ECE 6604 Homework # 6 Solutions

1. Since the PAPR is determined through simulations, your results may differ <u>slightly</u> from the answers below

N	μ	σ^2
256	6.12	1.50
512	6.84	1.61
1024	7.54	1.62

All values are in dB units.

5.10. Non-coherent FSK $\quad P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b/2} \quad \gamma_b = \alpha^2 \frac{E_b}{N_0}$

a) α Rayleigh

$$p_{\gamma_b}(x) = \frac{1}{\bar{\gamma}_b} e^{-x/\bar{\gamma}_b}, \qquad \bar{\gamma}_b = E[\alpha^2]\frac{E_b}{N_0}$$

$$P_b = \int_0^\infty P_b(x) p_{\gamma_b}(x)\,dx = \int_0^\infty \frac{1}{2} e^{-x/2} \cdot \frac{1}{\bar{\gamma}_b} e^{-x/\bar{\gamma}_b} dx = \frac{1}{2\bar{\gamma}_b}\int_0^\infty e^{-x\left(\frac{1}{2}+\frac{1}{\bar{\gamma}_b}\right)}dx$$

$$= \frac{1}{2+\bar{\gamma}_b}$$

(b) $$p_{\gamma_b}(x) = \frac{1}{\bar{\gamma}_b} e^{-K - \frac{(K+1)x}{\bar{\gamma}_b}} I_0\left(2\sqrt{\frac{K(K+1)x}{\bar{\gamma}_b}}\right)$$

$$P_b = \int_0^\infty P_b(x) p_{\gamma_b}(x)\,dx = \int_0^\infty \frac{1}{2} e^{-x/2} p_{\gamma_b}(x)\,dx = \frac{1}{2} E\left[e^{-\gamma_b/2}\right]$$

Noting that $\phi_{\gamma_b}(j\nu) = E[e^{j\nu\gamma_b}]$ is the characteristic function, we can use $\phi_{\gamma_b}(j\nu)|_{\nu=-1/2}$ to find the result.

$$\phi_{\gamma_b}(j\nu) = \frac{1}{1-j2\nu\sigma^2} \exp\left\{\frac{j\nu s^2}{1-j\nu 2\sigma^2}\right\} \qquad 2\sigma^2 = \frac{\bar{\gamma}_b}{K+1} \qquad s^2 = \frac{K\bar{\gamma}_b}{K+1}$$

$$\therefore \quad E\left[e^{-\gamma_b/2}\right] = \phi_{\gamma_b}(j\nu)|_{\nu=-1/2} = \frac{2}{2+2\sigma^2}\exp\left\{-\frac{s^2}{2+2\sigma^2}\right\}$$

$$= \frac{2}{2+\frac{\bar{\gamma}_b}{K+1}}\exp\left\{-\frac{\frac{K\bar{\gamma}_b}{K+1}}{2+\frac{\bar{\gamma}_b}{K+1}}\right\}$$

$$= \frac{2(K+1)}{2(K+1)+\bar{\gamma}_b}\exp\left\{-\frac{K\bar{\gamma}_b}{2(K+1)+\bar{\gamma}_b}\right\}$$

Finally, $$P_b = \frac{1}{2}E\left[e^{-\gamma_b/2}\right]$$

$$= \frac{K+1}{2(K+1)+\bar{\gamma}_b}\exp\left\{-\frac{K\bar{\gamma}_b}{2(K+1)+\bar{\gamma}_b}\right\}$$

5.12. $D^2 = \lim_{N\to\infty} \int_0^{NT} [s(t;\underline{x}^{(i)}) - s(t;\underline{x}^{(j)})]^2 dt$

$= \lim_{N\to\infty} \left\{ 2N\mathcal{E} - 2\int_0^{NT} s(t;\underline{x}^{(i)})\, s(t;\underline{x}^{(j)})\, dt \right\}$

$= \lim_{N\to\infty} \left\{ 2N\mathcal{E} - 2\int_0^{NT} \frac{2\mathcal{E}}{T} \cos[2\pi f_c t + \phi_i(t)] \cos[2\pi f_c t + \phi_j(t)]\, dt \right\}$

$= \lim_{N\to\infty} \left\{ 2N\mathcal{E} - \frac{2\mathcal{E}}{T} \int_0^{NT} \cos\Delta\phi(t)\, dt \right\}$ $\quad < f_c T \gg 1 >$

$= \frac{2\mathcal{E}}{T} \int_0^{\infty} [1 - \cos\Delta\phi(t)]\, dt$

$= 2(\log_2 M)\, E_b \frac{1}{T} \int_0^{\infty} [1 - \cos\Delta\phi(t)]\, dt$

6.2. $p_{\gamma_1}(x) = \frac{1}{\bar{\gamma}_1} e^{-\frac{x}{\bar{\gamma}_1}}$ and $p_{\gamma_2}(x) = \frac{1}{\bar{\gamma}_2} e^{-\frac{x}{\bar{\gamma}_2}}$

Predetection selective combining $\Longrightarrow \gamma_b^s = \max\{\gamma_1, \gamma_2\} \iff F_{\gamma_b^s}(x) = P_r[(\gamma_1 \le x) \cap (\gamma_2 \le x)]$

γ_1 and γ_2 are independent $\Longrightarrow F_{\gamma_b^s}(x) = (1 - e^{-\frac{x}{\bar{\gamma}_1}})(1 - e^{-\frac{x}{\bar{\gamma}_2}})$

$\bar{\gamma}_t = \frac{\bar{\gamma}_1 + \bar{\gamma}_2}{2}$, $\xi = \frac{\bar{\gamma}_1}{\bar{\gamma}_2}$ $\Longrightarrow$ $\bar{\gamma}_1 = \frac{2\xi}{1+\xi}\bar{\gamma}_t$ and $\bar{\gamma}_2 = \frac{2}{1+\xi}\bar{\gamma}_t$

thus $F_{\gamma_b^s}(x) = \left\{1 - e^{-\frac{1+\xi}{2\xi\bar{\gamma}_t}x}\right\} \cdot \left\{1 - e^{-\frac{1+\xi}{2\bar{\gamma}_t}x}\right\}$

A plot of $F_{\gamma_b^s}(x)$ against $10\log_{10}(\frac{x}{\bar{\gamma}_t})$ is shown below; the parameter R is the symbol ξ.

Note that when $F_{\gamma_b^s}(x) = 10^{-4}$, then $10\log_{10}(\frac{x}{\bar{\gamma}_t}) \approx -37$ dB. In Figure 5.14, $F_{\gamma_b^s}(x) = 10^{-4}$ at -40 dB. The factor of 3dB arises because we are plotting $\bar{\gamma}_t/2$.

Finally, note that when $\xi = 0.5$, we still get most of the diversity gain.