= X(z) + X(z)z^{-2} \\
Y(z)(1 - 0.5z^{-1}) &= X(z)(1 + z^{-2}) \\
H(z) &= \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - 0.5z^{-1}}
\end{align*}
\]

To find the poles and zeros, we rewrite \( H(z) \) as a function of \( z \) instead of \( z^{-1} \)

\[
H(z) = \frac{1 + z^{-2}}{1 - 0.5z^{-1}} = \frac{(1 + z^{-2})}{1} \frac{1}{(1 - 0.5z^{-1})} = \frac{(z^2 + 1)}{z^2} \frac{z}{(z - 0.5)} = \frac{(z^2 + 1)}{z(z - 0.5)}
\]

Therefore, there are zeros at: \( z = j \) and \( z = -j \)

And there are poles at: \( z = 0 \) and \( z = 0.5 \)
PROBLEM

You work for the local power generation company BuzzPower (BP), which owns two large generators. The most important factor determining how many generators they need to run to meet demand is the weather (due to air conditioning usage). If $X$ is a random variable denoting the high temperature today, a meteorological model assigns the following probabilities:

$$p_X(x) = \begin{cases} 
\frac{1}{8} & x = 79 \\
\frac{1}{2} & x = 80 \\
\frac{3}{8} & x = 81 \\
0 & \text{otherwise}
\end{cases}$$

If $Y \in \{1, 2\}$ is a random variable denoting the number of generators that must be used, BP knows from past history that the number of required generators follows the following probabilities that depend on the high temperature. In particular, BP’s model says that:

$$\Pr(Y = 1|X = x) = \frac{82 - x}{4} \quad \text{for } x \in \{79, 80, 81\}.$$

1. What is the probability of BP using two generators today?

2. What is the expected number of generators BP will use today?

3. You haven’t been outside today, but you know 1 generator is running. What are the maximum likelihood (ML) $\hat{X}_{\text{ML}}$ and maximum a posteriori (MAP) $\hat{X}_{\text{MAP}}$ estimates for the high temperature?
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If \( Y \in \{1, 2\} \) is a random variable denoting the number of generators that must be used, BP knows from past history that the number of required generators follows the following probabilities that depend on the high temperature. In particular, BP’s model says that:

\[
\Pr(Y = 1|X = x) = \frac{82 - x}{4} \quad \text{for } x \in \{79, 80, 81\}.
\]

1. What is the probability of BP using two generators today?

\[
\Pr(Y = 2) = 1 - \left( \sum_x \Pr(Y = 1|X = x)\Pr(X = x) \right) = 1 - \left( \frac{31}{48} + \frac{11}{22} + \frac{13}{48} \right) = 0.5625
\]

2. What is the expected number of generators BP will use today?

\[
E[Y] = \sum_x E[Y|X = x]\Pr(X = x) = \frac{51}{48} + \frac{31}{22} + \frac{73}{48} = \frac{25}{16} = 1.5625
\]

3. You haven’t been outside today, but you know 1 generator is running. What are the maximum likelihood (ML) \( \hat{X}_{\text{ML}} \) and maximum a posteriori (MAP) \( \hat{X}_{\text{MAP}} \) estimates for the high temperature?

\[
\hat{X}_{\text{ML}} = \arg \max_x \Pr(Y = 1|X = x) = \arg \max_x \left( \frac{82 - x}{4} \right) = 79
\]

\[
\hat{X}_{\text{MAP}} = \arg \max_x \Pr(Y = 1|X = x)\Pr(X = x) = \left( \arg \max_x \left( \begin{array}{c} \frac{31}{48} \quad \frac{3}{32} \\ \frac{11}{22} = \frac{1}{4} \\ \frac{13}{48} \quad \frac{3}{32} \end{array} \right) \right) = 80
\]
1. (2 points) Consider the signal

\[ x(t) = \begin{cases} 
1, & \text{if } |t| \leq 1 \\
0, & \text{if } |t| > 1. 
\end{cases} \]

Derive a formula for its Fourier transform \( X(\omega) \) as a function of \( \omega \in \mathbb{R} \).

2. (5 points) Consider the signal

\[ f(t) = \frac{\sin(t)}{t}. \]

Derive a formula for its Fourier transform \( F(\omega) \) and plot it as a function of \( \omega \in \mathbb{R} \).

3. (3 points) Let \( f(t) \) be as in Part 2, and let the signal \( g(t) \) be defined as

\[ g(t) = f(t) \cos(12\pi t). \]

Let \( h(t) = f(t) \ast g(t) \) be the convolution of \( f(t) \) and \( g(t) \). Derive a formula for \( h(t) \) and plot it as a function of \( t \in \mathbb{R} \).
1. \[ X(\omega) = \int e^{-j\omega t} \, dt = \]

\[ = \left. \frac{1}{j\omega} e^{-j\omega t} \right|_{-1}^{1} = \frac{1}{j\omega} (e^{-j\omega} - e^{j\omega}) \]

\[ = \frac{e^{j\omega} - e^{-j\omega}}{2j\omega} = 2 \frac{\sin \omega}{\omega} \]

2. By part 1, \( \frac{1}{2} x(t) \xrightarrow{F} \frac{\sin \omega}{\omega} \)

Apply the inverse formula:

\[ \frac{1}{2} x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin \omega}{\omega} e^{j\omega t} \, d\omega \]
Hence

\[
\int_{-\infty}^{\infty} \frac{\sin \omega}{\omega} e^{j\omega t} d\omega = \pi X(t)
\]

Take complex conjugates of both sides, and note that \( X(t) \) is real-valued:

\[
\int_{-\infty}^{\infty} \frac{\sin \omega}{\omega} e^{-j\omega t} d\omega = \pi X(t)
\]

Swap \( t \) with \( \omega \):

\[
\int_{-\infty}^{\infty} \frac{\sin t}{t} e^{-j\omega t} dt = \pi \tilde{X}(\omega) = \pi X(\omega)
\]

\[
F(\omega) = \pi X(\omega) = \begin{cases} 
\pi, & \text{if } |t| \leq 1 \\
0, & \text{if } |t| > 1
\end{cases}
\]
3. \[ G(\omega) = \frac{1}{2} \left[ F(\omega - 12\pi) + F(\omega + 12\pi) \right] \]
We see that $H(w) = F(w) G(w) = 0$.

Hence $h(t) = 0 \quad \forall t \in \mathbb{R}$. 

\[ h(t) \]

\[ t \]
Consider the feedback system shown below with plant \( G_p(s) \), controller \( G_c(s) \), reference input \( r(t) \) and output \( y(t) \). The controller transfer function is \( G_c(s) = K_p \) (proportional control) with \( K_p > 0 \) for all parts of this problem.

For parts (a) through (d), assume that the plant transfer function is \( G_p(s) = \frac{1}{s(s + 4)} \).

(a) Find the transfer function and poles of the closed loop system, \( G_c(s) \).

(b) Find \( y(t) \) for \( K_p = 3 \) and \( r(t) = u(t) \) (where \( u(t) \) is the unit step function).

(c) Find \( y(t) \) for \( K_p = 20 \) and \( r(t) = \cos(4t) \).

(d) Indicate if the following statements are “True”, “False”, or “Can’t Tell”. Select “Can’t Tell” if there is not enough information to determine if it is true or false. Briefly explain/justify your answers.

(d1) The plant \( G_p(s) \) is unstable (in the sense of bounded input – bounded output).
(d2) The closed loop system is underdamped for \( K_p = 3 \).
(d3) If \( K_p \) increases indefinitely, the closed loop system will eventually become unstable.
(d4) It is possible to find a value of \( K_p \) so that \( G_c(s) \) is critically damped.
(d5) The closed loop system is a high pass filter for some values of \( K_p > 0 \).

(e) Now assume that the transfer function of the plant is \( G_p(s) = \frac{1}{s(s - 4)} \) and \( G_c(s) = K_p \). Show mathematically whether or not proportional control (with \( K_p > 0 \)) can stabilize the closed loop system.
**Problem 15 (Core: S&C-ECE3550)**

**Solution**

\( G_{cl}(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \frac{K_p}{s(s+4)} = \frac{K_p}{s^2 + 4s + K_p} = \frac{K_p}{(s+2)^2 - 4 + K_p} \)

Poles are \( p_{1,2} = -2 \pm \sqrt{4-K_p} \).

(b) \( G_{cl}(s) = \frac{3}{s^2 + 4s + 3} = \frac{3}{(s+1)(s+3)} \)

\[ Y(s) = \frac{1}{s} G_{cl}(s) = \frac{3}{s(s+1)(s+3)} = \frac{c_1}{s} + \frac{c_2}{(s+1)} + \frac{c_3}{(s+3)} \]

\[ c_1 = 1; \quad c_2 = \frac{3}{(-1)(-1+3)} = -\frac{3}{2}; \quad c_3 = \frac{3}{(-3)(-3+1)} = \frac{1}{2} \]

\[ y(t) = u(t) - \frac{3}{2} e^{-t} u(t) + \frac{1}{2} e^{-3t} u(t) \]

(c) Work in the frequency domain.

\[ G_{cl}(j\omega) = \frac{20}{-\omega^2 + 4j\omega + 20} \]

\[ G_{cl}(j4) = \frac{20}{-16 + 16j + 20} = \frac{20}{4+16j} = 0.2941 - 1.1765j = 1.2127R - 1.3258 \]

\[ y(t) = 1.2127 \cos(4t - 1.3258) \]

(d1) True (due to the pole at \( s = 0 \) on the \( j\omega \) axis).

(d2) False, it is overdamped (two distinct, real poles).

(d3) False, it becomes increasingly underdamped but not unstable since the poles remain in the left half plane.

(d4) True, \( K_p = 4 \) results in two poles at \( s = -2 \).

(d5) False, it is a low pass filter; \( G(j0) = 1 \) and \( \lim_{\omega \to \infty} |G_{cl}(j\omega)| = 0 \forall K_p \).

(e) \( G_{cl}(s) = \frac{K_p}{s(s-4)} = \frac{K_p}{s^2 - 4s + K_p} \)

This system has poles in the right half plane for all values of \( K_p \), so proportional control cannot stabilize the plant.
Consider a 32-byte cache organized as 4-way set-associative with 4-byte blocks.

(a) Assuming 16-bit addresses, show how the address bits are used for accessing the cache.

(b) For the following *hexadecimal* byte addresses, label each reference a hit or miss (assume the most significant byte of each address is 00).

(c) Show the final contents of the cache (i.e. which memory locations are stored in the cache) by showing the contents of the TAG for each cache block.

Assume the cache is initially empty and LRU (least recently used) replacement is used.

<table>
<thead>
<tr>
<th>H/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C</td>
</tr>
<tr>
<td>05</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>0A</td>
</tr>
<tr>
<td>07</td>
</tr>
<tr>
<td>00</td>
</tr>
<tr>
<td>23</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>00</td>
</tr>
<tr>
<td>22</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>2A</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>1B</td>
</tr>
<tr>
<td>2B</td>
</tr>
</tbody>
</table>
Consider a 32-byte cache organized as 4-way set-associative with 4-byte blocks.

(a) Assuming 16-bit addresses, show how the address bits are used for accessing the cache.

```
<table>
<thead>
<tr>
<th>H/M</th>
<th>M</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>05</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0A</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>07</td>
<td>Hit</td>
<td>1</td>
</tr>
<tr>
<td>00</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>M</td>
<td>0</td>
</tr>
<tr>
<td>00</td>
<td>Hit</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>Hit</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>M</td>
<td>0</td>
</tr>
<tr>
<td>2A</td>
<td>Hit</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>Hit</td>
<td>0</td>
</tr>
<tr>
<td>1B</td>
<td>M</td>
<td>0</td>
</tr>
<tr>
<td>00</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>010</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>011</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
```

(b) For the following hexadecimal byte addresses, label each reference a hit or miss (assume the most significant byte of each address is 00).

(c) Show the final contents of the cache (i.e. which memory locations are stored in the cache) by showing the contents of the TAG for each cache block.

Assume the cache is initially empty and LRU (least recently used) replacement is used.
The question is related to function call mechanisms, let’s assume the following convention is used when implementing function calls:

1. The stack grows downwards unless specified otherwise.
2. The stack contains the following items, in the order that they are allocated on the stack:
   - Return address
   - Frame Pointer
   - Input Parameters
   - Return Value
   - Local Variables of the Callee
3. All parameters are passed via the stack (none are passed via registers)
4. Caller only allocates part of the activation frame for the callee before calling 'jal' - it only allocates until the return value (including return value) and updates the stack pointer accordingly (so it points to the return value).
5. Caller pushes input parameters onto the stack in the reverse order as the parameters are declared.
6. Callee is responsible of populating the rest of the activation frame (just the local variables).
7. Callee is responsible of deallocating the local variable part of the activation frame when it returns
8. Caller is responsible of deallocating the rest of the callee’s activation frame.

In addition to the above, we also have the following:

9. The local variables are allocated in the same order as they were declared.
10. Stack pointer points to the last occupied slot on the stack.

The questions are on the next 2 pages.
Problem 17 (Core: CSS-ECE2035) Solution

Part A: Suppose we have the following program written in C.

```c
int foo(int n) {
    int *y, i;
    y = (int*)malloc(n * 1024 * sizeof(int)); //sizeof(int) is 4
    //some usages of y
    return 0;
}

int main() {
    int i;
    for(i=1;i<10000;i++) {
        foo(2048);
    }
    return 0;
}
```

The above program has the classical problem of memory leak: the program keeps allocating more and more memory, and never frees. Eventually, the system will run out of memory.

We have the following assumptions about the above program:
1. The code and global variables take 1Mbytes
2. The stack of main() takes 1Mbytes
3. Stack starts from the top of the address space and grows downward
4. Heap starts right on top of the code and global variable, and grows upward
5. The address space is 32-bit, and a process can use up the entire 4GB address space.

1. Where does ‘malloc()’ get the memory allocation, heap or stack?

2. During which iteration will the above program deplete all the usable memory? Answer this question using the value of index i in the for loop in main.
**Problem 17 (Core: CSS-ECE2035) Solution**

**Part B:** suppose the program now changes to:

```c
int foo(int n) {
    int y[n*1024], i;
    //some usages of y;
    return 0;
}

int main() {
    for(i=1;i<10000;i++) {
        foo(2048);
    }
    return 0;
}
```

This program uses the Variable Length Array (VLA) feature of C99.

1. Where does array `y` get its memory allocation, Heap or Stack?

2. Does the program still have the memory leak problem (that its memory usage will grow continuously)?

3.A If the answer is yes, will a garbage collector solve the memory leak problem for this program?

3.B If the answer is no, what is the maximum amount of memory that will be consumed by the above program?
Part A: Suppose we have the following program written in C.

```c
int foo(int n) {
    int *y, i;
    y = (int *)malloc(n * 1024 * sizeof(int)); // sizeof(int) is 4
    // some usages of y
    return 0;
}

int main() {
    int i;
    for(i=1; i<10000; i++) {
        foo(2048);
    }
    return 0;
}
```

The above program has the classical problem of memory leak: the program keeps allocating more and more memory, and never frees. Eventually, the system will run out of memory.

We have the following assumptions about the above program:
1. The code and global variables take 1Mbytes
2. The stack of main() takes 1Mbytes
3. Stack starts from the top of the address space and grows downward
4. Heap starts right on top of the code and global variable, and grows upward
5. The address space is 32-bit, and a process can use up the entire 4GB address space.

1. Where does ‘malloc()’ get the memory allocation, heap or stack?

**Heap**

2. During which iteration will the above program deplete all the usable memory? Answer this question using the value of index i in the for loop in main.

<table>
<thead>
<tr>
<th>code and global:</th>
<th>1MB</th>
</tr>
</thead>
<tbody>
<tr>
<td>stack of main:</td>
<td>1 MB</td>
</tr>
<tr>
<td>stack of each call to foo:</td>
<td>8MB + 24B (1 return address, 1 frame pointer, 1 input, 1 return value, 2 local variables)</td>
</tr>
<tr>
<td>so we have:</td>
<td>1MB + 1MB + n(8M+24B) &lt; 4096MB</td>
</tr>
<tr>
<td>so:</td>
<td>n &lt;511.7</td>
</tr>
<tr>
<td>so:</td>
<td>we can do at most 511 iterations.</td>
</tr>
</tbody>
</table>
Part B: suppose the program now changes to:

```c
int foo(int n) {
    int y[n*1024], i;
    //some usages of y;
    return 0;
}

int main() {
    int i;
    for(i=1;i<10000;i++) {
        foo(2048);
    }
    return 0;
}
```

This program uses the Variable Length Array (VLA) feature of C99.

1. Where does array y get its memory allocation, Heap or Stack?

**stack**

2. Does the program still have the memory leak problem (that its memory usage will grow continuously)?

**NO**

3.A If the answer is yes, will a garbage collector solve the memory leak problem for this program?

**N/A**

3.B If the answer is no, what is the maximum amount of memory that will be consumed by the above program?

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>code + global</td>
<td>1MB</td>
</tr>
<tr>
<td>stack of main</td>
<td>1MB</td>
</tr>
<tr>
<td>each call to foo</td>
<td>8MB + 20B</td>
</tr>
<tr>
<td><strong>Total consumption</strong></td>
<td>8MB + 20B</td>
</tr>
</tbody>
</table>
Between hosts A in Atlanta and B in Las Vegas there are 2 routers (X,Y). The link between routers (---) is 10 Mbps. The access links (LANs, ===) are 1000 Mbps. The distance from A to B is 2500 km. A starts to send a large file using TCP, sending 1500 byte packets to B. B ACKs with 40 byte packets. There is no other traffic on this network.

\[ A ===X---Y===B \]

1. What is the time required to transmit a 1200 Byte datagram at 10 Mbit/s? _________________ ms

2. What is the propagation delay one-way in milliseconds (ms): _________________ ms

3. What is the average transport rate for a 5 kByte Window size and this RTT? _________________ Mbit/s

4. Other traffic builds up the average level in X's X-to-Y output buffer to 100 kBytes. What does this queuing delay add to the RTT? _______________ ms

5. Before sending an IP datagram, what protocol is used to find the right Ethernet address? ____________

What four items of information that a host needs before it can operate normally on the network?

6. _________________

7. _________________

8. _________________

9. _________________

10. If these are not configured manually, what protocol can be used to get them over the network? _________________
Between hosts A in Atlanta and B in Las Vegas there are 2 routers (X, Y). The link between routers (---) is 10 Mbps. The access links (LANs, ===) are 1000 Mbps. The distance from A to B is 2500 km. A starts to send a large file using TCP, sending 1500 byte packets to B. B ACKs** with 40 byte packets. There is no other traffic on this network.

\[ A \text{---} X \text{---} Y \text{===} B \]

1. What is the time required to transmit a 1200 Byte datagram at 10 Mbit/s? ______________ 0.96 _____ ms

2. What is the propagation delay one-way in milliseconds (ms): ______ 12.5 ______ ms (or 8.33*)

3. What is the average transport rate for a 5 kByte Window size and this RTT? ___ 1.5 Mbps ___(or 2.3 *) Mbit/s

4. Other traffic builds up the average level in X's X-to-Y output buffer to 50 kBytes. What does this queuing delay add to the RTT? ____ 80 ____ms (note: 79 if they subtract answer 1)

5. Before sending an IP datagram, what protocol is used to find the right Ethernet address? ___ ARP ___
   (or Address Resolution Protocol)

   What four items of information that a host needs before it can operate normally on the network?

6. ___ IP Address for itself

7. ___ Network Mask ___

8. ___ IP Address for the Gateway (or Subnet, or Default) Router ___

9. ___ IP Address for the Local (or Resolver) DNS Server ___

10. If these are not configured manually, what protocol can be used to get them over the network?

   ______ DCHP (or mDNS, or Zeroconf)_________________________

   The speed of a signal in fiber or cable is 2E8 m/s.

   * Answers if student assumed radio where speed is 3E8 m/s (no penalty).
Problem 19 (Specialized: Optics-ECE4500) Solution

PROBLEM

Refractive Index and Absorption Coefficient in a Lossy Medium

When a light beam propagates in a lossy material, the light power \( P \) decays exponentially following the Beer’s law: \( P(z) = P(0)e^{-\alpha z} \), where \( P(0) \) is the level of the initial power, \( \alpha \) is the absorption coefficient, and \( z \) is the distance the light has travelled. The electric field of the light wave can be expressed as \( E = E_0e^{-i\omega t}e^{ikz} \), where \( \omega \) is the angular frequency, and \( k = 2\pi n/\lambda_0 \) is the wave vector of light in the medium. The refractive index, \( n = n' + in'' \), is generally complex in a lossy material.

(1) Based on the above information, develop, showing all work, a formula that describes the relation between the absorption coefficient \( \alpha \) and the imaginary part of the refractive index \( n'' \). Your result should contain only \( \alpha \), \( n'' \), the free-space wavelength \( \lambda_0 \), and mathematical constants (\( \pi \), \( e \), etc.).

(2) The table below shows the complex refractive index of silicon for a series of photon energies \( h\nu \).

<table>
<thead>
<tr>
<th>( h\nu ) (eV)</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n' )</td>
<td>3.517</td>
<td>3.565</td>
<td>3.625</td>
<td>3.699</td>
<td>3.788</td>
<td>3.898</td>
</tr>
<tr>
<td>( n'' )</td>
<td>0</td>
<td>1.9×10^{-4}</td>
<td>1.7×10^{-3}</td>
<td>5.1×10^{-3}</td>
<td>0.010</td>
<td>0.016</td>
</tr>
</tbody>
</table>

A laser beam at the wavelength of \( \lambda_0 = 1.03 \mu m \) is incident upon a double-side polished silicon wafer of thickness \( d = 0.6 \) mm. The wafer is placed in the air. Estimate, showing all work, the transmission \( T = P_{out}/P_{in} \) of the light through the wafer.
Refractive Index and Absorption Coefficient in a Lossy Medium

(1) The electric field of light is given as \( E = E_0 e^{-i\omega t} e^{ikz} \), and
\[
e^{ikz} = \exp \left[ i \left( \frac{2\pi n}{\lambda_0} \right) z \right] = \exp \left[ i \left( \frac{2\pi (n' + in'\prime)}{\lambda_0} \right) z \right] = \exp[i (2\pi n'/\lambda_0)z] \cdot \exp[-(2\pi n''/\lambda_0)z].
\]
So the magnitude of electric field decays following \( |E(z)| \propto \exp[-(2\pi n''/\lambda_0)z] \), and the light power \( P(z) \propto |E(z)|^2 \propto \exp[-(4\pi n''/\lambda_0)z] \). By comparing this expression to the Beer’s law, we obtain: \( \alpha = 4\pi n''/\lambda_0 \).

(2) The wavelength of \( \lambda_0 = 1.03 \, \mu m \) corresponds to the photon energy \( h\nu = 1.24/1.03 = 1.20 \, eV \). By looking up the table we find the complex refractive index \( n = n' + in'' = 3.565 + 1.9\times10^{-4}i \) at this wavelength. So the absorption coefficient is \( \alpha = 4\pi n''/\lambda_0 = 2.3\times10^3 \, m^{-1} \).

At the front (air-silicon) interface, the power reflection coefficient is
\[
R_1 = \left| \frac{n-1}{n+1} \right|^2 \approx \left| \frac{3.565 - 1}{3.565 + 1} \right|^2 \approx 0.316
\]
And the power transmission coefficient is \( T_1 = 1 - R_1 = 0.684 \).

The internal transmission of light through the wafer is given by the Beer’s law:
\[
T_{\text{internal}} = e^{-\alpha z} = \exp[-(2.3\times10^3) \times (0.6\times10^{-3})] = 0.252
\]
At the back (silicon-air) interface, the power reflection coefficient is again
\( R_2 = 0.316 \), and the power transmission coefficient is \( T_2 = 1 - R_2 = 0.684 \).

So the overall transmission of light through the wafer is
\[
T = T_1 T_{\text{internal}} T_2 = 0.684 \times 0.252 \times 0.684 = 0.118.
\]
Note that the contribution to \( T \) from light experiencing multiple-reflection in the wafer can be ignored.
Consider the optical data link shown below, in which the total span length is \( L = L_1 + L_2 = 150 \text{ km} \).

The optical transmitter provides average power \( P_t = 10 \text{ dBm} \) at carrier wavelength \( \lambda_m \); the receiver has sensitivity \( P_{rec} = -30 \text{ dBm} \), which yields a specified BER. Two single mode fibers make up the span, and these have loss and dispersion specifications as indicated. The second fiber is dispersion-compensating fiber (DCF); this fiber, while designed to have high negative dispersion, exhibits relatively high loss.

\[
\begin{array}{c|c|c}
\text{Tx} & D_1 = +6.0 \text{ ps/nm-km} & D_2 = -100 \text{ ps/nm-km} \\
\text{L}_1 & a_1 = 0.2 \text{ dB/km} & a_2 = 3.0 \text{ dB/km} \\
\text{Rx} & -30.0 \text{ dBm} & \text{L}_2 \\
\end{array}
\]

To achieve an overall dispersion penalty for the link that is 1 dB (the dispersion limit) or less, the path average dispersion magnitude, \( |D_{avg}| \), must be less than or equal to 2.0 ps/nm-km, where

\[
D_{avg} = \frac{D_1 L_1 + D_2 L_2}{L_1 + L_2}
\]

a. Find the lengths, \( L_1 \) and \( L_2 \), such that the link exhibits the minimum loss within the dispersion limiting constraint as defined above.

b. Determine whether or not the link is viable from a loss standpoint, as a result of your part a computations. If not, specify the minimum required gain in dB of an optical amplifier that should be positioned in line to compensate loss.

c. With the link constructed as per your results, and with the amplifier (if needed) installed, will this link be dispersion-limited, or loss-limited, or both? Explain.

d. Suppose the optical carrier wavelength is shifted to a different value, and that loss is to first order invariant with wavelength. It is known, however, that dispersion in both fibers will vary significantly with wavelength. What must be true about the relation between fiber dispersions in this link so that your previous results will hold true at all wavelengths? You should display this requirement in a simple formula.
Consider the optical data link shown below, in which the total span length is \( L = L_1 + L_2 = 150 \text{ km} \). The optical transmitter provides average power \( P_t = 10 \text{ dBm} \) at carrier wavelength \( \lambda_m \); the receiver has sensitivity \( P_{rec} = -30 \text{ dBm} \). Two single mode fibers make up the span, and these have loss and dispersion specifications as indicated. The second fiber is dispersion-compensating fiber (DCF); this fiber, while designed to have high negative dispersion, exhibits relatively high loss.

\[ D_1 = +6.0 \text{ ps/nm-km} \quad D_2 = -100 \text{ ps/nm-km} \quad a_1 = 0.2 \text{ dB/km} \quad a_2 = 3.0 \text{ dB/km} \]

To achieve an overall dispersion penalty for the link that is 1 dB (the dispersion limit) or less, the path average dispersion magnitude, \( |D_{avg}| \), must be less than or equal to 2.0 ps/nm-km, where

\[ D_{avg} = \frac{D_1 L_1 + D_2 L_2}{L_1 + L_2} \]

a. Find the lengths, \( L_1 \) and \( L_2 \), such that the link exhibits the minimum loss within the dispersion limiting constraint as defined above.

We set up:

\[ |D_{avg}|L = |6.0L_1 - 100(L - L_1)| \leq 2(150) \]

For minimum loss, \( L_2 \) should be as short as possible, meaning that we need \( |D_{avg}|L = 300 \). Therefore, \( L_1 = 144.3 \text{ km} \), and \( L_2 = 5.7 \text{ km} \).

b. Determine whether or not the link is viable from a loss standpoint, as a result of your part a computations. If not, specify the minimum required gain in dB of an optical amplifier that should be positioned in line to compensate loss.

With the lengths as found in part a, the net loss becomes (including the 1-dB dispersion penalty):

\[ \text{Loss} = 0.2(144.3) + 3.0(5.7) + 1.0 = 47 \text{ dB} \]

This value is greater than the allowed budget, which is 10dBm − (−30dBm) = 40dB. Therefore, a 7dB amplifier is needed at minimum to satisfy the power budget.

c. With the link constructed as per your results, and with the amplifier (if needed) installed, will this link be dispersion-limited, or loss-limited, or both? Explain.

The link will be both dispersion and loss limited, because the overall length is the maximum allowable within the dispersion and loss constraints.

d. Suppose the optical carrier wavelength is shifted to a different value, and that loss is to first order invariant with wavelength. It is known, however, that dispersion in both fibers will vary significantly with wavelength. What must be true about the relation between fiber dispersions in this link so that your previous results will hold true at all wavelengths? Display this requirement in a simple formula.

The change in dispersion with wavelength in the first fiber must be just compensated by the change in dispersion in the second, leading to a path average dispersion that is invariant with wavelength. In terms of the original wavelength \( \lambda_m \) and wavelength shift \( \Delta \lambda \), we have:

\[ D_{avg} = \frac{1}{L} \left[ \left( D_1(\lambda_m) + \Delta \lambda \frac{dD_1}{d\lambda} |_{\lambda_m} \right) L_1 + \left( D_2(\lambda_m) + \Delta \lambda \frac{dD_2}{d\lambda} |_{\lambda_m} \right) L_2 \right] = 2 \]

To make this result independent of \( \Delta \lambda \), the slopes must compensate, or

\[ L_1 \frac{dD_1}{d\lambda} |_{\lambda_m} = -L_2 \frac{dD_2}{d\lambda} |_{\lambda_m} \]
(2 pts) An electrocardiogram (ECG) represents the electrical activity of the heart, measured on the surface of the body. How can you calculate average heart rate from a ten second long ECG recording?

(3 pts) *Instantaneous* heart rate is defined as the inverse of the heartbeat period from one beat to the next – do you expect for a healthy adult to have any variability in their instantaneous heartbeat over the course of a 10 second recording? Why or why not?

(2 pts) Is an arrhythmia with a wide QRS complex a ventricular arrhythmia or a supra-ventricular arrhythmia? Why?

A patient is having premature atrial contractions (PACs), and otherwise has a completely healthy heart. The PACs are originating from several different ectopic foci within the atrium, causing the degree of prematurity to vary drastically from one PAC to the next.

(1 pt) Do you expect the PAC heartbeats to have more or less stroke volume compared to the normal rhythm beats?

(2 pts) Do you expect any relationship between the degree of prematurity of the PAC and the ensuing heartbeat’s stroke volume? Specifically, if the time interval from the current beat’s R-wave to the previous beat’s R-wave is shorter for one PAC compared to another, would you expect the stroke volume to be different, and, if so, higher or lower?
SOLUTION

(2 pts) An electrocardiogram (ECG) represents the electrical activity of the heart, measured on the surface of the body. How can you calculate average heart rate from a ten second long ECG recording?
   Answer: You can measure the R-R interval for each beat, then average these intervals over all beats in the recording.

(3 pts) *Instantaneous* heart rate is defined as the inverse of the heartbeat period from one beat to the next – do you expect for a healthy adult to have any variability in their instantaneous heartbeat over the course of a 10 second recording? Why or why not?
   Answer: Yes, it will change from one beat to the next due to respiratory sinus arrhythmia (RSA), otherwise known as heart rate variability (HRV). As one breathes, the baroreceptors in the aorta will experience varying forces which will cause the autonomic nervous system to change the heart rate accordingly; there are also other mechanisms responsible for this variability, including the changing of preload and afterload for the heart, but as long as HRV / RSA is mentioned, this is sufficient.

(2 pts) Is an arrhythmia with a wide QRS complex a ventricular arrhythmia or a supra-ventricular arrhythmia? Why?
   Wide QRS complex indicates that the arrhythmia originated in the ventricles; a narrow QRS indicates that the conduction propagated through the Purkinje fibers, which would mean that it propagated though the AV node (supra-ventricular).

A patient is having premature atrial contractions (PACs), and otherwise has a completely healthy heart. The PACs are originating from several different ectopic foci within the atrium, causing the degree of prematurity to vary drastically from one PAC to the next.

(1 pt) Do you expect the PAC heartbeats to have more or less stroke volume compared to the normal rhythm beats?
   Less stroke volume since there is less time to fill the heart (decreased preload).

(2 pts) Do you expect any relationship between the degree of prematurity of the PAC and the ensuing heartbeat’s stroke volume? Specifically, if the time interval from the current beat’s R-wave to the previous beat’s R-wave is shorter for one PAC compared to another, would you expect the stroke volume to be different, and, if so, higher or lower?
   Decreasing the filling time will lead to decreased preload and thus decreased stroke volume. Thus, increased prematurity (i.e. decreased preceding R-R interval) will lead to decreased ensuing stroke volume.
The intracellular and extracellular concentrations and conductances for the axon of a newly discovered squid species at rest are given below.

A) Find the Nernst potential for Na$^+$, K$^+$, and Cl$^-$. 
B) Find the membrane resting potential.

\[ k = 1.38 \times 10^{-23} \text{ J/K} \]
\[ q = 1.6 \times 10^{-19} \text{ C} \]
\[ T = 37 \degree C \]

<table>
<thead>
<tr>
<th>Species</th>
<th>Intracellular (mM)</th>
<th>Extracellular (mM)</th>
<th>Conductances (mS/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>500</td>
<td>10</td>
<td>0.415</td>
</tr>
<tr>
<td>Na</td>
<td>70</td>
<td>350</td>
<td>0.010</td>
</tr>
<tr>
<td>Cl</td>
<td>24</td>
<td>350</td>
<td>0.582</td>
</tr>
</tbody>
</table>
A) $E_{Na^+} = -\frac{kT}{e} \ln \left( \frac{70}{350} \right) = 43.0 \text{ mV}$

$E_{K^+} = -\frac{kT}{e} \ln \left( \frac{500}{10} \right) = -104.5 \text{ mV}$

$E_{Cl^-} = -\frac{kT}{e} \ln \left( \frac{2.4}{350} \right) = -71.6 \text{ mV}$

B) $V_{rest} = \frac{S_K E_K + S_{Na^+} E_{Na^+} + S_{Cl^-} E_{Cl^-}}{S_K + S_{Na^+} + S_{Cl^-}}$

$= \frac{(-104.5)(0.415) + (45.0)(0,010) + (-71.6)(0.582)}{0.415 + 0.010 + 0.582}$

$= -84.6 \text{ mV}$