ECE 6604 Assignment #4 Solutions

1

1 2.52 a) We are given that $T = 0.1$ s, $v = 30$ km/h and $\sigma_\Omega = 8$ dB. In order to have a shadow decorrelation/correlation of 0.1 at a distance of 30 m, we need to have ζ_D=0.1 for $D = 30$ m in the autocorrelation of Ω_p, i.e.

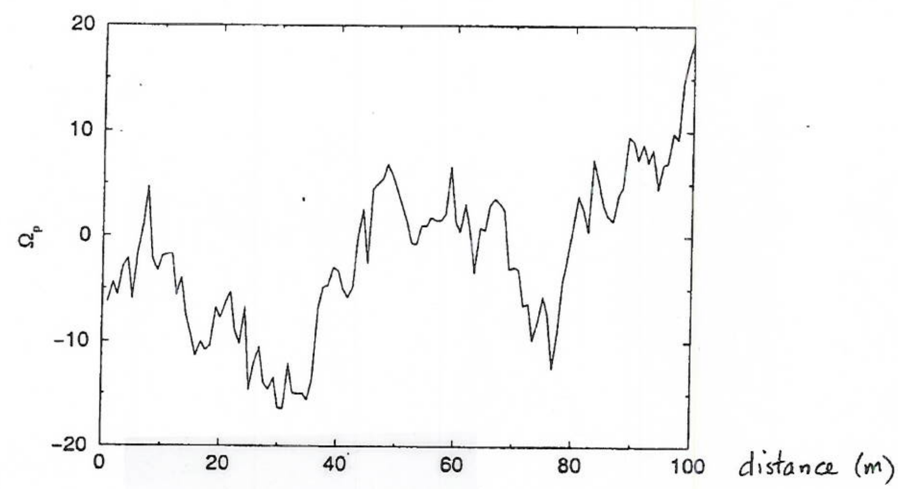
$$\begin{aligned}\phi_{\Omega_{(dB)}\Omega_{(dB)}}(n) &= \sigma_\Omega^2 \zeta_D^{(vT/D)|n|} \\ &= \sigma_\Omega^2 \zeta^{|n|}.\end{aligned}$$

When we solve the above equations for ζ, i.e.

$$\begin{aligned}\zeta &= \zeta_D^{vT/D} \\ &= 0.1^{(30/3.6)\cdot 0.1/30} = 0.938.\end{aligned}$$

b) Figure shows the variations in the local mean Ω_p due to shadowing for $\sigma_\Omega = 8$ dB over 100 m distance. Note that the variance of the process $\{v(kT)\}$ is

$$\sigma^2 = \frac{1+\zeta}{1-\zeta}\sigma_\Omega^2 = 2000.51 \tag{1}$$



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3.1

(a). $\because \mu_{\hat{Z}} = \dfrac{\sigma^2_{\hat{\Omega}} - \sigma^2_{\hat{Z}}}{2} + \ln\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)$

$\sigma^2_{\hat{Z}} = \ln\left[(e^{\sigma^2_{\hat{\Omega}}} - 1)\dfrac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)^2} + 1\right]$

$\because \mu_{\hat{\Omega}_k} = \xi \mu_{\Omega_k(dBm)} = 0.23026 \cdot \mu_{\Omega_k(dBm)} = \begin{cases} k=1, & -2.3026 \\ k=2, & -3.6539 \\ k=3, & -4.6052 \end{cases}$

$\sigma^2_{\hat{\Omega}_k} = \xi^2 \sigma^2_{\Omega_k(dB)} = \xi^2 \cdot 64 = 3.39$

$\therefore \begin{cases} \sigma^2_{\hat{Z}} = \ln\left[(e^{3.39} - 1)\dfrac{e^{2\cdot(-2.3026)} + e^{2\cdot(-3.6535)} + e^{2(-4.6052)}}{[e^{-2.3026} + e^{-3.6535} + e^{-4.6052}]^2} + 1\right] = 2.8251 \\ \mu_{\hat{Z}} = \dfrac{3.39 - 2.8251}{2} + \ln\left(e^{-2.3026} + e^{-3.6535} + e^{-4.6052}\right) = -1.6721 \end{cases}$

$\therefore \mu_{Z(dBm)} = \xi^{-1}\mu_{\hat{Z}} = -7.26\ (dBm) \qquad \sigma_Z^2 = \xi^{-2}\sigma^2_{\hat{Z}} = 53.284\ (dB)$

$\therefore Z \sim N(-7.26\ dBm,\ \sigma_Z^2 = 53.284_{(dB)})$

(b)

$\because \Lambda = \dfrac{C}{I} \cdot \quad \therefore \Lambda_{(dB)} = C_{(dB)} - I_{(dB)}$

$\therefore \mu_{\Lambda(dB)} = \mu_{C(dBm)} - \mu_{I(dBm)} = 0_{dBm} - (-7.26)_{dBm} = 7.26$

$\sigma_\Lambda^2 = \sigma_C^2 + \sigma_I^2 = 64 + 53.284 = 117.284\ dB$

$\therefore \Lambda_{(dB)} \sim N(7.26,\ \underset{(\sigma_\Lambda^2)}{117.284})$

3.2 a) $I = I_1 + I_2$

use Fenton-Wilkinson method

$I_1 \sim N(\mu_{\Omega_1}, \sigma_\Omega^2)$ $\quad \sigma_\Omega = 8$ dB

$I_2 \sim N(\mu_{\Omega_2}, \sigma_\Omega^2)$

$$\mu_{\Omega_p}(d) = -30 - 35\log_{10}(d)$$

$$\mu_{\Omega_1} = -30 - 35\log_{10}(10) = -65 \text{ dBm}$$

$$\mu_{\Omega_2} = -30 - 35\log_{10}(15) = -71.16 \text{ dBm}$$

$$\mu_{\hat{\Omega}_k} = \xi \mu_{\Omega_k} \quad , \quad \xi = \frac{\ln 10}{10} = .23026$$

$$\mu_{\hat{\Omega}_1} = (.23026)(-65) = -14.9668$$

$$\mu_{\hat{\Omega}_2} = (.23026)(-71.16) = -16.3852$$

$$\sigma_{\hat{\Omega}}^2 = \xi^2 \sigma_\Omega^2 = (.23026)^2(64) = 3.3932$$

$$\sigma_{\hat{Z}}^2 = \ln\left((e^{\sigma_{\hat{\Omega}}^2} - 1) \frac{e^{2\mu_{\hat{\Omega}_1}} + e^{2\mu_{\hat{\Omega}_2}}}{(e^{\mu_{\hat{\Omega}_1}} + e^{\mu_{\hat{\Omega}_2}})^2} + 1 \right)$$

$$= \ln\left((e^{3.3932} - 1) \frac{e^{-29.9336} + e^{-32.7704}}{(e^{-14.9668} + e^{-16.3852})^2} + 1 \right)$$

$$= \ln\left(28.7610 \times 0.686157 + 1 \right)$$

$$= \ln(20.7346) = 3.0318$$

$$\sigma_Z^2 = \sigma_{\hat{Z}}^2 / \xi^2 = 57.1833 \quad , \quad \sigma_Z = 7.5620$$

$$\mu_{\hat{Z}} = \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln\left(e^{\mu_{\hat{\Omega}_1}} + e^{\mu_{\hat{\Omega}_2}}\right)$$

$$= \frac{3.3932 - 3.0318}{2} + \ln\left(e^{-14.9668} + e^{-16.3852}\right)$$

$$= 0.1807 \qquad -14.7500$$

$$= -14.5693$$

$$\mu_Z = \mu_{\hat{Z}}/\xi = -63.2736$$

$$I_{dBm} \sim N(-63.2736,\ 57.1833)$$

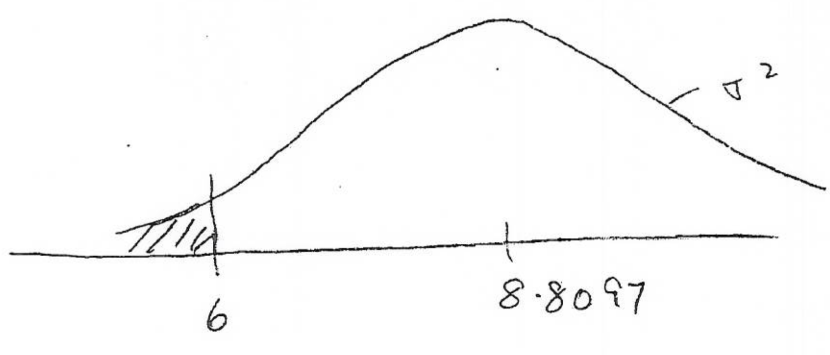
b) $$\Lambda_{dB} = C_{(dBm)} - I_{(dBm)}$$

$$\mu_{\Omega_d} = -30 - 35\log_{10}(5) = -54.4640$$

$$C_{(dBm)} \sim N(-54.4640,\ 64)$$

$$\therefore\ \Lambda_{dB} \sim N(8.8097,\ \underbrace{121.1833}_{=\sigma^2})$$

c)



$$P_0 = Q\left(\frac{8.8097 - 6}{11.0083}\right)$$

$$= Q(.2552)$$

$$= 0.400$$

4/ 3.4. From the Fenton-Wilkinson method,

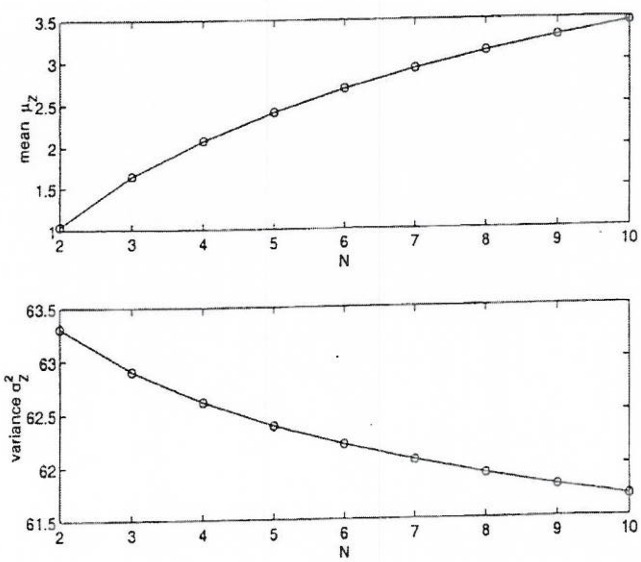
$$\begin{aligned}
\mu_L &= \mathrm{E}[L] = \sum_{k=1}^{N} \mathrm{E}[e^{\hat{\Omega}_k}] \\
\sigma_L^2 &= \mathrm{E}[L^2] - \mu_L^2.
\end{aligned}$$

Since $e^{\hat{Z}} = \sum_{k=0}^{N} e^{\hat{\Omega}_k} = L$, and $\hat{\Omega}_k$ are independent zero-mean Gaussian random variables with $\sigma_{\hat{\Omega}} = 8\mathrm{dB}$, i.e., $\hat{\Omega}_k \sim \mathrm{N}(0, 64)$, we have,

$$\begin{aligned}
\mu_L &= \Big(\sum_{k=1}^{N} e^{\mu_{\hat{\Omega}_k}}\Big) e^{\frac{1}{2}\sigma_{\hat{\Omega}}^2} = e^{\mu_{\hat{Z}} + \frac{1}{2}\sigma_{\hat{Z}}^2} \\
\sigma_L^2 &= \Big(\sum_{k=1}^{N} e^{2\mu_{\hat{\Omega}_k}}\Big) e^{\sigma_{\hat{\Omega}}^2}(e^{\sigma_{\hat{\Omega}}^2} - 1) = e^{2\mu_{\hat{Z}}} e^{\sigma_{\hat{Z}}^2}(e^{\sigma_{\hat{Z}}^2} - 1)
\end{aligned}$$

Then,

$$\begin{aligned}
\sigma_{\hat{Z}}^2 &= \ln\Big((e^{\sigma_{\hat{\Omega}}^2} - 1)\frac{\sum_{k=1}^{N} e^{2\mu_{\hat{\Omega}_k}}}{(\sum_{k=1}^{N} e^{\mu_{\hat{\Omega}_k}})^2} + 1\Big) = \ln(1 + (e^{64} - 1)\frac{N}{N^2}) = \ln(1 + (e^{64} - 1)/N) \\
\mu_{\hat{Z}} &= \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln(\sum_{k=1}^{N} e^{\mu_{\hat{\Omega}_k}}) = \frac{64 - \sigma_{\hat{Z}}^2}{2} + \ln N
\end{aligned}$$

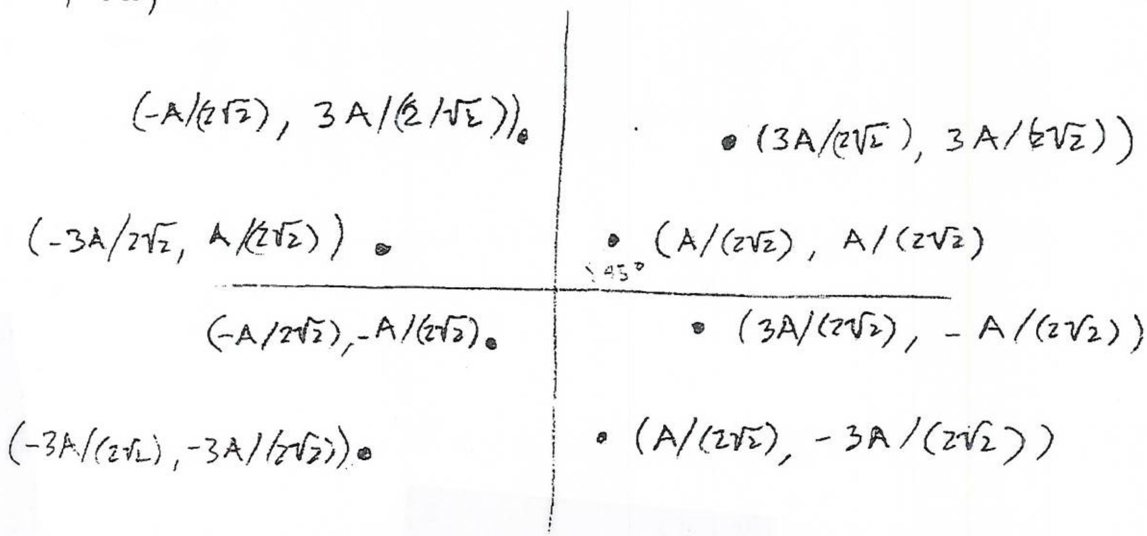


Mean $\mu_{\hat{Z}}$ and variance $\sigma_{\hat{Z}}^2$ as a function of N.

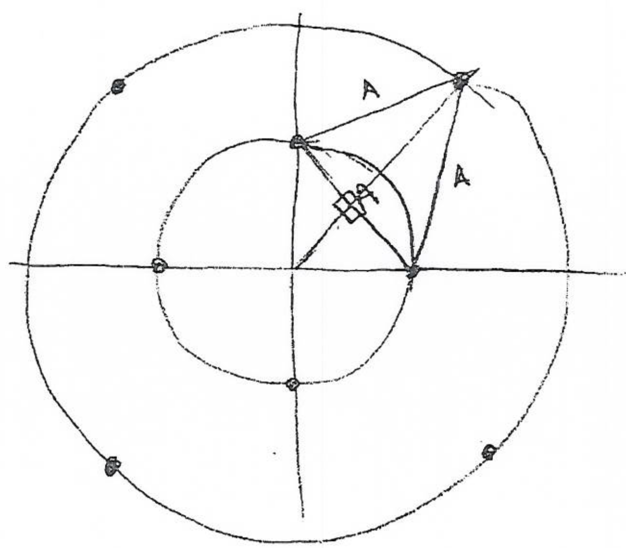
5/

5

4.3 a)



b)



$$r_i = A/\sqrt{2}$$

$$r_o = \frac{\sqrt{3}}{2}A + \frac{A}{2} = \frac{(1+\sqrt{3})A}{2}$$

c) For constellation a)

$$E_{av} = \frac{1}{4}\cdot\frac{1}{2}\left(\frac{A^2}{8}+\frac{A^2}{8}\right) + \frac{1}{4}\cdot\frac{1}{2}\left(\frac{9A^2}{8}+\frac{9A^2}{8}\right) + \frac{1}{2}\cdot\frac{1}{2}\left(\frac{9A^2}{8}+\frac{A^2}{8}\right)$$

$$= \frac{1}{8}\left(\frac{2A^2}{8}+\frac{18A^2}{8}+\frac{20A^2}{8}\right)$$

$$= \frac{1}{8}\,\frac{40A^2}{8} = \frac{5A^2}{8} = 0.625A^2$$

For constellation b)

$$E_{av} = \frac{1}{2}\cdot\frac{1}{2}\frac{A^2}{2} + \frac{1}{2}\cdot\frac{1}{2}\left(\frac{4+2\sqrt{3}}{4}\right)A^2$$

$$= \frac{A^2}{8} + \frac{(4+2\sqrt{3})A^2}{16}$$

$$= \frac{A^2}{8} + \frac{2+\sqrt{3}}{8}A^2 = \frac{3+\sqrt{3}}{8}A^2$$

$$= .591506\,A^2$$

Constellation b) is more energy efficient.

8/

6/ 4.5 You need to generate a long sample function of the complex envelope. Then compute PAPR numerically. The result is shown below