INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Note that Problem 2 occupies 3 pages, Problem 4 occupies 2 pages, Problem 14 occupies 2 pages, Problem 17 occupies 2 pages, and Problem 20 occupies 2 pages. Including the cover sheet, you should have 61 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:

   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in EVERY sheet you are turning in.
   (c) On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.
   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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</table>
A function $F(A,B,C,D)$ is defined according to the truth table below. Here ‘x’ means ‘don’t care’.

<table>
<thead>
<tr>
<th>A B C D</th>
<th>F</th>
<th>A B C D</th>
<th>F</th>
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<tbody>
<tr>
<td>0 0 0 0</td>
<td>1</td>
<td>1 0 0 0</td>
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<tr>
<td>0 0 0 1</td>
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<td>1 0 1 1</td>
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<tr>
<td>0 1 0 0</td>
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<td>0 1 0 1</td>
<td>0</td>
<td>1 1 0 1</td>
<td>x</td>
</tr>
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<td>0 1 1 0</td>
<td>1</td>
<td>1 1 1 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1 1 1</td>
<td>x</td>
<td>1 1 1 1</td>
<td>x</td>
</tr>
</tbody>
</table>

(a) Express the function in a sum-of-minterms and ‘don’t cares’ form.

$$F(A,B,C,D) = \text{_______________________________}$$

(b) Show the function in a Karnaugh map, and identify (circle) all Prime Implicants.

(c) Express the function as a minimized sum-of-products form.

$$F(A,B,C,D) = \text{_______________________________}$$

(d) List the Essential Prime Implicants.
A function $F(A,B,C,D)$ is defined according to the truth table below. Here 'x' means 'don’t care'.

<table>
<thead>
<tr>
<th>A B C D</th>
<th>F</th>
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<tbody>
<tr>
<td>0 0 0 0</td>
<td>1</td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>1</td>
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<tr>
<td>0 0 1 0</td>
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<td>0 0 1 1</td>
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<td>0 1 0 0</td>
<td>x</td>
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<td>0 1 1 1</td>
<td>x</td>
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<td>1 0 1 0</td>
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<td>1 1 0 0</td>
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<tr>
<td>1 1 0 1</td>
<td>x</td>
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<tr>
<td>1 1 1 0</td>
<td>0</td>
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<tr>
<td>1 1 1 1</td>
<td>x</td>
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</tbody>
</table>

(a) Express the function in a sum-of-minterms and 'don’t cares' form.

$$F(A,B,C,D) = \sum m(0,1,2,6,8,9,10) + \sum d(4,7,13,15)$$

(b) Show the function in a Karnaugh map, and identify (circle) all Prime Implicants.

![Karnaugh Map]

(c) Express the function as a minimized sum-of-products form.

$$F(A,B,C,D) = B'D' + B'C' + A'D'$$

(d) List the Essential Prime Implicants.

$$B'D' \quad B'C'$$
The state transition table of a 4-state FSM (2 flip flops) is given below.

<table>
<thead>
<tr>
<th>Present State = A(t), B(t)</th>
<th>Input = I</th>
<th>Next State = A(t+1), B(t+1)</th>
<th>Output = Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
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<td>0</td>
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<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

You are to design a finite state machine that realizes the above state transition diagram/state transition with one toggle flip flop (realizing the state A) and one D flip flop (realizing the state B) instead of two D flip flops. A toggle flip flop works as follows. When the input T(A) of the toggle flip flop is 1, the output A(t+1) of the flip flop becomes the complement of A(t). When the input T(A) of the flip flop is 0, the output A(t+1) of the flip flop is the same as its previous value A(t). This is shown in the state transition table for the T flip flop below. Both flip flops are positive edge triggered.

<table>
<thead>
<tr>
<th>T(A)</th>
<th>A(t)</th>
<th>A(t+1)</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>hold</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>hold</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>toggle</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>toggle</td>
</tr>
</tbody>
</table>

In contrast, the state transition table of a D flip flop is as follows:

<table>
<thead>
<tr>
<th>D(B)</th>
<th>B(t)</th>
<th>B(t+1)</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>B(t+1) = D(B)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>B(t+1) = D(B)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>B(t+1) = D(B)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>B(t+1) = D(B)</td>
</tr>
</tbody>
</table>
(a) Below, fill in the K-maps for $T(A)$, $D(B)$ and $Z$ and write the minimal Boolean expressions for the same.

$T(A) = \begin{array}{cccc}
A & B & I \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{array}$

$D(B) = \begin{array}{cccc}
A & B & I \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{array}$

$Z = \begin{array}{cccc}
A & B & I \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{array}$
(b) Draw a circuit diagram for the finite state machine showing all logic, FSM input and output and the two flip flops corresponding to A and B.
(a) Below, fill in the K-maps for T(A), D(B) and Z and write the minimal Boolean expressions for the same.

\[ T(A) = \overline{B} + I \]

\[ D(B) = \overline{I} \]

\[ Z = \overline{BI} + AI \]
(b) Draw a circuit diagram for the finite state machine showing all logic, FSM input and output and the two flip flops corresponding to A and B.
**Problem 3 (Core: CSS-ECE3055)**

The following RISC assembly language program is executed on a 32-bit MIPS processor. Fill in the register values that will be present, after execution of this program. A summary of MIPS instructions is included at the bottom of the page – for anyone unfamiliar with the MIPS instruction set. Prior to execution of the program, memory location 0x04000 contains 0x20313055. **Note:** 0x indicates hexadecimal and all answers must be in hexadecimal, default is decimal in the MIPS assembly language source file. A MIPS memory word or register contains 32-bits. Use XXXXXXXX for an undefined value.

lw $3, 0x04000
sll $4, $3, 7
sub $2, $4, $3
xor $3, $4, $2
lui $5, 10
ori $5, $5, 12561
add $6, $4, $3
bne $5, $6, LABEL1
addi $6, $0, -2035
LABEL1:
sw $6, 0x04000

After execution of the MIPS code sequence above,

R2 = 0x_____________________ (in hexadecimal)

R3 = 0x_____________________ (in hexadecimal)

R4 = 0x_____________________ (in hexadecimal)

R5 = 0x_____________________ (in hexadecimal)

Memory Location 0x04000 contains: 0x_____________________ (in hexadecimal)

The MIPS processor contains thirty-two 32-bit registers, $0 through $31. $0 always contains a zero. By default, all arithmetic operations use two’s complement arithmetic. Assume no branch delay slot is present.

<table>
<thead>
<tr>
<th>MIPS Instruction</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADDI Rd, Rs, Immed</td>
<td>Rd = Rs + Immediate value</td>
</tr>
<tr>
<td>ADD Rd, Rs, Rt</td>
<td>Rd = Rs + Rt (R – register ($))</td>
</tr>
<tr>
<td>ORI Rd, Rs, Immed</td>
<td>Rd = Rs low 16-bits bitwise logical OR Immediate value</td>
</tr>
<tr>
<td>LUI Rd, Immed</td>
<td>Rd = 16-bit Immediate value high 16-bits, 0's low 16-bits</td>
</tr>
<tr>
<td>BNE Rs, Rt, address</td>
<td>Branch to address, only if Rs not equal to Rt</td>
</tr>
<tr>
<td>LW Rd, address</td>
<td>LOAD - Rd gets contents of memory at address</td>
</tr>
<tr>
<td>SLL Rd, Rs, count</td>
<td>Shift left logical (use 0 fill) by count bits</td>
</tr>
<tr>
<td>SUB Rd, Rs, Rt</td>
<td>Rd = Rs - Rt</td>
</tr>
<tr>
<td>SW Rd, address</td>
<td>STORE - memory at address gets contents of Rd</td>
</tr>
<tr>
<td>XOR Rd, Rs, Rt</td>
<td>Rd = Rs bitwise logical XOR Rt</td>
</tr>
</tbody>
</table>
Problem 3 (Core: CSS-ECE3055) Solution

Solution
Problem 4 (Core: VSDD-ECE3060) Solution

Problem

Consider the following logic gate:

![Logic Gate Diagram]

(a) Size the NMOS and PMOS devices so that the output resistance is the same as that of an inverter with an NMOS of $W/L = 1$ and PMOS of $W/L = 2$. (You can put the sizing numbers on the transistor diagram). (5 point)
(b) For the previous circuit and your sizing solution, compute the RC delay for the following transitions. Clearly show the values of all junction capacitances and the resistances of each device in the circuit schematic of part-a. Show your RC delay computation to receive full credits. (5 points)

<table>
<thead>
<tr>
<th>Transition</th>
<th>RC Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ABCD: (0111) → (0110)</td>
<td></td>
</tr>
<tr>
<td>(ii) ABCD: (1011) → (1010)</td>
<td></td>
</tr>
</tbody>
</table>

Make the following assumptions:

- The junction capacitance of NMOS and PMOS with width W is C.
- The resistance of NMOS of size W is R.
- The electron mobility is 2X higher than the hole mobility ($\mu_n/\mu_p = 2$).
- Threshold voltage, channel length, and oxide thickness of PMOS and NMOS are same.
- The total load capacitance at output need to consider this external load cap and the junction capacitances of the NMOS/PMOS connected to the output.
Part (a):

![Circuit Diagram]

Part (b)

<table>
<thead>
<tr>
<th>Transition</th>
<th>RC Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ABCD: (0111) → (0110)</td>
<td>R(15C)</td>
</tr>
<tr>
<td>(ii) ABCD: (1011) → (1010)</td>
<td>R(15C) + 8C(R+R/2) = 27RC</td>
</tr>
</tbody>
</table>
**Problem**

A square loop of current with side lengths $L$ lies on the $xy$-plane, centered on the origin. Given a line current $I$ within the loop, what is the magnetic field $H$ at points along the $z$-axis, i.e., what is $H(0,0,z)$?

*Hint:* The following integral might be useful:

$$
\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 + u^2}} + C
$$
Square-loop current:

(a) Pick an integral based on the charge/current distribution:

\[ \vec{H}(\vec{r}) = \oint L \frac{I \vec{d} \times (\vec{r} - \vec{r}')} {4\pi|\vec{r} - \vec{r}'|^3} \]

(b) Expand variables of integration and point of observation:

Variable of Integration: \( \vec{r}' = x'\hat{x} + y'\hat{y} \)

Point of Observation: \( \vec{r} = z\hat{z} \)

Note that for this problem, all of our integration occurs in the xy-plane and all of our observation points are limited to the z-axis.

(c) Pick a differential element of integration:

Current in x-direction: \( \vec{d}I = dx'\hat{x} \)

Current in y-direction: \( \vec{d}I = dy'\hat{y} \)

This problem actually consists of 4 different current segments that travel in two different directions. Two travel along \( x \) and two travel along \( y \).

(d) Pick limits of integration:

\[
\oint L \frac{I \vec{d} \times (\vec{r} - \vec{r}')} {4\pi|\vec{r} - \vec{r}'|^3} = \left[ \int_{-L/2}^{L/2} \frac{I dx'\hat{x} \times (z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y})} {4\pi|z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y}|^3} \right]_{\text{Segment 1}} + \left[ \int_{-L/2}^{L/2} \frac{I dy'\hat{y} \times (z\hat{z} - \frac{L}{2}\hat{x} - y'\hat{y})} {4\pi|z\hat{z} - \frac{L}{2}\hat{x} - y'\hat{y}|^3} \right]_{\text{Segment 2}}
\]

\[
\left[ \int_{-L/2}^{L/2} \frac{I dx'\hat{x} \times (z\hat{z} - x'\hat{x} - \frac{L}{2}\hat{y})} {4\pi|z\hat{z} - x'\hat{x} - \frac{L}{2}\hat{y}|^3} \right]_{\text{Segment 3}} + \left[ \int_{-L/2}^{L/2} \frac{I dy'\hat{y} \times (z\hat{z} + \frac{L}{2}\hat{x} - y'\hat{y})} {4\pi|z\hat{z} + \frac{L}{2}\hat{x} - y'\hat{y}|^3} \right]_{\text{Segment 4}}
\]

For this problem, our integral breaks into four pieces.
Albert lives in a quiet valley in North Georgia where he operates a winery and vineyard. Set apart from civilization, he has no access to cable, DSL, phone lines, or any other wired conduit of internet access. But Albert is a crafty graduate of the Georgia Institute of Technology and devises a clever way to steal wireless internet service from the nearby town of Unprotectedlinksville, population 53. This town is 10 kilometers away from Albert, on the other side of a large mountain, and has several unprotected home WiFi servers broadcasting local internet service. Albert’s plan is to purchase 3 identical dish antennas that operate at 2.45 GHz and arrange them in the following configuration:

With this set-up, a signal will propagate from the town to the first dish in the link, which is pointed toward the town. The received power of this dish is piped directly to another dish which is pointed towards Albert’s vineyard. Thus, this pair of dish antennas acts like a passive repeater that does not require any power or maintenance. A third dish is mounted on top of Albert’s home, where a minimum value of -95 dBm must be received in order to maintain a wireless internet link on his home computer. Answer the following questions assuming matched and lossless cables. Assume that the antenna gain of the WiFi access point in town is 5 dBi, that the transmit power of this link is 30 dBm, and that both links are essentially free space (no excess loss).

(a) What is the minimum gain in dBi of these antennas to make this system work? (Ignore the effects of small-scale fading.) (7 points)

(b) If these are ideal circular dishes with 100% efficiency, what is the minimum radius based on your answer in part (a)? (3 points)

Useful Formulas:

\[ P_R = P_T + G_T + G_R - 20 \log_{10} \left( \frac{4\pi}{\lambda} \right) - 20 \log_{10}(r) - \text{Extra Loss} \]

\[ G_{\text{peak}} = \frac{4\pi A_{\text{em}}}{\lambda^2} \]

\[ f\lambda = c \]
(a) To start this problem, we recognize that there are two different radio links in this system. In the linear scale, we can write two different link budget equations:

\[
\begin{align*}
\text{Town to Passive Repeater: } P_{R1} &= P_T \frac{\lambda^2 G_T G_D}{(4\pi r_1)^2} \\
\text{Passive Repeater to Albert's House: } P_{R2} &= P_T \frac{\lambda^2 G_D^2}{(4\pi r_2)^2}
\end{align*}
\]

where \(G_D\) is the gain of the dish antenna (transmit or receive), \(r_1 = r_2 = 5000\) meters for this link, \(G_T = 5\) dBi, \(P_T = 30\) dBm, and the power received by the passive repeater \(P_{R1}\) becomes the transmit power for the second link \(P_{R2}\) (provided there are no additional losses. Thus, we can write one cumulative link budget:

\[
P_{R2} = P_T \frac{\lambda^4 G_T G_D^3}{(4\pi)^4} \quad \text{or} \quad G_D = \left( \frac{P_{R2} (4\pi r)^4}{G_T P_T (\lambda^4)} \right)^{\frac{1}{5}}
\]

If \(P_{R2}\) must be greater than \(3.2 \times 10^{-10}\) mW (-95 dBm) to operate correctly, then dish gain must be at least 32.8 dBi. This problem could have been just as easily solved with the dB-link budget equations.

(b) Using the following relationship (which is linear-scale):

\[
G = \frac{4\pi}{\lambda^2} A_{em}
\]

we see that the minimum electromagnetic area is about 2.27 m². A circle of radius 0.85m will make this dish.
Find $R$ and $C$ in the circuit below, so that $R$ consumes maximum power.

$V_s(t) = \cos t [V]$
Problem 7 (Core: EDA-ECE2040) Solution

**Solution**

![Circuit Diagram]

\[ V_s(t) = \cos t \, [V] \]

Note that \( \omega = 1 \).

\[ Z'_{th} = (1 + j\omega) \]
\[ Z_L = R \parallel \frac{1}{j\omega C} = \frac{1}{\frac{1}{R} + j\omega C} \]

Power is maximized when \( Z''_{th} = Z_L \)

\[ Z''_{th} = Z_L = 1 - j = \frac{1}{\frac{1}{R} + jC} \]

\[ \frac{1}{\frac{1}{R} + jC} = \frac{1}{1 - j} = \frac{1}{2} + \frac{1}{2}j \]

Therefore \( C = \frac{1}{2} \, F \)

Furthermore, \( R = 2 \, \Omega \)
The figure below is a BJT differential amplifier. It is given that $V_{CC} = 12$ V, $V_{EE} = 12$ V, $I_{EE} = 400 \, \mu$A, $R_{EE} = 200 \, k\Omega$, $V_T = 0.025$ V, $R_L = 2 \, k\Omega$. For all transistors, $\beta = 100$, $r_o = 300 \, k\Omega$, and $|V_{BE}| = 0.7$ V.

1. Draw the small-signal circuit.
2. Assume the current mirror and the current source $I_{EE}$ are ideal. If $R_L$ is not connected, derive and calculate the transconductance gain of the differential amplifier for differential-mode input signals.
3. With $R_L$ connected as in the circuit, calculate the differential-mode voltage gain.
Solution:

2. Since the current mirror and the current source are ideal, the DC bias conditions are:

\[ I_{E1} = I_{E2} = I_{E3} = I_{E4} = \frac{I_{EE}}{2} \]

\[ I_{C1} = I_{C2} = I_{C3} = I_{C4} = \frac{I_{EE}}{2} \times \frac{100}{101} \times \frac{400\,\text{mA}}{2} = 198\,\text{mA} \]

For all transistors,

\[ g_m = \frac{I_{C1}}{V_T} = 7.92\,\text{mS} \]

\[ r_{\pi} = \frac{\beta}{g_m} = 12.63\,\text{k}\Omega \]
Let $r_3$ be the total resistance connected between the base and the emitter of $q_3$.

$$r_3 = r_{o3} || \frac{1}{g_m} || r_{\pi3} || r_{\pi4}$$

$$\approx \frac{1}{g_m}$$

$$= 0.126 \text{ k}\Omega$$

Since $r_o$ is very large, it is neglected for transconductance derivation.

Apply KCL at node $v_e$,

$$\left(\frac{V_{id}}{2} - v_e\right) \left(\frac{1}{r_2} + g_m\right) + \left(-\frac{V_{id}}{2} - v_e\right) \left(\frac{1}{r_z} + g_m\right) = 0$$

$$\Rightarrow v_e = 0$$
Then
\[ i_1 = g_m \frac{V_{id}}{2} \]
\[ i_2 = -g_m \frac{V_{id}}{2} \]
\[ i_3 = -i_1 = i_4 \]
\[ i_{out} = -i_2 - i_4 \]
\[ = g_m V_{id} \]
Therefore, the transconductance is
\[ G_{m,dlm} = \frac{i_{out}}{V_{id}} \bigg|_{V_{out}=0} = g_m = 7.92 \text{ ms} \]

3. The output resistance of the differential amplifier is
\[ R_{out} = R_{o2} \frac{1}{1+g_m R_L} = 150 \text{ k}\Omega \]
The output voltage across \( R_L \) is
\[ V_{out} = i_{out} \cdot \frac{V_{out}}{V_{out}+R_L} \cdot R_L = g_m V_{id} \cdot \frac{V_{out}+R_L}{V_{out}+R_L} \]
The differential-mode voltage gain is
\[ A_{v,dlm} = \frac{V_{out}}{V_{id}} \bigg|_{V_{ic}=0} = g_m \cdot \frac{V_{out}+R_L}{V_{out}+R_L} = 15.63 \text{ V/V} \]
PROBLEM

A 100 hp (74,570 W) three-phase, wye-connected, 60 Hz, 4-pole, cylindrical rotor synchronous motor is operating at rated conditions and 80 percent power factor (leading). The efficiency, excluding field and stator losses, is 96 percent, and the synchronous reactance is 2.72 Ohms/phase.

Determine:

(a) developed torque;
(b) armature current;
(c) excitation voltage;
(d) power angle;
(e) maximum torque (also called pull-out torque).
(a) \[ n_s = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800 \text{ r/min} \]

\[ P_{\text{mech}} = \frac{T_n}{5252} = \Rightarrow \quad T_{\text{dev}} = \frac{5252 \cdot P_{\text{mech}}}{n} \]

\[ T_{\text{dev}} = \frac{5252 \times 100/0.96}{1800} = 304 \text{ lb-ft} = 412 \text{ N-m} \]

(b) \[ S = \frac{P_{\text{shaft}} \times 746}{\eta \times F_p} = \frac{100 \times 746}{0.96 \times 0.80} = 97,135 \text{ VA} \]

\[ \theta = -\cos^{-1} 0.80 = -36.87^\circ \]

\[ V_{1\phi} = \frac{460}{\sqrt{3}} = 265.581 \text{ V} \]

\[ S_{1\phi} = V_T I_a^* \quad \Rightarrow \quad \frac{97,135}{3} / -36.87^\circ = 265.581/0^\circ \times I_a^* \]

\[ I_a^* = 121.92 / -36.87^\circ \quad \Rightarrow \quad I_a = 121.92 / 36.87^\circ \text{ A} \]

(c) \[ E_f = V_T - I_a j X_s = 265.581/0^\circ - (121.92/36.87^\circ \times 2.72/90^\circ) \]

\[ E_f = 265.58 - 331.62 / 126.87^\circ \]

\[ E_f = 534.96 / -29.73^\circ \quad \Rightarrow \quad 535 / -29.7^\circ \text{ V} \]

(d) \[ \delta = -29.7^\circ \]

(e) Pull-out torque occurs at \( \delta = -90^\circ \).

\[ P_{\text{in}} = 3 \cdot \frac{-V_f E_f}{X_s} \cdot \sin \delta = 3 \cdot \frac{-265.581 \times 534.96}{2.72} \cdot \sin(-90^\circ) \]

\[ P_{\text{in}} = 156,700 \text{ W} \]

\[ P = \frac{T_n}{5252} \quad \Rightarrow \quad T_{\text{pull-out}} = \frac{5252 \cdot P}{n} \]

\[ T_{\text{pull-out}} = \frac{5252 \times 156,700}{746 \times 1800} = 613 \text{ lb-ft} = 831 \text{ N-m} \]
Problem 10 (Core: Power-ECE3070) Solution

PROBLEM

TRANSFORMER PROBLEM

A 15 kVA, 2300/230 volt, 60 Hz, single phase transformer has the following equivalent circuit with the following parameters referred to the 2300 volt side:

\[ R_{\text{eq}} = 4.45 \text{ ohm}; \quad X_{\text{eq}} = 6.45 \text{ ohm}; \quad R_c = 105 \text{ kilo ohm}; \quad X_m = 11 \text{ kilo ohm}; \]

(a) Calculate the magnitude of the input voltage \( V_{\text{in}} \) to the transformer when it supplies rated kVA to a load at a power factor of 0.8 lagging, at rated voltage.

(b) Calculate the efficiency for the conditions in (a).
Problem 10 (Core: Power-ECE3070) Solution

(a) \( I'_2 = \frac{15000}{2300} = 6.52 \text{ A} \) and the power factor angle is \( \cos^{-1} 0.8 = -36.9 \text{ degr.} = \theta \)

\[ V_{in} = 2300/0^\circ + (6.52/-36.9)(4.45 + j 6.45) = 2348.5/0^\circ \]

Magnitude of input voltage is **2348.5 volts**

(b) The copper loss \( P_c = (I'_2)^2 R_{eq} = (6.52^2)(4.45) = 189 \text{ watts} \)

Core losses are \( P_c = (V_{in})^2/(R_c) = (2348.5^2)/(105\times10^3) = 52.5 \text{ watts} \)

Output of transformer is \( P_{out} = 2300 (I'_2) \cos \theta = 15000 (0.8) = 12000 \text{ watts} \)

Input power = 189 +52.5+12000

Efficiency = \[12000/(189+52.5+12000)]100 \% = 98.03\%\]
In the diagram below, sketch and label the energy vs. distance ($E$ vs. $x$) diagram for an $N^+P^+N^-$ BJT in the **NORMAL ACTIVE REGION** of operation. Assume that the BASE terminal is grounded (i.e., the base terminal is the reference terminal) and show the arrows corresponding to the energies related to the BASE-EMITTER and BASE-COLLECTOR voltages, $V_{BE}$ and $V_{BC}$ and their polarities. Draw and label the “quasi-Fermi level energies”, $F_n$ or $F_p$, for MAJORITY carriers in each region outside the depletion regions (DRs). I have given you the energies $E_C$ and $E_V$ in the base.
Solution: Band diagram for N⁺PN⁻ BJT in Normal Active Mode
Problem 12 (Core: Microsystems-ECE3040) Solution

PROBLEM

The doping concentration in an ideal Si MOS capacitor is $N_D=10^{15}$ cm$^{-3}$ and its SiO$_2$ insulator is 1nm thick. The device is at room temperature. Ideal here means no charges in oxide or interface and flat-band condition.

1. What is the Fermi potential of the substrate in this MOS capacitor? (Reminder: Fermi potential is defined as $(E_{i-Bulk}-E_F)/q$).

2. What is the Si surface potential when the device is at the onset of inversion?

3. If the electric field at the surface of Si is $-10$KV/cm, what would be the electric field at the middle of the insulator?

4. Assume a second MOS capacitor that is identical with this MOS capacitor except for a charge that is trapped at the interface of the Si and SiO$_2$ regions of the second device. If the charge density is $3.4\times10^{-6}$ C/cm$^2$, what would be the difference between the threshold voltages of the two MOS capacitors?

$kT=26$ meV at room temperature. $k_{Si}=11.8$, $k_{SiO2}=3.9$, $n_i=10^{10}$ cm$^{-3}$
Solution

1. \[ n_{\text{bulk}} = n_i \exp \left( \frac{E_F - E_{\text{i, bulk}}}{kT} \right) = N_D \quad \Rightarrow \quad \phi_p = -\frac{kT}{q} \ln \left[ \frac{N_o}{n_i} \right] = -0.3 \text{V} \]

2. \[ \phi_s = 2 \phi_F = -0.6 \text{V} \]

3. Since the charge density inside the insulator is zero, the electric field would be constant. Hence, the electric field at the middle of the insulator is the same as the electric field at the interface with Si.

\[ D_{\text{oxide}} = D_{\text{semi}, x=0} \quad \Rightarrow \quad \varepsilon_{\text{oxide}} = \frac{K_{\text{Si}}}{K_{\text{SiO}_2}} \varepsilon_{\text{semi}} = -30.25 \text{KV/cm} \]

4. \[ C_a = \frac{K_{\text{SiO}_2} \varepsilon_0}{t_{\text{SiO}_2}} = 3.4 \times 10^{-6} \text{ F/cm}^2 \quad , \quad V_{FB} = -\frac{Q}{C_o} = -1 \text{V} \]
Problem 13 (Core: DSP-ECE2026)

PROBLEM

A Linear and Time-Invariant system is described by a difference equation

\[ y[n] = 0.7x[n-1] - 0.4y[n-1] \]

where \( y[n] \) is the output signal and \( x[n] \) is the input.

(a) This system is **FIR** **IIR** (circle one)

(b) Determine the system function, \( H(z) = \)

(c) Determine an equation for the impulse response, \( h[n] = \)

(d) Determine the output when the input \( x[n] = 10\cos(n \cdot 0.2) \). \( y[n] = \)

(e) Determine the input signal so that the output is \( y[n] = \delta[n-3] \). \( x[n] = \)
A Linear and Time-Invariant system is described by a difference equation
\[ y[n] = 0.7x[n - 1] - 0.4y[n - 1] \]
where \( y[n] \) is the output signal and \( x[n] \) is the input.

(a) This system is **FIR** **IIR** (circle one)

(b) Determine the system function, 
\[ H(z) = \frac{0.7z^{-1}}{1 - 0.4z^{-1}} \]

\[ H(z) = \frac{Y(z)}{X(z)} \text{ and } Y(z) = 0.7z^{-1}X(z) + 0.4z^{-1}Y(z) \]

(c) Determine an equation for the impulse response, 
\[ h[n] = 0.7(-0.4)^{n-1}u[n - 1] \]

\[ \begin{align*}
   a^n u[n] & \mapsto \frac{1}{1 - az^{-1}} \\
   Ga^{n-1}u[n - 1] & \mapsto \frac{Gz^{-1}}{1 - az^{-1}}
\end{align*} \]

(d) Determine the output when the input \( x[n] = 10\cos(\pi n - 0.2\pi) \), \( y[n] = -5\cos(\pi n - 0.2\pi) = 5\cos(\pi n + 0.8\pi) \)

\[ H(e^{j\hat{\omega}}) \bigg|_{\hat{\omega} = \pi} = \frac{0.7e^{-j0.8\pi}}{1 - 0.4e^{-j\pi}} = \frac{-0.7}{1.4} = -0.5 \]

(e) Determine the input signal so that the output is \( y[n] = \delta[n-3] \). \( x[n] = (10/7)\delta[n - 2] - (4/7)\delta[n - 3] \)

\[ H(z) = \frac{Y(z)}{X(z)} \Rightarrow X(z) = \frac{Y(z)}{H(z)} \Rightarrow X(z) = z^{-3} \frac{1 - 0.4z^{-1}}{0.7z^{-1}} \]
\[ \Rightarrow X(z) = \frac{z^{-2} - 0.4z^{-3}}{0.7} \Rightarrow X(z) = \frac{10}{7}z^{-2} - \frac{4}{7}z^{-3} \]
Two indistinguishable but biased coins lie on the table in front of you. When Coin A is flipped, it comes up heads ($H$) with probability $3/4$. When Coin B is flipped, it comes up $H$ with probability $2/5$. You choose one of them at random and start flipping it.

1. What is the probability that you see the first $H$ on the fourth flip?

2. Suppose you do see the first $H$ on the fourth flip. What is the probability you are holding Coin A?
3. Given the observation of the first $H$ coming on the fourth flip, what is the probability that the fifth flip will also be $H$?
1. The probability you observe the sequence $TTTH$ is

$$P(TTTH) = P(\text{Coin A})P(TTTH|\text{Coin A}) + P(\text{Coin B})P(TTTH|\text{Coin B})$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{2}{5}$$

$$= 0.0491$$

2. Using Bayes rule

$$P(\text{Coin A}|TTTH) = \frac{P(TTTH|\text{Coin A})P(\text{Coin A})}{P(TTTH)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4}}{0.0491}$$

$$= 0.1432$$

3. With $TTTH$ observed, the probability you are holding Coin A is 0.1432, and the probability you are holding Coin B is 0.8568. Thus

$$P(\text{fifth flip H}|TTTH) = (0.1432) \cdot \frac{3}{4} + (0.8568) \cdot \frac{2}{5} = 0.4501$$
Consider the feedback configuration below. Assume that $C(s)$ is chosen cleverly enough such that the output $y$ of the closed-loop system is bounded for all bounded reference signals $r$, no matter what $k$ is. The plant and controller are of the form

$$G(s) = \frac{P(s)}{Q(s)}, \quad C(s) = \frac{P_C(s)}{Q(s)},$$

where $P, Q, P_C, Q_C$ are polynomials in $s$, and where $P(0) \neq 0, Q_C(0) \neq 0$.

**a**

For what values of $k$ can perfect tracking be guaranteed, in the sense that the steady-state tracking error of a constant reference signal is zero?

**b**

Same question as in **a** but now the reference signal $r(t)$ is a decaying exponential,

$$r(t) = 1 - e^{-t}.$$

**c**

Same question as in **a** but now the reference signal $r(t)$ is a sinusoid,

$$r(t) = \sin(t).$$
Problem 15 (Core: S&C-ECE3550) Solution

a

We first observe that the combined controller is

\[ C'(s) = C(s) + k \frac{s}{s} = \frac{sP_C(s) + kQ_C(s)}{sQ_C(s)}, \]

which gives the closed-loop transfer function

\[ G_{CL}(s) = \frac{G(s)C'(s)}{1 + G(s)C'(s)} = \frac{PP_C + kPQ_C}{sQ_C + sP_C + kPQ_C}. \]

Since the closed-loop system is BIBO, the Final Value Theorem applies and we have that

\[ \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} sG_{CL}(s)R(s) = \lim_{s \to 0} sG_{CL}(s) \frac{1}{s} = G_{CL}(0). \]

But, the closed-loop DC-gain \( G_{CL}(0) \) is equal to

\[ \frac{kP(0)Q_C(0)}{kP(0)Q_C(0)} = 1 \]

for all non-zero values of \( k \). If \( k = 0 \) we cannot guarantee that tracking is achieved. Hence, tracking is achieved for all \( k \neq 0 \).

b

Since \( r(t) \to 1 \) as \( t \to \infty \) this is indeed the same (asymptotically) situation as in a, i.e., perfect tracking is achieved also in this case for any \( k \neq 0 \).

c

As this reference signal does not have a well-defined steady-state, the output of the system will also not have a steady-state value, i.e., the final value theorem does not apply and the tracking error can never be guaranteed (in general) to be zero in steady-state no matter what \( k \) is.
Problem 16 (Core: S&C-ECE3550) Solution

**Problem**

The feedback gains of the control system shown below have been chosen to place both eigenvalues of the overall system at $s = -1$ under the false assumption that the actuator transfer function is equal to unity. Determine the full range of actuator eigenvalue $\lambda$ for which the actual third-order overall system is internally stable.

![Control System Diagram]
The overall system is described by

\[ Y(s) = \left( \frac{\lambda}{s + \lambda} \right) \left( \frac{1}{s - 1} \right) \left( -3Y(s) + \frac{1}{s} (R(s) - Y(s)) + W(s) \right). \]

Clearing fractions results in

\[ s (s + \lambda) (s - 1) Y(s) = \lambda (-3sY(s) + R(s) - Y(s) + sW(s)) \]

and grouping terms results in

\[ \underbrace{(s^3 + (\lambda - 1)s^2 + (2\lambda)s + \lambda)}_{\text{characteristic polynomial}} Y(s) = \lambda (R(s) + sW(s)). \]

Application of the Routh-Hurwitz criterion involves the table

\[
\begin{array}{c|cc}
\lambda & 1 & 2\lambda \\
\lambda - 1 & \lambda & 1 \\
\lambda & * & \lambda \\
0 & \lambda & \\
\end{array}
\]

where

\[ * = \frac{\lambda(2\lambda - 3)}{\lambda - 1} \]

along with the inequalities

\[ \lambda > 0 \text{ and } \lambda > 1 \text{ and } \lambda > \frac{3}{2}. \]

Therefore, for this particular design, the overall system is internally stable if and only if

\[ \lambda > \frac{3}{2}, \]

i.e. if and only if the actuator itself is stable and sufficiently fast.
Question 1:

This question involves stack frame and function calls. The following convention is used:

1. The system is a 32-bit MIPS computer. Function calls are implemented via ‘jal’ and function call returns are implemented via ‘jr $31’.
2. The stack grows downwards (towards the lower end of the address space).
3. Frame pointer is not used, so it doesn't need to be preserved and restored.
4. All parameters are passed via the stack (none are passed via registers).
5. Caller only allocates part of the activation frame for the callee before calling 'jal' - it only allocates the input parameters on the stack, and updates the stack pointer accordingly (right on top of the input parameters).
6. Caller pushes input parameters onto the stack in the reverse order as the parameters are declared.
7. Callee is responsible of populating the rest of the activation frame, which includes the return address, return value, and local variables.
8. Caller is responsible of deallocating the entire activation frame (of the callee) when the callee returns.
9. The activation frame of a function is populated in the following order:
   1) input parameters (allocates only if there are any input parameters)
   2) return address
   3) return value (allocates only if the function does return a value)
   4) local variables (if any)
10. The local variables are allocated in the same order as they were defined.
11. Stack pointer points to the next available slot on the stack.

The question is on the next page:
We have the following program:

```c
#define N 10

int foo (int A) {
    int w[10];
    int i;
    for(i=0;i<10;i++) w[i] = i;
    for(i=0;i<N; i++) w[i] = w[i] - 4;
    return w[0];
}

int main () {
    return foo(3);
}
```

What may happen to the code if we change the definition of N to the following values?

1. #define N 11

2. #define N 12
We have the following program:

```c
#define N 10
int foo (int A) {
    int w[10];
    int i;
    for(i=0;i<10;i++) w[i] = i;
    for(i=0;i<N;i++) w[i] = w[i] - 4;
    return w[0];
}

int main () {
    return foo(3);
}
```

What may happen to the code if we change the definition of `N` to the following values?

1. `#define N 11`

   When N=11, the second for loop will overflow the array by one element, when modifying w[10], it actually changes the return value on the stack. But right after the change, the return value will be updated to w[0], and the function program will proceed successfully.

2. `#define N 12`

   When N=12, the second for loop will overflow the array by two elements. When modifying w[10], it changes the return value on the stack, but this will be updated back to w[0] when the program tries to execute ‘jr $31’ so this is not an issue. However, when modifying w[11], the program actually reduces the return address by 4, which will cause the ‘jr $31’ instruction to jump back to the ‘jal foo’ instruction upon returning from ‘foo’. This will cause an infinite loop.
PROBLEM

A priority queue is an abstract data structure that stores numeric values as they arrive to the queue and, when a value is requested from the queue, it dequeues (removes) the largest value currently in the queue.

a) Describe a (real) data structure that can be used to implement a priority queue and which requires $O(\log n)$ time for each operation in an arbitrary sequence of enqueue and dequeue operations, where $n$ is the length of the queue. The enqueue operation adds an arbitrary item to the queue and the dequeue operation removes the largest item. Be sure to justify why the enqueue and dequeue operations require $O(\log n)$ time for your data structure.

b) Write pseudo-code for the enqueue and dequeue operations for your data structure.
a) Describe a (real) data structure that can be used to implement a priority queue and which requires $O(\log n)$ time for each operation in an arbitrary sequence of enqueue and dequeue operations, where $n$ is the length of the queue. The enqueue operation adds an arbitrary item to the queue and the dequeue operation removes the largest item. Be sure to justify why the enqueue and dequeue operations require $O(\log n)$ time for your data structure.

Use a heap, which is a partially ordered tree, i.e. a tree with the following property. The value of the item stored in the root of every subtree is greater than or equal to the values of all items stored in the subtree.

The tree is maintained as a complete binary tree up to a certain level with nodes filled in from left to right in the final partially filled level. The enqueue operation places the item to be added in the next open position in the tree and then “bubbles it up” to a position where the heap property is satisfied, i.e. it repeatedly swaps the new item with its parent in the tree until its parent’s value is greater than or equal to its value. The dequeue operation removes the root, which is by definition the largest item in the tree. It then takes the item at the last position in the tree, moves it to the root, and “bubbles it down” to a position where the heap property is satisfied. The “bubble down” operation compares the item newly placed at the root with its children and, if it is smaller than either child, swaps it with the larger child. This operation is repeated until the item reaches a position where its value is greater than or equal to all its children’s values.

Since “bubble up” and “bubble down” operations both perform a number of swaps that is at most the height of the tree and the tree is maintained as a complete binary tree plus one partial level, the operations both require $O(n)$ time.
Problem 18 (Specialized: CSS-Operating Syst.)

This code is in C and uses an array implementation of a complete binary tree. It also makes use of a standard swap function that swaps two numbers.

```c
void enqueue(int pqueue[], int item, int *length) {
    (*length)++;  
    pqueue[*length] = item; 
    bubbleUp(pqueue, *length); 
}

void bubbleUp(int pqueue[], int position) {
    if ((position > 1) && (pqueue[position] > pqueue[position/2])) {
        swap(pqueue, position, position/2); 
        bubbleUp(pqueue, position/2); 
    }
}

int dequeue(int pqueue[], int *length) {
    int max; 
    max = pqueue[1]; 
    swap(pqueue, 1, *length); 
    --(*length); 
    bubbleDown(pqueue, 1, *length); 
    return max; 
}

void bubbleDown(int pqueue[], int position, int length) {
    int childPosition;   
    childPosition = 2*position; 
    if ((childPosition < length) && (pqueue[childPosition+1] > pqueue[childPosition])) 
        childPosition++;   
    if ((childPosition <= length) && (pqueue[position] < pqueue[childPosition])) {
        swap(pqueue, position, childPosition); 
        bubbleDown(pqueue, childPosition, length); 
    }
} 
```
PROBLEM

What four items of information are needed before a Internet host can operate normally on the network:
1. ______________________
2. ______________________
3. ______________________
4. ______________________

If these are not configured manually, what protocol can be used to get them over the network?
5. ______________________

6. Every Internet Autonomous System must have an Authoritative DNS. What does it do?

Why has Georgia Tech migrated its campus WiFi wireless network from WEP to WPA-Enterprise?
7. ________________________________________________________________

What circumstance has made the immediate transition from IP version 4 to IP version 6 necessary?
8. ________________________________________________________________

If Georgia Tech OIT assigns you the 130.209.123.0/24 block of IP addresses for the subnet in your lab:

9. What is the lowest address that you can assign to a host? ______________________

10. What is the IP subnet broadcast address? ______________________
What four items of information are needed before a Internet host can operate normally on the network:

1. ___ Assigned IP address___
2. ___ Network mask___
3. ___ Gateway router IP address___
4. ___ DNS server IP address ____

If these are not configured manually, what protocol can be used to get them over the network?
5. _ DHCP (Dynamic Host Configuration Protocol) ________________

6. Every Internet Autonomous System must have an Authoritative DNS. What does it do?
   ___ It is the authoritative source for the IP addresses of hosts with names assigned by the AS ___

Why has Georgia Tech migrated its campus WiFi wireless network from WEP to WPA-Enterprise?
7. ___ To improve security and privacy. ___

What circumstance has made the immediate transition from IP version 4 to IP version 6 necessary?
8. _____ Exhaustion of the IPv4 network addresses. ______

If Georgia Tech OIT assigns you the 130.209.123.0/24 block of IP addresses for the subnet in your lab:

9. What is the lowest address that you can assign to a host?   ___ 130.209.123.1 _____

10. What is the IP subnet broadcast address? ___ 130.209.123.255 _____
Problem 20 (Specialized: Optics-ECE4500) Solution

**Problem**

Grating Spectrometer - Wavelength Demultiplexing

A waveguide reflection grating spectrometer is used for wavelength division demultiplexing. Three telecommunication signal waves from the waveguide are incident in air upon the metallic grating at an angle of 15° counter-clockwise from the normal as shown in figure. The three waves have frequencies of 195.5 TeraHz, 196.0 TeraHz, and 196.5 TeraHz which are frequencies on the ITU standard grid. The three wavelengths are dispersed by the grating into $i = +1$-order backward-diffracted waves as shown in the figure. The grating is designated as having “800 lines per mm.” An array of photodetectors is oriented normal to the 196.0 TeraHz (no. 2) beam as shown.

1) Calculate, showing all work, the wavelengths in nanometers of the waves numbered 1, 2, and 3 in the figure.

2) Calculate, showing all work, the backward-diffracted wave angles (measured counter-clockwise from the normal) in degrees for these waves.
Express wavelengths accurately to five significant figures. Express the angles in degrees accurately to within ±0.01°. Put your answers in the spaces provided.

<table>
<thead>
<tr>
<th>Wave No.</th>
<th>Wavelength (nm)</th>
<th>Angle of Diffraction (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \theta' = +15^\circ \]

\[ n_1 = 1.000 \]

\[ \Lambda = \frac{1}{800} \text{lines/mm} = 0.00125 \text{mm} = 1.25 \mu m \]

\[ D = 10.0 \text{mm} \]

\[ f_3 = 195.5 \times 10^{12} \text{Hz} \quad \lambda_3 = \frac{c}{f_3} = 1533.46 \text{nm} \]

\[ f_2 = 196.0 \times 10^{12} \text{Hz} \quad \lambda_2 = \frac{c}{f_2} = 1529.55 \text{nm} \]

\[ f_1 = 196.5 \times 10^{12} \text{Hz} \quad \lambda_1 = \frac{c}{f_1} = 1525.66 \text{nm} \]

Grating equation for backward-diffracted orders

\[ \sin \theta' + \sin \theta'_i = \frac{i\lambda}{n_1 \Lambda} \]

For \( i = 1 \) and \( n_1 = 1 \)

\[ \theta'_1 = \sin^{-1} \left( \frac{\lambda}{\Lambda} - \sin \theta' \right) \]

\[ (\theta'_1)_3 = 75.4555^\circ \]

\[ (\theta'_1)_2 = 74.7579^\circ \]

\[ (\theta'_1)_1 = 74.0934^\circ \]
Problem 21 (Specialized: Optics-ECE4501) Solution

**Problem**

An optical fiber communication link is assembled using binary on-off keying with direct detection. The optical transmitter consists of a cw (continuous wave) laser whose output is externally modulated to produce the binary NRZ signal. At the other end, an optical receiver is used in which thermal noise is the dominant noise contributor (all other noise mechanisms are negligible). At room temperature (300° K), the noise current is found to be $\sigma_T = 60.0$ nA. The average optical power at the receiver is $P_{\text{rec}} = 1.5 \mu W$, with incomplete extinction – that is, the modulator does not completely block the laser power. As a result, 20% of the optical power resides in the “zero” bit slots, with the rest in the “one” bits. Upon detection, the resulting current levels in the “one” and “zero” bits are $I_1 = 0.4 \mu A$ and $I_0 = 0.1 \mu A$ respectively.

a. Evaluate $Q = (I_1 - I_0)/(\sigma_1 + \sigma_0)$.

It is desired to operate the link at a BER of $10^{-9}$, which means that $Q$ must attain a value of 6. This is to be accomplished by doing two things simultaneously: 1) cool the receiver with liquid nitrogen ($T = 77° K$), and 2) increase the average optical power.

b. What is the new required average optical power, and what is the equivalent decibel reduction in link loss needed to accomplish this?

c. Suppose an ideal modulator is used, such that complete extinction is achieved (zero power in the “zero” bit slots), but with the laser output power unchanged. With this accomplished, in addition to the aforementioned temperature reduction, re-answer part b.
Problem 21 (Specialized: Optics-ECE4501) Solution

**SOLUTION**

**a.** Evaluate \( Q = \frac{(I_1 - I_0)}{(\sigma_1 + \sigma_0)} \).

*answer:* With thermal noise dominant, \( \sigma_1 = \sigma_0 = \sigma_T \), and so

\[
Q = \frac{I_1 - I_0}{2\sigma_T} = \frac{(0.4 - 0.1) \mu\text{A}}{2(0.060)\mu\text{A}} = 2.5
\]

It is desired to operate the link at a BER of \( 10^{-9} \), which means that \( Q \) must attain a value of 6. This is to be accomplished by doing two things simultaneously: 1) cool the receiver with liquid nitrogen \( (T = 77^\circ \text{K}) \), and 2) increase the average optical power.

**b.** What is the new required average optical power, and what is the equivalent decibel reduction in link loss needed to accomplish this?

*answer:* Thermal noise current is in proportion to \( \sqrt{T} \), so the new noise current will be

\[
\sigma'_T = \sigma_T \sqrt{\frac{T'}{T}} = 60 \sqrt{\frac{77}{300}} = 30.4 \text{nA}
\]

With this change alone, \( Q \) is now increased to

\[
Q' = Q \frac{\sigma_T}{\sigma'_T} = 2.5 \frac{60.0}{30.4} = 4.93
\]

To further increase \( Q \) to a value of 6, the currents must be increased by a factor of \( 6/4.93 = 1.22 \). The received optical power must therefore be increased by this same factor. So the new received power will be \( P_{\text{rec}} = 1.5 \times 1.22 = 1.83 \mu\text{W} \). The required loss reduction in the link is thus

\[
P_{\text{red}}[\text{dB}] = 10 \log_{10} (1.22) = 0.86 \text{dB}
\]

**c.** Suppose an ideal modulator is used, such that complete extinction is achieved (zero power in the “zero” bit slots), but with the laser output power unchanged. With this accomplished, in addition to the aforementioned temperature reduction, re-answer part b.

*answer:* With complete extinction, \( I_0 = 0 \), and \( Q \) with temperature reduction becomes:

\[
Q = \frac{I_1}{2\sigma'_T} = \frac{(0.4) \mu\text{A}}{2(0.0304)\mu\text{A}} = 6.58
\]

So the use of an ideal modulator plus temperature reduction has more than solved the problem! With complete modulation, the new average received power is \( 0.8 \times 1.5 = 1.2 \mu\text{W} \). To achieve \( Q = 6 \), this power can be reduced to

\[
P'_{\text{rec}} = 1.2 \times \frac{6.00}{6.58} = 1.09 \mu\text{W}
\]

So the link could actually tolerate an *increased* loss of

\[
P_{\text{red}} = 10 \log_{10} \left( \frac{6.00}{6.58} \right) = -0.4 \text{dB}
\]
Design a process flow using to pattern a thin film deposited layer of copper on a silicon substrate using negative photoresist. The process flow should consist of a step-by-step cross sectional flow diagram of the process with a short description of each step next to the cross section. The process flow should include a cross section of the mask design at the appropriate point in the flow diagram.
Problem Statement: Design a process flow using to pattern a thin film deposited layer of copper on a silicon substrate using negative photoresist. The process flow should consist of a step by step cross sectional diagram of the process with a short description of each step next to the cross section. The process flow should include a cross section of the mask design at the appropriate point in the flow diagram.
Problem 23 (Specialized: BioEng-ECE4781) 

**Problem**

Draw a sketch of a typical motor neuron in humans, labeling five different anatomical parts. (2 points)

Why do motor neurons have more than one leg? (2 points)

What happens inside a motor neuron that makes naturally occurring Action Potentials travel in one direction down the motor neuron? (2 points)

Design a data acquisition system for measuring the human EMG using skin electrodes. You can represent each stage of your data acquisition system using a block diagram, but specify details like the gain and other features of each amplifier stage, filters with corners specified, and exactly how you will isolate the patient from line voltage and line ground. (4 points)
Draw a sketch of a typical motor neuron in humans, labeling five different anatomical parts. (2 points)

*The drawing could show one or more dendrites, one or more axons, the cell body, a synaptic ending on each axon and the fifth part could be any internal organelle, e.g. Nucleus, RER, Golgi Apparatus, etc.*

Why do motor neurons have more than one leg? (2 points)

*Because one motor neuron usually innervates multiple muscle cells.*

What happens inside a motor neuron that makes naturally occurring Action Potentials travel in one direction down the motor neuron? (2 points)

*Each segment of the axon becomes refractory after the segment produces an action potential. During this period, the distributions of sodium and potassium ions return to their resting concentrations via a pump mechanism in the cell membrane.*

Design a data acquisition system for measuring the human EMG using skin electrodes. You can represent each stage of your data acquisition system using a block diagram, but specify details like the gain and other features of each amplifier stage, filters (specify the corners), and exactly how you will isolate the patient from line voltage and line ground. (4 points)

*The first stage should be a differential amp with high input resistance and high common-mode rejection. The gain should be around 100x. The next stage should be a bandpass filter with corners of approximately .5 Hz and 30 Hz. The next stage should be a single-input amplifier with a gain of approximately 100x. The next stage should be an isolation device, e.g. an opto-isolator or a transformer type isolator. The A-to-D details should be also described.*
Why do hospital diagnostic labs use pulse stimuli when they know that all biological systems are nonlinear, and what can they do to make the system appear to be more linear? (2 points)

How does adaptation affect the convolution integral representation of the system? (2 points)

What changes in the convolution integral if the system is noncausal? (2 points)

What is the best way to deal with periodic noise when using pulse stimuli? (2 points)

Why is it difficult to measure a Visual Evoked Response using pulse stimuli of light and surface electrodes on the scalp? (2 points)
Why do hospital diagnostic labs use pulse stimuli when they know that all biological systems are nonlinear, and what can they do to make the system appear to be more linear? (2 points)

Pulse stimuli are used to identify approximate details of a linear transfer function representation of the system. If the system is linear and time-invariant, and if the stimulus is close to a Dirac Delta Function, then the measured pulse response is the transfer function. The big problem is that biological systems are not linear or time-invariant, but a very small pulse stimulus tends to make the system look more linear.

How does adaptation affect the convolution integral representation of the system? (2 points)

The h in the convolution integral changes during the adaptation process, so it is better to test a system after adaptation has concluded. For example, if the background light level suddenly changes by a large amount, the h representing the retina will change drastically during the next few minutes. A pulse stimulus could be used to characterize the system after the system has completed the adaptation process.

What changes in the convolution integral if the system is noncausal? (2 points)

The transfer function $h$ will have +/- values, so the integration will need to be $+/-\;\infty$.

What is the best way to deal with periodic noise when using pulse stimuli? (2 points)

Use pulse stimuli with random intervals, and then find the desired ensemble statistic, e.g. ensemble mean.

Why is it difficult to measure a Visual Evoked Response using pulse stimuli of light and surface electrodes on the scalp? (2 points)

The answer to this question requires a lot of deep thinking. When a flash of light enters the eye, a small percentage of the light is scattered to the peripheral retina where the retina is predominantly composed of rods. The rods are much more sensitive than the cones, which are concentrated in the fovea. The VER is mostly a cone response, so it is very difficult to measure VER using a flash stimulus. In hospitals, the VER is usually measured using a toggling checkerboard pattern, which produces a constant amount of scattered light to the peripheral retina.
For a voltage clamped membrane, find the minimum seal resistance to achieve a signal to noise ratio of 75 for a potassium channel with the following characteristics.

\( g_k = 20 \text{ pS}, \ V_m = -40 \text{ mV}, \ E_k = -90 \text{ mV}, \ T = 25 \text{ C}, \ \text{frequency range} = 100 \text{ Hz} \)
For a voltage-clamped membrane, find the minimum equivalent seal resistance to achieve a signal/noise ratio of \( \frac{75}{\delta} \) for a potassium channel with the following characteristics.

\[ g_k = 20 \text{ pS}, \ V_m = -40 \text{ mV}, \ E_k = -90 \text{ mV}, \ i = 25^\circ \text{C} \]

The frequency range is \( \frac{f}{100} \),

\[ L_k = g_k (V_m - E_k) = (20 \times 10^{-12}) (-90 - (-90)) \]

\[ = 2.0 \text{ pA} \]

\[ \Delta V = \sqrt{\frac{4kT}{R}} \]

if \( S/N = \frac{75}{\delta} \) then \( \Delta V = \frac{1.0 \text{ pA}}{75} \)

\[ = 0.0133 \text{ pA} \]

\[ 0.0133 \times 10^{-12} = \frac{4 (1.38 \times 10^{-23}) (1 \times 8.9)}{R} \]

\[ 9.3 \times 10^{-9} = \frac{4.2 \times 10^{-12}}{R} \]

\[ R = \frac{1.38 \times 10^{-9}}{4.2 \times 10^{-12}} \]

\[ = \frac{1.38 \times 10^3}{4.2 \times 10^{-12}} \]

\[ \Rightarrow R = 3.24 \times 10^8 \text{ m} \]

\[ R_{eq} = R_{channel} \parallel R_{seal} \]

\[ R_{channel} = \frac{1}{2R_1} = 5 \times 10^{-10} \]

\[ R_{seal} = \frac{(5 \pm 10) \times 10^5 \Omega}{(5 \times 10^4 - 10^5 \Omega)} \]

\[ R_{seal, min} = 3.23 \times 10^{-9} \Omega \]

\[ 1.14 \times 10^5 \Omega \]

\[ R_{seal, max} = 3.23 \times 10^{-9} \Omega \]

\[ 1.14 \times 10^5 \Omega \]