INSTRUCTIONS

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Note that Problem 3 occupies 2 pages, Problem 17 occupies 3 pages. Including the cover sheet, you should have 62 pages. There should be no blank pages in the booklet.

2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.

3. All wireless devices must be turned off for the entire duration of the exam.

4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.

5. Your examination code number MUST APPEAR ON EVERY SHEET. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. DO NOT write your name on any of these sheets. Use the preprinted numbers whenever possible, or WRITE LEGIBLY!!!

6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!

7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM EST.

8. When you hand in the exam:

   (a) Separate the 8 problems to be graded as explained above.

   (b) Check to see that your Code Number is in EVERY sheet you are turning in.

   (c) On the section at the bottom of this page, CIRCLE the problem numbers that you are turning in for grading.

   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.

   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

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Problem 1 (Core: VSDD-ECE2020)  

Problem

a. (3pts) Use algebraic techniques to simplify: $F = (a \cdot c + b \cdot c \cdot d + \bar{a} \cdot d)$

b. (2pts) Draw the CMOS pull-down network of $F$ corresponding to the pull-up network

c. (2pts) Express $F$ in a sum of product (SOP) form. Remove any duplicate terms.

d. (3pts) Use a 16X1 multiplexer to implement the function $F$. 
(a) \( F = (a \cdot c + b \cdot c \cdot d + \bar{a} \cdot d) \)
\[ = (a \cdot c + b \cdot c \cdot d + \bar{a} \cdot d + c \cdot d) \]
\[ = (a \cdot c + \bar{a} \cdot d) \]

(b)
Problem 1 (Core: VSDD-ECE2020) Solution

(c) \( F = \left( (\overline{a} + \overline{b}) \cdot \overline{d} + \overline{c} \cdot \overline{d} + \overline{b} \cdot \overline{d} \right) \\
= (\overline{a} \cdot \overline{d} + \overline{b} \cdot \overline{d} + \overline{c} \cdot \overline{d}) \\

(d) 

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Problem 2 (Core: VSDD-ECE2030)  

**PROBLEM**

For each part implement the specified device. **Label all inputs and outputs.**

| Part A | Implement the following expression using N and P type switches (NFETS and PFETS).  

\[ \text{OUT}_X = \overline{A} \cdot B + C \cdot (\overline{D} + \overline{E}) \]  

| Part B | Implement a 1-to-2 decoder using basic gates (AND, OR, NAND, NOR, & NOT) only. Assume only true (non-complemented) inputs are available.  

| Part C | Determine the appropriate expression for this mixed logic design. How many transistors are required?  

| Part D | Reimplement the design in Part C using only AND gates and inverters. How many transistors are required?  

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**Solution**

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\[ \text{OUT} = \]  

\# transistors =  

\# transistors =
For each part implement the specified device. **Label all inputs and outputs.**

**Part A** Implement the following expression using N and P type switches (NFETS and PFETS).

\[
OUT_X = \overline{A} \cdot B + C \cdot (\overline{D} + \overline{E})
\]

**Part B** Implement a 1-to-2 decoder using basic gates (AND, OR, NAND, NOR, & NOT) only. Assume only true (non-complemented) inputs are available.

**Part C** Determine the appropriate expression for this mixed logic design. How many transistors are required?

\[
OUT = A \cdot (\overline{B} + C) + D
\]

# transistors = 20

**Part D** Reimplement the design in Part C using only AND gates and inverters. How many transistors are required?

# transistors = 24
Problem 3 (Core: CSS-ECE3056) Solution

**Problem**

We have a canonical five stage pipelined 32-bit MIPS core - i) instruction fetch, ii) instruction decode and register fetch, iii) execute and memory address compute, iv) memory operation, and v) write-back. Register write operations occur in the first half of a clock cycle, and read operations in the second half. Now consider the execution of the following assembly language code block. **Assume that the first instruction is fetched in cycle 0!** Assume full hardware support for forwarding. Branch addresses and condition are computed in instruction decode. Branches are assumed not taken. Assume all instructions are native instructions. The first instruction is at address 0x00400000 and the registers $a0 and $a1 have the values 0x10010000 and 0x10010040 respectively.

1. Proc: move $t0, $a0  #load start address of array
2.    addi $t2, $0, 0  #initialize count using immediate
3.    move $t1, $a1  #load end address of the array
4.    addi $t1, $t1, 4  #point to first word after array
5.    addi $t3, $zero, $zero  #initialize sum
6.  loop: lw $t4, 0($t0)  #fetch array element
7.     addi $t2, $t2, 1  #increment count
8.    addi $t0, $t0, 4  #point to next word
9.    bne $t1, $t0, loop  #if not done, start next iteration
10.    move $v0, $t3
11.    jr $31

a. (4 pts) Identify location of all idle cycles in the above procedure
b. (3 pts) What operation or instruction is in each stage of the pipeline during cycle 10 after the procedure starts executing.

c. (3 pts) The execution of the \texttt{lw} instruction generates a page fault exception on its first execution. What are the values in register $t1$ and $t2$ while the page fault is being serviced?
We have a canonical five stage pipelined 32-bit MIPS core - i) instruction fetch, ii) instruction decode and register fetch, iii) execute and memory address compute, iv) memory operation, and v) write-back. Register write operations occur in the first half of a clock cycle, and read operations in the second half. Now consider the execution of the following assembly language code block. Assume that the first instruction is fetched in cycle 0! Assume full hardware support for forwarding. Branch addresses and condition are computed in instruction decode. Branches are assumed not taken. Assume all instructions are native instructions. The first instruction is at address 0x00400000 and the registers $a0 and $a1 have the values 0x10010000 and 0x10010040 respectively.

1. Proc: move $t0, $a0 #load start address of array
2. addi $t2, $0, 0 #initialize count using immediate
3. move $t1, $a1 #load end address of the array
4. addi $t1, $t1, 4 #point to first word after array
5. add $t3, $zero, $zero #initialize sum
6. loop: lw $t4, 0($t0) #fetch array element
7. add $t3, $t3, $t4 #update sum
8. addi $t2, $t2, 1 #increment count
9. addi $t0, $t0, 4 #point to next word
10. bne $t1, $t0, loop #if not done, start next iteration
11. move $v0, $t3
12. jr $31

a. (4 pts) Identify location of all idle cycles in the above procedure

1. Proc: move $t0, $a0 #load start address of array
2. addi $t2, $0, 0 #initialize count using immediate
3. move $t1, $a1 #load end address of the array
4. addi $t1, $t1, 4 #point to first word after array
5. add $t3, $zero, $zero #initialize sum
6. loop: lw $t4, 0($t0) #fetch array element <stall> #load-to-use hazard
7. add $t3, $t3, $t4 #update sum
8. addi $t2, $t2, 1 #increment count
9. addi $t0, $t0, 4 #point to next word <stall> #branch condition computed in ID
10. bne $t1, $t0, loop #if not done, start next iteration <stall/flush> #for all but the last iteration
11. move $v0, $t3
12. jr $31 <stall/flush>
Problem 3 (Core: CSS-ECE3056) Solution

b. (3 pts) What operation or instruction is in each stage of the pipeline during cycle 10 after the procedure starts executing.

<table>
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<tr>
<th>Stage</th>
<th>Instruction</th>
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<tr>
<td>IF</td>
<td>bne $t1, $t0, loop</td>
</tr>
<tr>
<td>ID</td>
<td>addi $t0, $t0, 4</td>
</tr>
<tr>
<td>EX</td>
<td>addi $t2, $t2, 1</td>
</tr>
<tr>
<td>MEM</td>
<td>add $t3, $t3, $t4</td>
</tr>
<tr>
<td>WB</td>
<td>&lt;stall&gt;</td>
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c. (3 pts) The execution of the lw instruction generates a page fault exception on its first execution. What are the values in register $t1 and $t2 while the page fault is being serviced?

$t1 = 0x10010044$
$t2 = 0x00000000$
Consider the following binary function \( F = (a + bc + d) \).

**a.** (3pts) Draw the CMOS transistor-level schematic of \( F \). Use \( \text{\begin{tikzpicture}[baseline] \draw (0,0) -- (0.5,0); \end{tikzpicture}} \) for pFETs and \( \text{\begin{tikzpicture}[baseline] \draw (0,0) -- (0.5,0); \end{tikzpicture}} \) for nFETs.

**b.** (4pts) Assuming that the width of all transistors used in \( F \) is 2\( w \), compute the worst-case delay in terms of \( \tau \) when \( F \) is driving a load of 10\( C_{\text{inv}} \). The meaning of the symbols is: \( w \) is the width of the minimum-size nFET, \( C_{\text{inv}} \) is the input capacitance of a minimum-sized inverter with equal rise and fall times, \( R \) is the resistance of the minimum-size nFET, and \( \tau = R \cdot C_{\text{inv}} \). Assume that the mobility of electrons is twice as high as that of holes.

**c.** (3pts) Perform resizing of all transistors in \( F \) in terms of \( w \) such that the rise and fall times are matched and also the worst-case delay of driving a load of 1\( C_{\text{inv}} \) becomes \( \tau \).
Problem 4 (Core: VSDD-ECE3060)

Solution

1. (10pts) Consider the following binary function \( F = (a + bc + d) \).

   a. (3pts) Draw the CMOS transistor-level schematic of \( F \). Use \( \quad \) for pFETs and \( \quad \) for nFETs.

   b. (4pts) Assuming that the width of all transistors used in \( F \) is \( 2w \), compute the worst-case delay in terms of \( \tau \) when \( F \) is driving a load of \( 10C_{\text{inv}} \). The meaning of the symbols is: \( w \) is the width of the minimum-size nFET, \( C_{\text{inv}} \) is the input capacitance of a minimum-sized inverter with equal rise and fall times, \( R \) is the resistance of the minimum-size nFET, and \( \tau = R \cdot C_{\text{inv}} \). Assume that the mobility of electrons is twice as high as that of holes.

   c. (3pts) Perform resizing of all transistors in \( F \) in terms of \( w \) such that the rise and fall times are matched and also the worst-case delay of driving a load of \( 1C_{\text{inv}} \) becomes \( \tau \).

Solution: \( F = \overline{a + bc + d} = \overline{a} \overline{bc}d = \overline{a} (\overline{b} + \overline{c})d \)

(b) Pull-up: worse delay path is \( V_{dd} - a - b - d - F \) or \( V_{dd} - a - c - d - F \). So, delay = \( (R + R + R) \cdot 10C_{\text{inv}} = 30\tau \). Pull-down: worse delay path is \( F - b - c - Gnd \). So, delay = \( 2 \cdot 0.5 \cdot 10C_{\text{inv}} = 10\tau \). Thus, the worst-case delay is 30\( \tau \).
Consider a spherical charge distribution in free space as shown in the figure below where the charge density in the spherical region is given by:
\[
\rho_v = K_0 r \quad C/cm^3 \quad 0 \leq r \leq 2cm
\]
where \(K_0\) is a constant and “\(r\)” is the radius of the sphere.

The total charge in the spherical region is \(Q_{\text{Total}} = 5nC\).

a) Calculate the constant \(K_0\).

b) Calculate the displacement flux density \(D\) at a distance of \(r = 1cm\). Assume that \(D\) is radially directed and with a constant magnitude at a fixed distance.

c) Calculate the potential difference \(V\) between the points \(r = 3cm\) and \(r = 4cm\) due to the spherical charge distribution.
Problem 5 (Core: E&M-ECE3025) Solution

Solution

a) 
\[
\int_{0}^{2\pi} \int_{0}^{2\pi} K_0 r^2 \sin \theta d\theta d\phi = 5nC
\]
\[
K_0 \left[ \frac{r^4}{4} \right]_{0}^{1} = 4\pi K_0 \cdot 4 = 16\pi K_0 = 5nC \Rightarrow K_0 = 0.1nC / cm^4
\]

b) Apply Gauss’s Law
\[
\int \vec{D} \cdot dS = Q_{\text{encl}}
\]
\[
\int_{0}^{2\pi} \int_{0}^{2\pi} D_r r^2 \sin \theta d\theta d\phi = \int_{0}^{1} \int_{0}^{2\pi} 0.1r^2 \sin \theta d\theta d\phi
\]
\[
D_r 4\pi r^2 = 0.1 \left[ \frac{r^4}{4} \right]_{0}^{1} 4\pi \Rightarrow D = \frac{0.1 \times 1 \times 4\pi}{4\pi \times 1^2} = 0.025a_r (nC / cm^2)
\]

c) 
\[
V = -\int \vec{E} \cdot dl = -\int_{0}^{3} E_r dr
\]
\[
\vec{E} = \frac{5 \times 10^{-9}}{4\pi \varepsilon_0 r^2} a_r \quad r > 2cm
\]
\[
V = \int_{0}^{3} \frac{5 \times 10^{-9}}{4\pi \varepsilon_0 r^2} a_r \cdot dr = \frac{5 \times 10^{-9}}{4\pi \varepsilon_0} \left[ \frac{1}{r} \right]_{4}^{1} = 374V
\]
A left-hand circular polarized wave in air with an electric field phasor of the form

\[ E^i(z) = (\hat{x} + j \hat{y})E_0^i e^{-j\beta z} \]

where \( E_0^i \) is a given constant, \( \beta = \omega / c \), \( \omega \) is the radian frequency, and \( c = 1 / \sqrt{\mu_0 \varepsilon_0} \) is the speed of light, is incident on a perfectly conducting half space (infinite conductivity) whose surface is located at \( z = 0 \). The reflected electric field has the general form

\[ E^r(z) = (E_x^r \hat{x} + j E_y^r \hat{y}) e^{+j\beta z} \]

(a) Find the coefficients \( E_x^r \) and \( E_y^r \).

(b) Describe the polarization of the reflected wave by choosing one of the following: linear, right-hand circular, left-hand circular, right-hand elliptic, left-hand elliptic
A left-hand circular polarized wave in air with an electric field phasor of the form

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\[ \vec{E}'(z) = (E_x^r \hat{x} + j E_y^r \hat{y})e^{j\beta z} \]

(a) Find the coefficients \( E_x^r \) and \( E_y^r \).

By imposing the electromagnetic boundary condition

\[ \hat{n} \times \vec{E}|_{z=0} = \hat{n} \times \left( \vec{E}_i^i(0) + \vec{E}_1^r(0) \right) \]

\[ = -\hat{y}(E_0^i + E_x^i) - \hat{x}(jE_0^i + jE_y^i) \]

\[ = 0 \]

we obtain

\[ E_x^r = -E_x^i \]

\[ E_y^r = -E_y^i \]

(b) Describe the polarization of the reflected wave by choosing one of the following:

linear, right-hand circular, left-hand circular, right-hand elliptic, left-hand elliptic

The reflected field can be written as

\[ \vec{E}'(z) = -(\hat{x} + j \hat{y})E_0^i e^{j\beta z} \]

To investigate the polarization, consider the corresponding time function

\[ \vec{e}'(z,t) = -\hat{x} E_0^i \cos(\omega t + \beta z) + \hat{y}E_0^i \sin(\omega t + \beta z) \]

In the \( z = 0 \) plane, at \( t = 0 \), \( \vec{e}' \) points in the \(-\hat{x}\) direction. Slightly later in time at \( \omega t = \pi/2 \), \( \vec{e}' \) points in the \(+\hat{y}\) direction. Since the wave is propagating in the \(-\hat{z}\) direction, this wave exhibits right-hand circular polarization.
In the AC circuit below, the sources are labeled with their phasor values and the elements are labeled by their impedances. Suppose the frequency is $\omega = 3 \text{ rad/sec}$.

![AC circuit diagram](image)

a. Give the capacitance of the capacitor.

b. Give the AC steady state time domain expression, $v_L(t)$, for the voltage across the inductor, corresponding to the phasor $V_L$.

c. Give the average power supplied by the current source on the right, that is, the one labeled $\sqrt{2} \angle 45^\circ \text{V}$.

d. How can this circuit be simplified if the frequency, $\omega$, becomes arbitrarily large? For this part of the problem, you can ignore the phasor values of the sources.
In the AC circuit below, the sources are labeled with their phasor values and the elements are labeled by their impedances. Suppose the frequency is \( \omega = 3 \text{ rad/sec} \).

![AC Circuit Diagram]

a. Give the capacitance of the capacitor.

Capacitor impedance is \( Z_c = \frac{1}{j\omega C} \). Substituting for \( \omega \), we have

\[
\frac{1}{j\beta C} = -j6,
\]

which yields \( C = \frac{1}{18} \text{ F} \).

b. Give the AC steady state time domain expression, \( v_L(t) \), for the voltage across the inductor, corresponding to the phasor \( V_L \).

There are several choices for how to compute \( V_L \). Node Voltage Analysis (NVA) has the advantage that there will be only one equation, whereas Mesh Current Analysis will have two equations, therefore, NVA is used in this solution. First, label the node voltages. There are only three nodes total in this circuit. We assign one to be ground, choosing the node on the bottom for ground, since it is connected to a voltage source, which will eliminate a node equation. The other two nodes are labeled \( V_A \) and \( V_B \), however, because the voltage source is connected to ground, we just have \( V_A = 6\text{ V} \). That leaves only one unknown node voltage, \( V_B \), and, conveniently, that node voltage is also \( V_L \).
Write the node equation for $V_L$ (adding currents “out”):

$$\frac{V_L - 6}{-j6} + \frac{V_L}{j8} - \sqrt{2}e^{j\frac{\pi}{4}} = 0$$

Rearranging and using $\sqrt{2}e^{j\frac{\pi}{4}} = 1 + j$, we have

$$V_L \left(\frac{1}{-j6} + \frac{1}{j8}\right) = V_L \left(\frac{j}{24}\right) = 1 + 2j$$

and

$$V_L = 24(2 - j).$$

Using $1 + 2j = \sqrt{3}e^{j\frac{\pi}{3}}$ and solving for $V_L$ yields

$$V_L = -j24\sqrt{3}e^{j\frac{\pi}{3}} = 24\sqrt{3}e^{j\frac{\pi}{3}}e^{-j\frac{\pi}{2}} = 24\sqrt{3}e^{-j\frac{\pi}{6}}$$

Therefore, the time-domain expression is

$$v_L(t) = 24\sqrt{3} \cos \left(3t - \frac{\pi}{6}\right) \text{ volts}$$
c. Give the average power supplied by the current source on the right, that is, the one labeled $\sqrt{2} \angle 45^\circ$ V.

The formula for average power supplied, in terms of phasors, assuming the voltage and the current are defined in the non-passive convention and have the units of volts and amps, respectively, is

$$Avg \ Pwr \ Supplied = \frac{1}{2} Re\{VI^*\} \text{ Watts}$$

We already have both the V and the I, so

$$Avg \ Pwr \ Supplied = \frac{1}{2} Re\left\{\left(24\sqrt{3}e^{-j\frac{\pi}{6}}\right)\sqrt{2}e^{-j\frac{\pi}{4}}\right\}$$

$$= 12\sqrt{6} \cos\left(\frac{5\pi}{12}\right) = 7.6 \text{ Watts}$$

d. How can this circuit be simplified if the frequency, $\omega$, becomes arbitrarily large? For this part of the problem, you can ignore the phasor values of the sources.

The impedance of the capacitor will tend to zero, creating a short circuit and the impedance of the inductor will tend to infinity, creating an open circuit, so as the frequency grows asymptotically large, the circuit can be approximated as shown below.
The following circuit has been implemented in a CMOS process and all transistors are identical.

(a) What is the function of this circuit?

(b) What type of feedback network is the circuit in the gray box?

(c) What is the current gain \(i/O/i_{REF}\) of this circuit?

(d) What is the input impedance \(R_{in}\)?

(e) What is the output impedance \(R_{out}\)?

(f) Assuming \(g_m = 5 \text{ mS and } r_o = 50 \text{ k}\Omega\), find the numerical values for (c), (d), and (e).
Problem 8 (Core: EDA-ECE3400) Solution

(a) This is a Wilson Current Mirror
(b) Shunt-Series Feedback
(c) Small Signal models:

\[ KCL: i_{REF} = \frac{V_g}{R_0} + \frac{g_{m1}}{g_{m2}} i_0 \]
\[ V_{S3} = i_0 \frac{g_{m2}}{g_{m3}} \]
\[ V_{gs3} = V_g - \frac{i_0}{g_{m2}} \]
\[ V_g = \left( i_{REF} - \frac{g_{m1}}{g_{m2}} i_0 \right) R_0 \]
\[ V_{gs3} = \left( i_{REF} - \frac{g_{m1}}{g_{m2}} i_0 \right) R_0 - \frac{i_0}{g_{m2}} \]
\[ i_0 = g_{m3} V_{gs3}, \text{ if we ignore the current through } R_0 \]
\[ i_0 = g_{m3} \left( i_{REF} - \frac{g_{m1}}{g_{m2}} i_0 \right) R_0 - \frac{g_{m3}}{g_{m2}} i_0 \]
\[ i_{REF} = \frac{g_{m3} R_0}{1 + g_{m3}^2 + g_{m3} g_{m1} R_0 R_0} \]
\[ i_i = \frac{g_m V_0}{2 + g_m V_0} = \frac{g_m V_0}{2} \]
\[ i_i = \frac{2 + g_m V_0}{1 + g_m V_0} \]

(d) To find \( R_{in} \), Replace \( i_{REF} \) with a test voltage, \( V_t \) and short \( V_{out} \), then Redraw the Small Signal model and find \( i_t \). \( R_{in} = \frac{V_t}{i_t} \)
Problem 8 (Core: EDA-ECE3400) Solution

\[ R_{in} = R_{in}' || V_{o1} \]

So we find \( R_{in}' \):

\[ V_{g3} = V_t \]

\[ V_{s3} = \frac{V_{o1}}{g_{m2}} = g_{m3} V_{g3} \]

\[ \rightarrow V_{g3} = V_t - \frac{g_{m3} V_{g3}}{g_{m2}} \quad \rightarrow V_{g3} = \frac{V_t}{1 + \frac{g_{m3}}{g_{m2}}} = \frac{V_t}{2} \]

\[ R_{in}' = \frac{V_t}{g_{m1} || \frac{V_{o1}}{g_{m2}}} = \frac{V_t}{g_{m1} \cdot g_{m3} V_{g3} + 1 \cdot g_{m3} V_t} \approx \frac{2}{g_{m3}} = R_{in} \]

\[ \rightarrow R_{in} = V_{o1} || \frac{2}{g_{m3}} = \frac{2}{g_{m3}} = R_{in} \]

(c) To find \( R_{out} \), replace \( V_{out} \) with a test voltage, \( V_t \), and open \( i_{REF} \) to find \( i_t \):

\[ i_t = g_{m3} V_{g3} + \left( \frac{V_t - i_t}{g_{m2}} \right) \]

KCL:

\[ V_{o1} + \frac{g_{m1}}{g_{m2}} i_t = \left( \frac{V_{g3} + V_{s3}}{V_{o1}} \right) + \frac{g_{m1}}{g_{m2}} i_t = \Theta \]

\[ \rightarrow V_{g3} + V_{s3} = \Theta \]

\[ i_t = \Theta \left( \frac{V_{o1} + \frac{1}{g_{m2}}}{g_{m3}} \right) \]

\[ \rightarrow i_t = -g_{m1} \cdot i_t \left( V_{o1} + \frac{1}{g_{m2}} \right) + \left( \frac{V_t - i_t}{g_{m2}} \right) \]

\[ \text{(Substitute in)} \]
Problem 8 (Core: EDA-ECE3400) Solution

\[ V_t = \frac{V}{V_0} = \left( 1 + \frac{g_m V_0}{g_m} + \frac{1}{g_m V_0} \right) \]

\[ R_{out} = \frac{V_t}{i_t} = V_0 \left( 2 + \frac{g_m}{g_m V_0} \right) \]

\[ (f) \quad \frac{V_{in}}{V_{REF}} = \frac{5 \text{ mS} \times 50 \text{ k}\Omega}{2 + 5 \text{ mS} \times 50 \text{ k}\Omega} = \frac{250}{252} = 0.992 \]

\[ R_{in} = \frac{2}{5 \text{ mS}} = 400 \text{ S} \]

\[ R_{out} = 50 \text{ k}\Omega \left( 2 + 5 \text{ mS} \times 50 \text{ k}\Omega \right) = 50 \text{ k}\Omega \times 252 = 12,600 \text{ S} \]
A 3-phase, 230 V, 27 kVA, load operating at power factor 0.9 (lagging) is supplied by three 10 kVA, 1330/230 V, 60 Hz transformers connected in Y-Δ by means of a common 3-phase feeder whose impedance is 0.003 + j0.015 Ohms per phase. The transformers are supplied from a 3-phase source through a 3-phase feeder whose impedance is 0.8 + j5 Ohms per phase. The equivalent impedance of one transformer referred to the low-voltage side is 0.12 + j0.25 Ohms. Determine the required supply voltage (phase-to-phase) if the load voltage is 230 V.
Problem 9 (Core: Energy-ECE3300)

**Solution**

\[ V_S \]

\[ V_L = 133 \, /0^\circ \]

\[ I_L = 67.67 \, /-25.8^\circ \]

\[ V_S' = 133 \, /0^\circ + 67.67 \, /-25.8^\circ = 140.7 \, /-21^\circ \]

**eq. impedance of the transformer referred to hv side:**

\[ R_{eq} + jX_{eq} = \left( \frac{1330}{230} \right)^2 \left( 0.12 + j \times 0.25 \right) = 4.0 + j \times 8.36 \]

**turns ratio of the equivalent y-y bank is**

\[ \alpha' = \frac{\sqrt{3} \times 1330}{230} = 69.94 \]

**single phase cct shown in picture, by referring to lv side (second picture), we get:**

\[ R = \frac{0.80 + 4.01}{10} + 0.003 = 0.0552 \]

\[ X = (5 + 8.36) \frac{1}{10} + 0.015 = 0.149 \]

\[ V_L = \frac{230}{\sqrt{3}} \, /10^\circ = 133 \, /10^\circ \, V \]

\[ I_L = \frac{27 \times 1.03}{3 \times 133} = 67.67 \, A \]

\[ \varphi_L = - \cos^{-1} 0.9 = -25.8^\circ \]

\[ V_S' = 133 \, /0^\circ + 67.67 \, /-25.8^\circ \times (0.051 + j \times 0.149) \]

\[ = 13\, /0^\circ + 10.6571 \, /45.8^\circ = 140.7 \, /21^\circ \, V \]

**line (phase to phase) supply voltage is**

\[ V_{HV} = 1407 \times \sqrt{3} = 2437 \, V \]
A 460 V line to line, 60 Hz, 4 pole, 1740 rpm, Y-connected, three phase induction motor has the following equivalent circuit parameters per phase: \( R_1 = 0.25 \Omega \), \( R_2 = 0.2 \Omega \), \( X_1 = X_2 = 0.5 \Omega \). The core loss resistance and magnetizing inductance can be neglected.

a) Determine the starting current when started direct at rated voltage

b) Determine the starting torque.

c) Determine the rated slip.
Problem 10 (Core: Energy-ECE3300) Solution

SOLUTION

a) At start, the slip is 1, therefore the starting current is,

\[ I_{\text{start}} = \frac{V_s}{R_1 + \frac{R_2}{s} + j(X_1 + X_2)} = \frac{460 / \sqrt{3}}{0.25 + \frac{0.2}{1} + j(0.5 + 0.5)} \]

\[ = 242.2 \angle -65.8^\circ \text{ A} \]

b) Neglecting the magnetizing branch implies that the stator and rotor current are the same. Therefore the starting torque is,

\[ T_{\text{start}} = 3 \frac{1}{\omega_s} I_{\text{start}}^2 \frac{R_2}{s} = 3 \frac{1}{2\pi 60} \cdot 242.2^2 \cdot 0.2 = 93.34 \text{ Nm} \]

c) The rated slip is,

\[ s = \frac{n_s - n_r}{n_s} = \frac{1800 - 1740}{1800} = 0.0333 \]

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The equilibrium energy band diagram for a GaAs sample doped with $4 \times 10^{15}$ cm$^{-3}$ Si atoms is shown below. For GaAs at $T=300$ K, $n_i = 2.25 \times 10^6$ cm$^{-3}$. Knowing that Ga is a group III element, As is a group V element and Si is a group IV element:

a) Find electron and hole concentration ($n$ and $p$) at $T=300$ K.

b) Is this sample n-type or p-type? Why?

c) What atoms in GaAs do Si atoms replace, and with what concentrations?

d) Suppose 10% of Ga atoms are replaced by Al atoms (another group III element), but this process does not change the distribution of Si dopants in the structure. Under this new condition, does the Fermi level ($E_F$) move upward or downward with respect to the intrinsic level $E_i$? Why? For this part, you do not have to calculate anything; a qualitative reasoning is enough.
The equilibrium energy band diagram for a GaAs sample doped with $4 \times 10^{15} \text{ cm}^{-3}$ Si atoms is shown below. For GaAs at $T=300$ K, $n_i = 2.25 \times 10^6 \text{ cm}^{-3}$. Knowing that Ga is a group III element, As is a group V element and Si is a group IV element:

a) Find electron and hole concentration ($n$ and $p$) at $T=300$ K.

\[ n = n_i \exp \left( \frac{E_F - E_i}{kT} \right) = 2.25 \times 10^6 \times \exp \left( \frac{0.52}{0.0259} \right) = 1.1 \times 10^{15} \text{ cm}^{-3} \]

\[ p = \frac{n_i^2}{n} = \frac{(2.25 \times 10^6)^2}{1.1 \times 10^{15}} = 4.6 \times 10^{-3} \text{ cm}^{-3} \]

NOTE: Due to allowed approximations, answers with about 5% variation shall be accepted.

b) Is this sample n-type or p-type? Why?

n-type since $n \gg p$

c) What atoms in GaAs do Si atoms replace and with what concentrations?

Si atoms that replace Ga will act as donors and their concentration is represented by $N_D$.

Si atoms that replace As will act as acceptors and their concentration is represented by $N_A$.

\[ N_D + N_A = 4 \times 10^{15} \text{ (total Si concentration)} \]

\[ N_D - N_A = 1.1 \times 10^{15} \text{ (electron concentration, } n \text{)} \]

Thus, solving these two equations results in the concentration of Si atoms replacing GA and that replacing As to be $N_D = 2.55 \times 10^{15} \text{ cm}^{-3}$ and $N_A = 1.45 \times 10^{15} \text{ cm}^{-3}$, respectively.

d) Suppose 10% of Ga atoms are replaced by Al atoms (another group III element), but this process does not change the distribution of Si dopants in the structure. Under this new condition, does Fermi level ($E_F$) move upward or downward? Why? For this part, you do not have to calculate anything; a qualitative reasoning is enough.

Since Al has the same valance electrons as Ga, it will not act as a donor or an acceptor. Thus, the concentrations of donors and acceptors remain intact resulting in no change in electron and hole concentrations and thus, no change in the location of $E_F$ compared to $E_i$. 

\[ E_F - E_i = 0.52 \text{ eV} \]
The room-temperature \( (T = 300 \text{ K}) \) capacitance-voltage characteristics of a uniformly-doped silicon PN junction diode are shown below. *The graph is drawn to scale.* Assume that either side of the junction contains one and only one type of dopant. The doping concentration on the \( p \)-side \( (N_A) \) is 10 times higher than the doping concentration on the \( n \)-side \( (N_D) \). The dopants are 100\% ionized and the diode junction area is \( 2.1 \times 10^{-5} \text{ cm}^2 \).

The following physical constants might be needed in this problem: The free-space permittivity is \( 8.85 \times 10^{-14} \text{ F/cm} \); the electronic charge is \( 1.6 \times 10^{-19} \text{ C} \); The dielectric constant of Si is 11.9; the effective electron mass is \( 1.12m_0 \), where the free electron mass \( (m_0) \) is \( 9.1 \times 10^{-31} \text{ kg} \); and the effective hole mass is \( 0.8m_0 \). The Boltzmann constant is \( 1.38 \times 10^{-23} \text{ J/K} \) and the Planck constant is \( 6.626 \times 10^{-34} \text{ J-s} \).

Answer the following questions.

(a) (4 points) Find the built-in voltage \( (V_{bi}) \) of the junction in the units of volt. Show your work by including a proper discussion of the calculation methodology in less than 40 words. Lengthy discussion will not be considered for grading.

(b) (4 points) Calculate the dopant concentration on the lowly doped side of the diode in the units of \( \text{cm}^{-3} \).

(c) (2 points) Calculate the depletion width of the diode at \( V_a = -1.5 \text{V} \) in microns.

*(Hint: You may check the consistency of the answers for correctness after you work out the solutions.)*
(a) The depletion width and the junction capacitance has the following relationship:

\[ C_j = \frac{\varepsilon_s \varepsilon_0 A}{w} = \frac{\varepsilon_s \varepsilon_0 A}{w} \Rightarrow \frac{1}{C_j^2} = \frac{2(N_A + N_D)}{qN_A N_D \varepsilon_s \varepsilon_0 A^2} (V_{bi} - V_a) \]

The built-in voltage can be determined by extrapolating the \(1/C^2(V_a)\) curve at \(1/C^2 = 0\) in the graph (i.e., the x-axis intersect). So, \(V_{bi} = 0.8\ V\).

(b) The carrier concentration can be found from the slope of the \(1/C_j^2\) v.s. \(V_a\) plot by:

\[ \frac{\partial(\frac{1}{C_j^2})}{\partial V_a} = \frac{\Delta(\frac{1}{C_j^2})}{\Delta V_a} = \frac{-2(N_A + N_D)}{qN_A N_D \varepsilon_s \varepsilon_0 A^2} = \frac{(10 - 4) \times 10^{24}}{-5 - (-1.5)} = -1.714 \times 10^{24} \]

\[ N_A = 10N_D \Rightarrow \frac{-2(N_A + N_D)}{qN_A N_D \varepsilon_s \varepsilon_0 A^2} = \frac{-2 \times 11}{10N_D q\varepsilon_s \varepsilon_0 A^2} = -1.714 \times 10^{24} \]

\[ \Rightarrow N_D = \frac{22}{1.714 \times 10^{24} \cdot 10 \cdot 1.6 \times 10^{-19} \cdot 11.9 \cdot 8.85 \times 10^{-14} \cdot (2.1 \times 10^{-5})^2} = 1.73 \times 10^{16} cm^{-3} \]

(c) From the graph,

\[ C_j = \frac{\varepsilon_s \varepsilon_0 A}{w} \Rightarrow w = \frac{\varepsilon_s \varepsilon_0 A}{C_j} = \frac{11.9 \cdot 8.85 \times 10^{-14} \cdot 10^{-5}}{4.4 \times 10^{-5} (cm)} = 0.44 \mu m \]
The block diagram above defines a system for discrete-time filtering of continuous-time signals. 

Note: all parts of this question can be worked independently.

(a) In this part assume that the impulse response of the LTI system is $h[n] = \delta[n]$. Both converters have the same sampling rate; determine $f_s$ so that the following two conditions are satisfied:
1. when the input signal is $x(t) = 7 \cos(6000\pi t + 2\pi/3)$, the output is a constant, and
2. when $x(t) = \cos(3200\pi t)$ the frequency of the output signal $y(t)$ is 400 Hz.

$$f_s = \underline{\text{[solution]}}$$

(b) Suppose that the frequency response of the LTI system is $H(e^{j\hat{\omega}}) = \frac{2 + 5e^{-j\hat{\omega}}}{1 + 0.4e^{-j\hat{\omega}}}$. Determine the output $y[n]$ when the input signal is $x[n] = \pi \cos(n - 2)$. Simplify your answer by writing it as a sinusoid, i.e., $y[n] = A \cos(\hat{\omega}_0 n + \varphi)$

$$y[n] = \underline{\text{[solution]}}$$

(c) Suppose that the LTI system is a second-order IIR notch filter that has the following system function,

$$H(z) = \frac{B(z)}{A(z)} = \frac{(1 - e^{j\theta}z^{-1})(1 - e^{-j\theta}z^{-1})}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}.$$ 

Suppose that the impulse response of the notch filter is

$$h[n] = 1.5625(0.8)^n \cos(0.6\pi n + 3.106)u[n]$$

Make a pole-zero plot of the system function. Label the exact locations of the poles and zeros.
The block diagram above defines a system for discrete-time filtering of continuous-time signals. Note: all parts of this question can be worked independently.

(a) In this part assume that the impulse response of the LTI system is $h[n] = \delta[n]$. Both converters have the same sampling rate; determine $f_s$ so that the following two conditions are satisfied:

1. when the input signal is $x(t) = 7 \cos(6000\pi t + 2\pi/3)$, the output is a constant, and
2. when $x(t) = \cos(3200\pi t)$ the frequency of the output signal $y(t)$ is 400 Hz.

$f_s = 1000$ Hz

(b) Suppose that the frequency response of the LTI system is $H(e^{j\omega}) = \frac{2 + 5e^{-j\omega}}{1 + 0.4e^{-j\omega}}$.

Determine the output $y[n]$ when the input signal is $x[n] = \pi \cos(n - 2)$. Simplify your answer by writing it as a sinusoid, i.e., $y[n] = A \cos(\omega_0 n + \phi)$

$$y[n] = 5\pi \cos(n - 2.46) \quad H(e^{j\omega})\big|_{\omega=1} = 5e^{-j0.46}$$

(c) Suppose that the LTI system is a second-order IIR notch filter that has the following system function,

$$H(z) = \frac{B(z)}{A(z)} = \frac{(1 - e^{j\theta} z^{-1})(1 - e^{-j\theta} z^{-1})}{(1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})},$$

Suppose that the impulse response of the notch filter is

$$h[n] = 1.5625(0.8)^n \cos(0.6\pi n + 3.106)u[n]$$

Make a pole-zero plot plot of the system function. Label the exact locations of the poles and zeros.

$r = 0.8, \ \theta = 0.6\pi$

poles at $0.8e^{\pm 0.6\pi}$
zeros on unit circle at $1.0e^{\pm 0.6\pi}$
There are two slot machines in front of you. It costs $1 to play either, and if you win you get $3, otherwise you get $0. One of the machines pays out with probability 33%, but the other one is rigged and only pays out with probability 5%. You pick one of the slot machines at random, pull the arm, and lose.

1. Given that you lost, what is the probability that you picked the rigged machine?

2. Suppose you play the same slot machine again. Given that your first pull resulted in a loss, what is the probability that playing the same slot machine again will result in you winning?

3. Suppose instead that after losing on the first pull you decide to play the other machine. What is the expected amount of money you will win on the second pull?
Problem 14 (Core: DSP-ECE3077) Solution

There are two slot machines in front of you. It costs $1 to play either, and if you win you get $3, otherwise you get $0. One of the machines pays out with probability 33%, but the other one is rigged and only pays out with probability 5%. You pick one of the slot machines at random, pull the arm, and lose.

(a) Given that you lost, what is the probability that you picked the rigged machine?

Let \( A \) = the event that you picked the rigged machine
\( B \) = the event that your first pull resulted in a loss

We want \( P(A|B) \):

\[
P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{0.95 \cdot \frac{1}{2}}{0.9083} \approx 0.5876
\]

(b) Suppose you play the same slot machine again. Given that your first pull resulted in a loss, what is the probability that playing the same slot machine again will result in you winning?

Let \( C \) = the event that your second pull results in a win

\[
P(C|B) = P(C|A, B) \cdot P(A|B) + P(C|A^c, B) \cdot P(A^c|B)
\]

\[
= 0.05 \cdot 0.5876 + \frac{1}{2} \cdot (1 - 0.5876)
\]

\[
\approx 0.1668
\]

(c) Suppose instead that after losing on the first pull you decide to play the other machine. What is the expected amount of money you will win on the second pull?

Let \( D \) = the event that your second pull now results in a win

\[
P(D|B) = P(D|A, B) \cdot P(A|B) + P(D|A^c, B) \cdot P(A^c|B)
\]

\[
\approx \frac{1}{3} \cdot 0.5876 + 0.05 \cdot (1 - 0.5876)
\]

\[
\approx 0.2165
\]

\[
E[\text{winning}] = 3 \cdot P(D|B) + 0 \cdot P(D^c|B)
\]

\[
\approx 3 \cdot 0.2165 \approx $0.65
\]
Consider the set, \( S \), of smooth signals, modeled as infinitely differentiable functions. Two linear systems, \( P \) and \( Q \) act on \( S \), such that an anti-commutation relation holds:

\[
PQ - QP = I, \tag{1}
\]

\( I \) being the identity on \( S \). Assume that a nonzero signal \( e \) exists such that \( \forall t \colon Pe(t) = 0 \).

Show the following:

1. Show that the signal space cannot be finite dimensional if such \( P \) and \( Q \) act on it as given above.

2. Show that the systems represented by the action \( P_1 = \frac{d}{dt} \) and \( Q_1 \), defined by:

\[
\forall t, \ (Q_1 x)(t) = tx(t),
\]

satisfy the anti-commutation property (1).

The insight provided by this special case should connect the abstract problem to the problem of solving a homogeneous linear time invariant ordinary differential equation.

3. Let \( P_2 \) be a series and parallel combination of the systems in (1) such that the output \( y \), given the input \( u \), is \( y = P_2 u \), where \( P_2 = (Q_1^2 + I)P_1 \). Find the corresponding anti-commuting system \( Q_2 \), so that (1) holds.

Hint: try in analogy to \( Q_1 \), the multiplication by some time function, \( q \).

4. Prove the general “systems-in-series” relation, \( PQ^n = Q^nP + nQ^{n-1} \), for all non-negative integers \( n \). It is understood that \( Q^0 = I \).

Hint: Use mathematical induction.

5. Prove the general relation \( P^{n+1}Q^n e = 0 \) for all nonnegative integers \( n \).

6. If the pair \( P, Q \) satisfies the anti-commutation relation, show that \( P^n x = 0 \) is solved by a linear combination (weighted sum) of the signals \( e, Qe, \ldots, Q^{n-1}e \).
1. Linear operations on finite dimensional spaces can be represented by matrices. Thus, for \( \dim \mathcal{S} = n < \infty \), \( P \) and \( Q \) are square \( n \times n \) matrices. Taking the trace of the anti commutation relation, we find

\[
\text{tr} \ PQ - \text{tr} \ PQ = \text{tr} I_n = n.
\]

But since \( \text{tr}(PQ) = \text{tr}(QP) \), the left hand side is zero, leading to a contradiction.

2. In simple terms \( (P_1Q_1 - Q_1P_1)x \) is the signal \( \frac{d}{dt}(tx(t)) - t \frac{d}{dt}x(t) \). But this is directly seen to equal \( x(t) \).

3. We need to find \( Q_2 \) such that 

\[
(t^2 + 1) \frac{d}{dt}(Q_2x)(t) - Q_2(t^2 + 1)x(t) = x(t)
\]

for all \( x \). Try for \( Q_2 \) pointwise multiplication by some smooth function \( q(t) \). Then one needs to solve 

\[
(t^2 + 1)(q(t)x)' - q(t^2 + 1)x = x,
\]

which leads to \( (t^2 + 1)q'(t) = 1 \). This is a simple integral

\[
dq(t) = \frac{dt}{t^2 + 1} = d \arctan t.
\]

Hence \( (Q_2x)(t) = \arctan(t)x(t) \). The operator is the pointwise multiplication by \( \arctan \).

4. Use mathematical induction.
   i) The relation obviously holds for \( n = 0 \).
   ii) Induction step: Assume the relation holds for \( n = n_0 \), then

\[
PQ^{n_0+1} = PQ^{n_0}Q
= (Q^{n_0}P + n_0Q^{n_0-1})Q
= Q^{n_0}(QP) + n_0Q^{n_0}
= Q^{n_0}(QP + I) + n_0Q^{n_0}
= Q^{n_0+1}P + (n_0 + 1)Q^{n_0}.
\]

5. Use again mathematical induction.
   i) The relation obviously holds for \( n = 0 \).
   ii) Induction step: Assume the relation holds for \( n = n_0 \), then, using the above,

\[
P^{n_0+1}Q^{n_0}e = P^{n_0}(PQ^{n_0})e
= P^{n_0}(Q^{n_0}P + n_0Q^{n_0-1})e
= P^{n_0}Q^{n_0}Pe + n_0P^{n_0}Q^{n_0-1}e
= 0
\]

6. It follows directly from the previous part and the linearity of \( P \) implying \( P0 = 0 \), that \( P^n(a_0e + a_1 Qe + \ldots + a_{n-1}Q^{n-1}e) = 0 \). Hence all such combinations are solutions.
Consider the system shown below that is composed of three LTI systems with impulse responses of $h_1(t)$, $h_2(t)$, and $h_3(t)$ and corresponding frequency responses of $H_1(j\omega)$, $H_2(j\omega)$, and $H_3(j\omega)$. The input is $x(t)$ and the output is $y(t)$.

(a) Express $h(t)$, the impulse response of the overall system, in terms of $h_1(t)$, $h_2(t)$, and $h_3(t)$.

(b) Express $H(j\omega)$, the frequency response of the overall system, in terms of $H_1(j\omega)$, $H_2(j\omega)$, and $H_3(j\omega)$.

For parts (c), (d) and (e), assume $h_2(t) = \delta(t-t_d)$ and $h_3(t) = -h_1(t)$.

(c) Express $H(j\omega)$ in terms of $H_1(j\omega)$ and $t_d$.

(d) Let $x(t)$ be a completely general periodic signal with period $T_0$ and assume that $t_d > 0$. For what, if any, values of $T_0$ is the output $y(t)$ identically zero?

(e) For $h_1(t) = e^{-t}u(t)$, $t_d = 2$, and $x(t) = 10 + 3\cos(2t) + 2\delta(t-5)$, what is the output, $y(t)$?
(a) \( h(t) = h_1(t) + h_2(t) * h_3(t) \)

(b) \( H(j\omega) = H_1(j\omega) + H_2(j\omega)H_3(j\omega) \)

(c) \( H_3(j\omega) = -H_1(j\omega) \) and \( H_2(j\omega) = e^{-j\alpha_d} \)

\[
H(j\omega) = H_1(j\omega) - e^{-j\alpha_d}H_1(j\omega) \\
= (1 - e^{-j\alpha_d})H_1(j\omega) \\
= e^{-j\alpha_d/2}(e^{+j\omega/2} - e^{-j\omega/2})H_1(j\omega) \\
= 2je^{-j\omega/2}\sin(\alpha_d/2)H_1(j\omega) \\
= 2e^{-j(\alpha_d-\pi)/2}\sin(\alpha_d/2)H_1(j\omega)
\]

(d) The output is zero if all of the harmonics of \( x(t) \) coincide with the zeroes of the sine term of \( H(j\omega) \) in part (c). This condition is:

\[
\frac{\alpha_d}{2} = n\pi, \quad n = 1, 2, 3K \\
\frac{2\pi t_d}{2T_0} = n\pi, \quad n = 1, 2, 3K \\
T_0 = \frac{t_d}{n}, \quad n = 1, 2, 3K
\]

So any periodic signal \( x(t) \) with period \( T_0 = t_d/n \) (\( n = 1, 2, 3K \)) will result in \( y(t) = 0 \).

(e) Work in the frequency domain for the first two terms \( (\omega = 0 \text{ and } \omega = 2) \), and convolve in the time domain for the third term.

\[
H_1(j\omega) = \frac{1}{1 + j\omega} \\
H(j\omega) = \frac{2e^{-j\omega\pi/2}\sin(\omega)}{1 + j\omega} \\
H(j0) = 0 \\
H(j2) = \frac{2e^{-j(2\pi/2)}\sin(2)}{1 + j2} = \frac{2e^{-j(2-\pi/2)}\sin(2)(1-j2)}{5} = 0.0289 - j0.8128 = 0.8133e^{-j1.5364}
\]

\[
h(t) = 3(0.8133)\cos(2t - 1.5364) + 2e^{-t-5}u(t-5) \\
= 2.4399\cos(2t - 1.5364) + 2e^{-t-5}u(t-5)
\]
Problem 17 (Specialized: CSS-ECE3035) Solution

**Problem**

In this question, assume a MIPS R3000/R4000 ISA. A subset of MIPS instructions is attached. Use only those instructions.

Consider the following C code, where foo, bar and f1 are appropriately declared functions:

```c
int i, n;
int A[1000], z;

n = bar();
...

for (i = 0; i < n; i++) {
...
A[i] = foo(i);
z = f1(A[i]);
if (A[i] < z) break;
...
}
```

(a) (2) Consider the allocation of the variables i and z to either S (callee saved) or T (caller saved) registers. Based only on the code actually shown, tell which register type you would allocate each variable to, and why you would make that choice. Be specific with respect to advantages of your choice.
(b) Without performing any optimization, write MIPS code (with comments) for the for loop NOT including the three statements in the body of the loop. For this question assume that \( n \) is assigned to $8, and \( i \) is assigned to $9.

<table>
<thead>
<tr>
<th>Label</th>
<th>Instruction</th>
<th>Comment</th>
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<tbody>
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</tbody>
</table>

(c) Now optimize the loop (again not considering the three statements in the body) to minimize the loop overhead.

<table>
<thead>
<tr>
<th>Label</th>
<th>Instruction</th>
<th>Comment</th>
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<tbody>
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</tbody>
</table>

(d) Suppose the body of the loop requires ten instructions to implement (just assume this). What is percentage improvement in performance of your optimized code assuming a large number of iterations.
<table>
<thead>
<tr>
<th>instruction</th>
<th>example</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>arithmetic</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>add</td>
<td>add $1,$2,$3</td>
<td>$1 = $2 + $3</td>
</tr>
<tr>
<td>subtract</td>
<td>sub $1,$2,$3</td>
<td>$1 = $2 - $3</td>
</tr>
<tr>
<td>add immediate</td>
<td>add $1,$2,100</td>
<td>$1 = $2 + 100</td>
</tr>
<tr>
<td>add unsigned</td>
<td>addu $1,$2,$3</td>
<td>$1 = $2 + $3</td>
</tr>
<tr>
<td>subtract unsigned</td>
<td>subu $1,$2,$3</td>
<td>$1 = $2 - $3</td>
</tr>
<tr>
<td>add immediate unsigned</td>
<td>addiu $1,$2,$3</td>
<td>$1 = $2 + 100</td>
</tr>
<tr>
<td>set if less than</td>
<td>slt $1, $2, $3</td>
<td>if ($2 &lt; $3), $1 = 1 else $1 = 0</td>
</tr>
<tr>
<td>set if less than immediate</td>
<td>slti $1, $2, 100</td>
<td>if ($2 &lt; 100), $1 = 1 else $1 = 0</td>
</tr>
<tr>
<td>set if less than unsigned</td>
<td>sltu $1, $2, $3</td>
<td>if ($2 &lt; $3), $1 = 1 else $1 = 0</td>
</tr>
<tr>
<td>set if &lt; immediate unsigned</td>
<td>sltiu $1, $2, 100</td>
<td>if ($2 &lt; 100), $1 = 1 else $1 = 0</td>
</tr>
<tr>
<td>multiply</td>
<td>mult $2,$3</td>
<td>Hi, Lo = $2 * $3, 64-bit signed product</td>
</tr>
<tr>
<td>multiply unsigned</td>
<td>mulu $2,$3</td>
<td>Hi, Lo = $2 * $3, 64-bit unsigned product</td>
</tr>
<tr>
<td>divide</td>
<td>div $2,$3</td>
<td>Lo = $2 / $3, Hi = $2 mod $3</td>
</tr>
<tr>
<td>divide unsigned</td>
<td>divu $2,$3</td>
<td>Lo = $2 / $3, Hi = $2 mod $3, unsigned</td>
</tr>
<tr>
<td><strong>transfer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>move from Hi</td>
<td>mfhi $1</td>
<td>$1 = Hi</td>
</tr>
<tr>
<td>move from Lo</td>
<td>mflo $1</td>
<td>$1 = Lo</td>
</tr>
<tr>
<td>load upper immediate</td>
<td>lui $1,100</td>
<td>$1 = 100 x $2^{16}</td>
</tr>
<tr>
<td><strong>logic</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>and $1,$2,$3</td>
<td>$1 = $2 &amp; $3</td>
</tr>
<tr>
<td>or</td>
<td>or $1,$2,$3</td>
<td>$1 = $2</td>
</tr>
<tr>
<td>and immediate</td>
<td>andi $1,$2,100</td>
<td>$1 = $2 &amp; 100</td>
</tr>
<tr>
<td>or immediate</td>
<td>ori $1,$2,100</td>
<td>$1 = $2</td>
</tr>
<tr>
<td>nor</td>
<td>nor $1,$2,$3</td>
<td>$1 = not($2</td>
</tr>
<tr>
<td>xor</td>
<td>xor $1, $2, $3</td>
<td>$1 = $2 ⊕ $3</td>
</tr>
<tr>
<td>xor immediate</td>
<td>xor $1, $2, 255</td>
<td>$1 = $2 ⊕ 255</td>
</tr>
<tr>
<td><strong>shift</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>shift left logical</td>
<td>sll $1,$2,5</td>
<td>$1 = $2 &lt;&lt; 5 (logical)</td>
</tr>
<tr>
<td>shift left logical variable</td>
<td>sllv $1,$2,$3</td>
<td>$1 = $2 &lt;&lt; $3 (logical), variable shift amt</td>
</tr>
<tr>
<td>shift right logical</td>
<td>srl $1,$2,5</td>
<td>$1 = $2 &gt;&gt; 5 (logical)</td>
</tr>
<tr>
<td>shift right logical variable</td>
<td>srlv $1,$2,$3</td>
<td>$1 = $2 &gt;&gt; $3 (logical), variable shift amt</td>
</tr>
<tr>
<td>shift right arithmetic</td>
<td>sra $1,$2,5</td>
<td>$1 = $2 &gt;&gt; 5 (arithmetic)</td>
</tr>
<tr>
<td>shift right arithmetic variable</td>
<td>srav $1,$2,$3</td>
<td>$1 = $2 &gt;&gt; $3 (arithmetic), variable shift amt</td>
</tr>
<tr>
<td><strong>memory</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>load word</td>
<td>lw $1, 1000($2)</td>
<td>$1 = memory [2+1000]</td>
</tr>
<tr>
<td>store word</td>
<td>sw $1, 1000($2)</td>
<td>memory [2+1000] = $1</td>
</tr>
<tr>
<td>load byte</td>
<td>lb $1, 1002($2)</td>
<td>$1 = memory[2+1002] in least sig. byte</td>
</tr>
<tr>
<td>load byte unsigned</td>
<td>lbu $1, 1002($2)</td>
<td>$1 = memory[2+1002] in least sig. byte</td>
</tr>
<tr>
<td>store byte</td>
<td>sb $1, 1002($2)</td>
<td>memory[2+1002] = $1 (byte modified only)</td>
</tr>
<tr>
<td><strong>branch</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>branch if equal</td>
<td>beq $1,$2,100</td>
<td>if ($1 = $2), PC = PC + 4 + (100*4)</td>
</tr>
<tr>
<td>branch if not equal</td>
<td>bne $1,$2,100</td>
<td>if ($1 ≠ $2), PC = PC + 4 + (100*4)</td>
</tr>
<tr>
<td><strong>jump</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>jump</td>
<td>j 10000</td>
<td>PC = 10000*4</td>
</tr>
<tr>
<td>jump register</td>
<td>jr $31</td>
<td>PC = $31</td>
</tr>
<tr>
<td>jump and link</td>
<td>jal 10000</td>
<td>$31 = PC + 4; PC = 10000*4</td>
</tr>
</tbody>
</table>
In this question, assume a MIPS R3000/R4000 ISA. A subset of MIPS instructions is attached. Use only those instructions.

Consider the following C code, where foo, bar and f1 are appropriately declared functions:

```c
int i,n;
int A[1000],z;

n = bar();
...

for (i=0; i<n; i++) {
...
A[i] = foo(i);
z = f1(A[i]);
if (A[i] < z) break;
...
}
```

(a) (2) Consider the allocation of the variables i and z to either S (callee saved) or T (caller saved) registers. Based only on the code actually shown, tell which register type you would allocate each variable to, and why you would make that choice. Be specific with respect to advantages of your choice.

Because variable i is live across function calls foo and f1, it is best allocated to an S register, which would require one save to and restore from the stack in the enclosing function. If it were allocated to a T register, one save and restore would be required per iteration of the loop.

Because variable z is not live across any function calls, it can be allocated to a T register with no save/restore required.
(b) (3) Without performing any optimization, write MIPS code (with comments) for the for loop NOT including the three statements in the body of the loop. For this question assume that n is assigned to $8, and i is assigned to $9.

<table>
<thead>
<tr>
<th>Label</th>
<th>Instruction</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>addi $9, $0, 0</td>
<td># i = 0</td>
</tr>
<tr>
<td>loop:</td>
<td>slt $10, $9, $8</td>
<td># $10 = 1 if i &lt; n</td>
</tr>
<tr>
<td></td>
<td>beq $10, $0, done</td>
<td># branch to done if i &gt;= n</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#</td>
</tr>
<tr>
<td></td>
<td>&lt;loop body&gt;</td>
<td>#</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#</td>
</tr>
<tr>
<td></td>
<td>addi $9, $9, 1</td>
<td># i++</td>
</tr>
<tr>
<td></td>
<td>j loop</td>
<td># jump to loop</td>
</tr>
<tr>
<td>done:</td>
<td></td>
<td>#</td>
</tr>
</tbody>
</table>

(c) (3) Now optimize the loop (again not considering the three statements in the body) to minimize the loop overhead.

<table>
<thead>
<tr>
<th>Label</th>
<th>Instruction</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>addi $9, $0, 0</td>
<td># i = 0</td>
</tr>
<tr>
<td></td>
<td>slt $10, $9, $8</td>
<td># $10 = 1 if i &lt; n</td>
</tr>
<tr>
<td></td>
<td>beq $10, $0, done</td>
<td># branch to done if i &gt;= n</td>
</tr>
<tr>
<td>loop:</td>
<td>&lt;loop body&gt;</td>
<td>#</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#</td>
</tr>
<tr>
<td></td>
<td>addi $9, $9, 1</td>
<td># i++</td>
</tr>
<tr>
<td></td>
<td>bne $9, $8, done</td>
<td># branch to loop if i != n (provided i is not modified</td>
</tr>
<tr>
<td>done:</td>
<td></td>
<td>#</td>
</tr>
</tbody>
</table>

(d) (2) Suppose the body of the loop requires ten instructions to implement (just assume this). What is percentage improvement in performance of your optimized code assuming a large number of iterations?

original loop requires 14 instruction per iteration and optimized requires 12 instructions per iteration.

14/12 = 1.17 thus the optimized loop is 17% faster.
Problem 18 (Specialized: CSS-Operating Syst.)

Consider the following recursive Java method for sorting an array of \( n \) numbers, stored in \( DataArray \). To sort \( DataArray \), a program would call \( recSort(0, n) \).

```java
public static void recSort(int i, int n) {
    if (i != (n-1)) {
        recSort(i+1, n);
        int j = i+1;
        int valBeingInserted = DataArray[i];
        while ((j < n) && (valBeingInserted > DataArray[j])) {
            DataArray[j-1] = DataArray[j];
            j = j+1;
        }
        DataArray[j-1] = valBeingInserted;
    }
}
```

a) Write a recurrence equation for the worst-case running time of \( recSort(0, n) \) in terms of the array size \( n \).

b) Solve this recurrence equation to get a closed-form expression for the worst-case running time of \( recSort(0, n) \). No credit will be given for expressions derived from a direct evaluation of the code, i.e. you must solve a proper recurrence equation to get credit for this part.
SOLUTION

a) \( T(n) = T(n-1) + c \times (n-1) \), \( T(1) = d \), where \( c \) and \( d \) are constants

b) Using substitution:

\[
T(n) = T(n-2) + c(n-2) + c(n-1)
\]
\[
T(n) = T(n-3) + c(n-3) + c(n-2) + c(n-1)
\]

... 
\[
T(n) = T(n-i) + c \sum n-j \quad \text{(sum ranging from } j=1 \text{ to } i)\]

Let \( n-i = 1 \) and substitute \( T(1) = d \) to get
\[
T(n) = d + c \sum n-j = d + c \sum j \quad \text{(sums ranging from } j=1 \text{ to } n-1)\]
\[
T(n) = d + c \times n \times (n-1)/2
\]
\[
T(n) = \frac{1}{2} cn^2 - \frac{1}{2} cn + d
\]
Problem 19 (Specialized: Telecom-ECE3600) Solution

Given a message of 300000 bytes, and maximum segment size (MSS) of 1500 bytes. All segments are assumed to be of the same size (MSS). Assume each RTT is 20 milliseconds. Assume there is no loss.

(a) (4 pts) Assume 7 RTTs of slow start followed by at least \( n \) RTTs of additive increase are needed for TCP to send out the message. Write an expression for, and find integer \( n \).

(b) (4pts) Following up with Part (a), write an expressions for the average transmission rate of the TCP, where the average is over the time duration in RTTS when that TCP uses for sending the message. Calculate the average rate.

(c) (2pts) Now additional information is given about the network: All the bytes sent by TCP passes a slow link with bandwidth 120 Mbps (mega bits per second). Assume the round trip time (RTT) of 20 mili-seconds is the twice of the propagation delay for bits to travel on the bottleneck link. Write an expression on the total bits needed to “fill” the bottleneck link in one RTT. Then calculate the optimal window size for TCP in the number of MSS, relating to the total bits in the slow link.
Problem 19 (Specialized: Telecom-ECE3600) Solution

<table>
<thead>
<tr>
<th>SOLUTION</th>
</tr>
</thead>
</table>

(a) \( (1+\ldots+2^6) + (2^6+1) + (2^6+2) + \ldots + (2^6+n) \geq \frac{300000}{1500} \)

\( n=2 \)

(b) Average rate = total bytes sent / total time used

\[
\text{Average rate} \sim \frac{300000 \times 8}{(9 \times 20 \times 10^{-3})} \sim 13 \text{ Mbps}
\]

(Or to be more accurate: Total time used \( \sim (8 + 8/66) \) RTT.
Average rate \( \sim \frac{300000 \times 8}{[(8+8/66) \times 20 \times 10^{-3}]} \sim 14.7 \text{ Mbps} \)

(c) Total bits (in the bottleneck link in one RTT):

Link rate \times \text{propagation delay} \sim 120 \times 20 \times 10^{-3} = 2.4 \text{ M bits}

Optimal window size = \frac{\text{Total bits}}{\text{MSS}} = \frac{2.4 \times 10^6}{(1500 \times 8)} = 200 \text{ (MSS)}
Problem 20 (Specialized: Optics-ECE4500) Solution

Problem

Semiconductor Laser

A gallium arsenide semiconductor pn-junction diode laser emits light centered at a freespace wavelength of 850 nm. The spectral lineshape is homogeneously broadened. The spectral full-width of the emission is 0.1 nm. The refractive index of the gallium arsenide over this range of wavelengths is 3.6.

From the above information, state whether the lineshape function in frequency, $g(f)$, is Lorentzian or Gaussian or other shape. Calculate, showing all work, the energy gap of the semiconductor material in electron-volts ($eV$). Calculate, showing all work, the frequency full-width of the emission in Gigahertz ($GHz$). Calculate, showing all work, the exponential decay time constant of the laser emission in picoseconds ($ps$). Express your answers accurately to four significant figures and put your final answers in the spaces provided.

(Circle best choice.)
Lineshape function in frequency is (Lorentzian) (Gaussian) (Other Shape).

Energy gap = $\text{__________} eV$

Frequency full-width of emission = $\text{__________} GHz$

Decay time constant = $\text{__________} ps$
Homogeneous broadening → Lineshape function in frequency is Lorentzian. →

\[ g_L(f) = \frac{\Delta f/2 \pi}{(f - f_0)^2 + (\Delta f/2)^2}, \]

where \( f_0 \) is the center (oscillating) frequency and \( \Delta f \) is the full-width-at-half-maximum (FWHM) linewidth.

Gallium arsenide laser

\( \lambda = 850 \text{ nm} \)

\( \Delta \lambda = 0.10 \text{ nm} \)

\( E_G = \frac{hc}{\lambda} = 1.23984 \text{ eV} \mu\text{m}/0.850 \mu\text{m} = 1.4586 \text{ eV} \)

\( \tau = 1/(\pi \Delta f) \)

\( f = c/\lambda \)

\( df = -(c/\lambda^2) d\lambda \)

\( |\Delta f| = (c/\lambda^2) \Delta \lambda = 41.495 \times 10^9 \text{ Hz} = 41.495 \text{ GHz} \)

\( \tau = 1/(\pi \Delta f) = 7.6711 \times 10^{-12} \text{ sec.} = 7.6711 \text{ ps} \)
Problem 21 (Specialized: Optics-ECE4501) Solution

PROBLEM

A typical 10 dB optical tap is shown here, in which input power through the single mode fiber at the left, $P_a$, is split into the two output fibers on the right, the lower of which (port $c$) carries 10% of the input power, or is 10 dB down. The power in the direct (upper) port, $P_b$, is ideally 90% of $P_a$, but is typically lower. The reduction from the ideal 90% power level to the actual $P_b$ is specified as the insertion loss in dB. The device works the same way in reverse, with the power reduction in transmission from ports $b$ to $a$ the same as that for $a$ to $b$, and with the $c$ to $a$ reduction the same as in $a$ to $c$. The loss (negative), or gain (positive) in decibels (dB) is defined as $10 \log_{10} \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)$.

\[ a \quad P_a \quad \rightarrow \quad P_b \quad b \quad \rightarrow \quad P_c \quad c \]

a. With zero insertion loss, what power reduction in dB will occur between ports $a$ and $b$ for the 10 dB device?

b. If the measured loss between ports $a$ and $b$ is -2.46 dB, what is the insertion loss in decibels?

Now, consider a primitive add-drop multiplexer unit composed of two 10 dB taps arranged back-to-back as shown. This device would be used, for example, for each subscriber in a ring network, in which several units are connected through their ports $b'$ and $b$ in a loop, with power moving from left to right -- or clockwise around the ring. The laser transmitter (Tx), and optical receiver (Rx) are used respectively to transmit data to, or to receive data from, any other subscriber in the ring.

\[ b' \quad \rightarrow \quad a \quad \rightarrow \quad b \quad \rightarrow \quad c' \quad \rightarrow \quad a \quad \rightarrow \quad c \quad \rightarrow \quad b \]

c. If the insertion loss for each tap is specified as 1 dB, what is the power reduction in dB between ports $b'$ and $b$?

d. Again, with 1 dB insertion loss, what is the power reduction in dB between ports $b'$ and $c$ and between $c'$ and $b$?

e. Now, suppose that $N$ identical multiplexer units are arranged in a ring network as described above, and that each transmitter is rated as having power output, $P_{\text{out}} = 10$ dBm (decibels relative to a milliwatt), and that each receiver is specified as having sensitivity of $P_{\text{rec}} = -30$ dBm. If all fiber distributed loss and all splice losses are negligible, what is the maximum possible number of subscribers, $N_{\text{max}}$? Again, we have 1 dB insertion loss for each tap.
A typical 10 dB optical tap is shown here, in which input power through the single mode fiber at the left, $P_a$, is split into the two output fibers on the right, the lower of which (port $c$) carries 10% of the input power, or is 10 dB down. The power in the direct (upper) port, $P_b$, is ideally 90% of $P_a$, but is typically lower. The reduction from the ideal 90% power level to the actual $P_b$ is specified as the insertion loss in dB. The device works the same way in reverse, with the power reduction in transmission from ports $b$ to $a$ the same as that for $a$ to $b$, and with the $c$ to $a$ reduction the same as in $a$ to $c$. The loss (negative), or gain (positive) in decibels (dB) is defined as $10 \log_{10}(P_{out}/P_{in})$.

\[ L_{ab} = 10 \log_{10}(0.9) = -0.46 \text{ dB} \]

b. If the measured loss between ports $a$ and $b$ is -2.46 dB, what is the insertion loss in decibels?

\[ L_{ins} = -2.46 - (-0.46) = -2.0 \text{ dB}. \]

Now, consider a primitive add-drop multiplexer unit composed of two 10 dB taps arranged back-to-back as shown. This device would be used, for example, for each subscriber in a ring network, in which several units are connected through their ports $b'$ and $b$ in a loop, with power moving from left to right - or clockwise around the ring. The laser transmitter (Tx), and optical receiver (Rx) are used respectively to transmit data to, or to receive data from, any other subscriber in the ring.

c. If the insertion loss for each tap is specified as 1 dB, what is the power reduction in dB between ports $b'$ and $b$?

\[ L_{b' \rightarrow b} = 2(-1.46) = -2.92 \text{ dB} \]

d. Again, with 1 dB insertion loss, what is the power reduction in dB between ports $b'$ and $c$ and between $c'$ and $b$?

\[ L_{c' \rightarrow b} = -10(0.0 + 1.46) = -11.46 \text{ dB} \]

e. Now, suppose that $N$ identical multiplexer units are arranged in a ring network as described above, and that each transmitter is rated as having power output, $P_{out} = 10$ dBm (decibels relative to a milliwatt), and that each receiver is specified as having sensitivity of $P_{rec} = -30$ dBm. If all fiber distributed loss and all splice losses are negligible, what is the maximum possible number of subscribers, $N_{max}$? Again, we have 1 dB insertion loss for each tap.

\[
P_{out}[\text{dBm}] - P_{rec}[\text{dBm}] = 40 \text{ dB} = [11.46 + (N - 2)(2.92) + 11.46] \text{ dB}
\]

\[ \Rightarrow N = 7.85 \rightarrow N_{max} = 7 \]
A thermal oxidation process has been performed on a (100) silicon wafer. The process is a three-step oxidation as given below. Calculate the oxide thickness after each step in the oxidation process. Also, calculate the total thickness of silicon consumed during the oxidation process.

Step 1: Dry O2: 10 minutes, 1000°C; Step 2: Wet O2: 20 minutes, 1100°C; Step 3: Dry O2: 10 minutes, 1000°C

### Oxidation Coefficients for Silicon

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>Dry A(µm)</th>
<th>Dry B(µm²/hr)</th>
<th>Dry τ(h)</th>
<th>Wet A(µm)</th>
<th>Wet B(µm²/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.370</td>
<td>0.0011</td>
<td>9</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>920</td>
<td>0.235</td>
<td>0.0049</td>
<td>1.4</td>
<td>0.50</td>
<td>0.203</td>
</tr>
<tr>
<td>1000</td>
<td>0.165</td>
<td>0.0117</td>
<td>0.37</td>
<td>0.226</td>
<td>0.287</td>
</tr>
<tr>
<td>1100</td>
<td>0.090</td>
<td>0.027</td>
<td>0.076</td>
<td>0.11</td>
<td>0.510</td>
</tr>
<tr>
<td>1200</td>
<td>0.040</td>
<td>0.045</td>
<td>0.027</td>
<td>0.05</td>
<td>0.720</td>
</tr>
</tbody>
</table>

The τ parameter is used to compensate for the rapid growth regime for thin oxides (After Deal and Grove)
1. Dry O₂  \( A_i = 0.165, \beta_i = 0.0117, \tau = 0.37, t = 0.167 \text{ hr.} \)
   \[ t_{ox1} = 0.0319 \mu m \]

2. Wet O₂  \( A_z = 0.11, \beta_z = 0.510, \tau = \frac{t_{ox1}^2 + A_i t_{ox1}}{\beta_z}, t = 0.33 \text{ hr.} \)
   \[ t_{oxz} = 0.366 \mu m \]

3. Dry O₂  \( A_g = 0.165, \beta_g = 0.0117, \tau = \frac{t_{oxz}^2 + A_i t_{oxz}}{\beta_g}, t = 0.167 \text{ hr.} \)
   \[ t_{ox3} = 0.369 \mu m \]

Total Oxide Thickness = 0.368 \mu m
Total Si Consumed = 0.44 t_{ox3} = 0.162 \mu m
(1 pt) When Neurologists are studying a patient’s EMG, they are looking for data about SMU’s. What is an SMU?

(1 pt) Why are the number of “turn-arounds” significant?

(1 pt) Why is it difficult to automatically analyze the EMG signal to count turn-arounds?

(2 pts) Why is it impossible to perform this analysis via Surface EMG electrodes?

(2 pts) Why is it unwise to use an active band-pass filter to block 60Hz noise in some medical diagnostic protocols (e.g. ERG), but a band-pass filter can be used in EMG measurements?

(3 pts) Draw a block diagram that shows how you would design a data acquisition system for measuring the human EMG. The first stage is always a differential amplifier, which you can represent as a box without specifying any details accept for the approximate gain.
(1 pt) When Neurologists are studying a patient’s EMG, they are looking for data about SMU’s. What is an SMU?
Answer: A Skeletal Motor Unit is the response coming from all of the muscle cells innervated by the same motor neuron.

(1 pt) Why are the number of “turn-arounds” significant?
Answer: If the muscle cells in one SMU are contracting synchronously, then the number of up/down turn-arounds are small.

(1 pt) Why is it difficult to automatically analyze the EMG signal to count turn-arounds?
Answer: Because the signals coming from SMU’s overlap.

(2 pts) Why is it impossible to perform this analysis via Surface EMG electrodes?
Answer: Because the surface electrodes are a long distance away from the contracting muscle cells, and noise levels are huge.

(2 pts) Why is it unwise to use an active band-pass filter to block 60Hz noise in some medical diagnostic protocols (e.g. ERG), but a band-pass filter can be used in EMG measurements?
Answer: Because diagnostic protocols like ERG use a discrete stimulus and the ERG response is characterized by latencies between the stimulus and different components of the ERG response (e.g. A-wave, B-wave, etc). A 60Hz BP filter produces phase shifts that depend on the frequencies of interest; therefore the ERG waveform can have significant latency changes. The EMG protocol has no discrete stimulus.

(3 pts) Draw a block diagram that shows how you would design a data acquisition system for measuring the human EMG. The first stage is always a differential amplifier, which you can represent as a box without specifying any details accept for the approximate gain.
Answer: First stage—Differential Amplifier with a gain of 10x to 100x
Next stage—Band-pass filter
Next stage—Amplifier with gain of 10x to 100x
Next stage—Isolation device, e.g. opto-isolator
Next stages—Sample-and-Hold and A-to-D
PROBLEM

(2 pts) Why is a flash of light used as a stimulus for the human ERG?

(2 pts) What would you use as an ERG stimulus if the eye is filled with blood and light cannot reach the retina?

(3 pts) Draw a sketch of an ERG electrode that could be used for adults, and indicate what can be used as the second reference signal going into a first-stage differential amplifier.

(3 pts) When small amplitude flash stimuli are used to characterize the ERG, the response RMS signal-to-noise can be 1/1000. How is the protocol changed to deal with the noise problem?
(2 pts) Why is a flash of light used as a stimulus for the human ERG?
   Answer: A flash of light is similar to a Dirac Delta Function, which means if the system were linear and time invariant (which it is not), the flash response signal will be similar to the linear transfer function.

(2 pts) What would you use as an ERG stimulus if the eye is filled with blood and light cannot reach the retina?
   Answer: A pulse of electrical current can be used, but white-noise modulations of electrical current is a better stimulus.

(3 pts) Draw a sketch of an ERG electrode that could be used for adults, and indicate what can be used as the second reference signal going into a first-stage differential amplifier.
   Answer: Many electrode designs are possible, e.g. a contact lens with an off-center hole containing a cylindrical Ag wire. The most complicated design was invented by Burian and Allen, which had a small center contact lens supported via a spring attached to a large surrounding annular contact lens. FYI, this electrode used to cost about $400 and lost their appeal when Herpes and HIV infections became common among ophthalmology patients. The reference electrode can be an identical electrode in the unstimulated eye or a second conductive surface in the stimulated eye.

(3 pts) When small amplitude flash stimuli are used to characterize the ERG, the response RMS signal-to-noise can be 1/1000. How is the protocol changed to deal with the noise problem?
   Answer: The best way to deal with the noise problem is to use a large number of flash stimuli with random spacing between the flashes. The ERG is then calculated as the ensemble average response. Blinks are a huge problem when the diagnostic testing continues for several minutes.
Problem 25 (Specialized: BioEng-ECE4784) Solution

**PROBLEM**

a) (3pts) Using the values for channel resting conductance provided for a mammalian axon calculate the sodium and potassium membrane conductance of a section which has a diameter of 0.1mm and a length of 1mm. Assume the axon is at room temperature.

Na: channel conductance of 10pS. channel density of 1000 per $\mu$m$^2$.

K: channel conductance of 5pS. channel density of 300 per $\mu$m$^2$.

b) (3pts) You have the following intracellular and extracellular concentrations for Na$^+$ and K$^+$

$[K^+]_e = 10$mM and $[K^+]_i = 200$mM

$[Na^+]_e = 180$mM and $[Na^+]_i = 60$mM

First, calculate the Nernst potentials for sodium and potassium.

At a membrane voltage, $V_m$, of +70mV, find the sodium current and the potassium current through this patch of membrane.

c) (4pts) At a membrane voltage, $V_m$, of +70mV what will be the membrane current through a single Na channel. The equation for Johnson-Nyquist noise is $i_n = \sqrt{\frac{4kT\Delta f}{R}}$. Using this equation, calculate the signal to noise ratio of that Na channel in dB. Note: R is the resistance generating the noise and $\Delta f$ is the relevant noise bandwidth.

**Constants:**

$k = $ Boltzmann constant = $1.3806488 \times 10^{-23}$ J K$^{-1}$

$q = $ electronic charge = $1.6 \times 10^{-19}$ C
Problem 25 (Specialized: BioEng-ECE4784) Solution

**SOLUTION**

a)(3pts) Using the values for channel resting conductance provided for a mammalian axon calculate the sodium and potassium membrane conductance of a section which has a diameter of 0.1mm and a length of 1mm. Assume the axon is at room temperature.

- **Na**: channel conductance of 10pS, channel density of 1000 per µm^2.
- **K**: channel conductance of 5pS, channel density of 300 per µm^2.

b)(3pts) You have the following intracellular and extracellular concentrations for Na^+ and K^+:

- \([K^+]_e = 10mM\) and \([K^+]_i = 200mM\)
- \([Na^+]_e = 180mM\) and \([Na^+]_i = 60mM\)

First calculate the Nernst potentials for sodium and potassium.

At a membrane voltage, \(V_m\), of -40mV, find the sodium current and the potassium current through this patch of membrane.

c) (4pts) At a membrane voltage, \(V_m\), of -40mV what will be the membrane current through a single Na channel. The equation for Johnson-Nyquist noise is \(i_n = \sqrt{\frac{4kT\Delta f}{R}}\). Using this equation, calculate the signal to noise ratio of that Na channel in dB. Note: \(R\) is the resistance generating the noise and \(\Delta f\) is the relevant noise bandwidth.

**Constants:**

- \(k\) = Boltzmann constant = \(1.3806488 \times 10^{-23}\) J K\(^{-1}\)
- \(q\) = electronic charge = \(1.6 \times 10^{-19}\) C

\[a) g_{Na} = \left(10pS\right) \times \left(1000\text{per}\mu\text{m}^2\right) \times \left(100\mu\text{m}\right) \times \left(1000\mu\text{m}\right) = 1mS\]

\[g_K = \left(5pS\right) \times \left(300\text{per}\mu\text{m}^2\right) \times \left(100\mu\text{m}\right) \times \left(1000\mu\text{m}\right) = 0.15mS\]

\[b) V_K = \frac{RT}{F} \ln \left(\frac{[K^+]_e}{[K^+]_i}\right) = \frac{5k}{q} \ln \left(\frac{[K^+]_e}{[K^+]_i}\right) = 25mV \ln \left(\frac{180mM}{200mM}\right) = -75mV\]

\[V_{Na} = \frac{RT}{F} \ln \left(\frac{[Na^+]_e}{[Na^+]_i}\right) = \frac{kT}{q} \ln \left(\frac{[Na^+]_e}{[Na^+]_i}\right) = 25mV \ln \left(\frac{180mM}{60mM}\right) = 27mV\]

\[I_K = g_K(V_m - V_K) = 0.15mS(-40mV - -75mV) = +5.25\mu A\]

\[I_{Na} = g_{Na}(V_m - V_{Na}) = 1mS(-40mV - 27mV) = -67\mu A\]

c) \(i_{Na} = \gamma_{Na}(V_m - V_{Na}) = 10pS(-40mV - 27mV) = 0.67pA\)

\[\Delta f = 1kHz\text{ for standard axons, i.e. the data rate is at a 1kHz rate, roughly}\]

\[i_n = \sqrt{4kT\Delta f\gamma_{Na}} = \sqrt{4 \times 1.3806488 \times 10^{-23}JK^{-1}\times 300K(1kHz)10pS} = 12.87f ASNR = 20\log\frac{i_{Na}}{i_n} = 20\log\frac{0.6710^{-12}}{12.8710^{-15}} = 34dB\]