Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2017

Thursday December 14, 8:00am - 10:50am

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Consider a linear time-invariant channel having the impulse response

$$g(t, \tau) = \delta(\tau) + 2\delta(\tau - \tau_1) + \delta(\tau - 2\tau_1) \ .$$

a) 4 marks: Derive a closed-form expression for magnitude response of the channel $|T(f, t)|$ and sketch showing all important points.

b) 2 marks: Repeat part a) for the phase response of the channel $\angle T(f, t)$.

c) 4 marks: What is the mean delay and rms delay spread of the channel.

Extra sheet

Extra sheet

2) The power delay profile for a WSSUS channel is given by

$$\psi_g(\tau) = Ae^{-a\tau}u(\tau) + \frac{A}{2}e^{-a(\tau-\tau_d)}u(\tau-\tau_d) \ , \quad 0 \leq \tau \leq \infty$$

a) 3 marks: Find the channel frequency correlation function.

b) 2 marks: Find the total envelope power Ω_p.

c) 2 marks: Find the mean delay μ_τ.

d) 3 marks: Find the rms delay spread σ_τ.

Extra sheet

Extra sheet

3) Consider a cellular system that uses a 3-cell hexagonal reuse cluster. The base stations employ 120° wide-beam directional antennas and they all have the same antenna height and transmit with the same power level. The cell radii are assumed to be 5 km.

The propagation path loss follows the model

$$\mu_{\Omega_p\ (\mathrm{dBm})}(d) = \mu_{\Omega_p\ (\mathrm{dBm})}(d_o) - 10\beta \log_{10}(d/d_o) \quad (\mathrm{dBm})$$

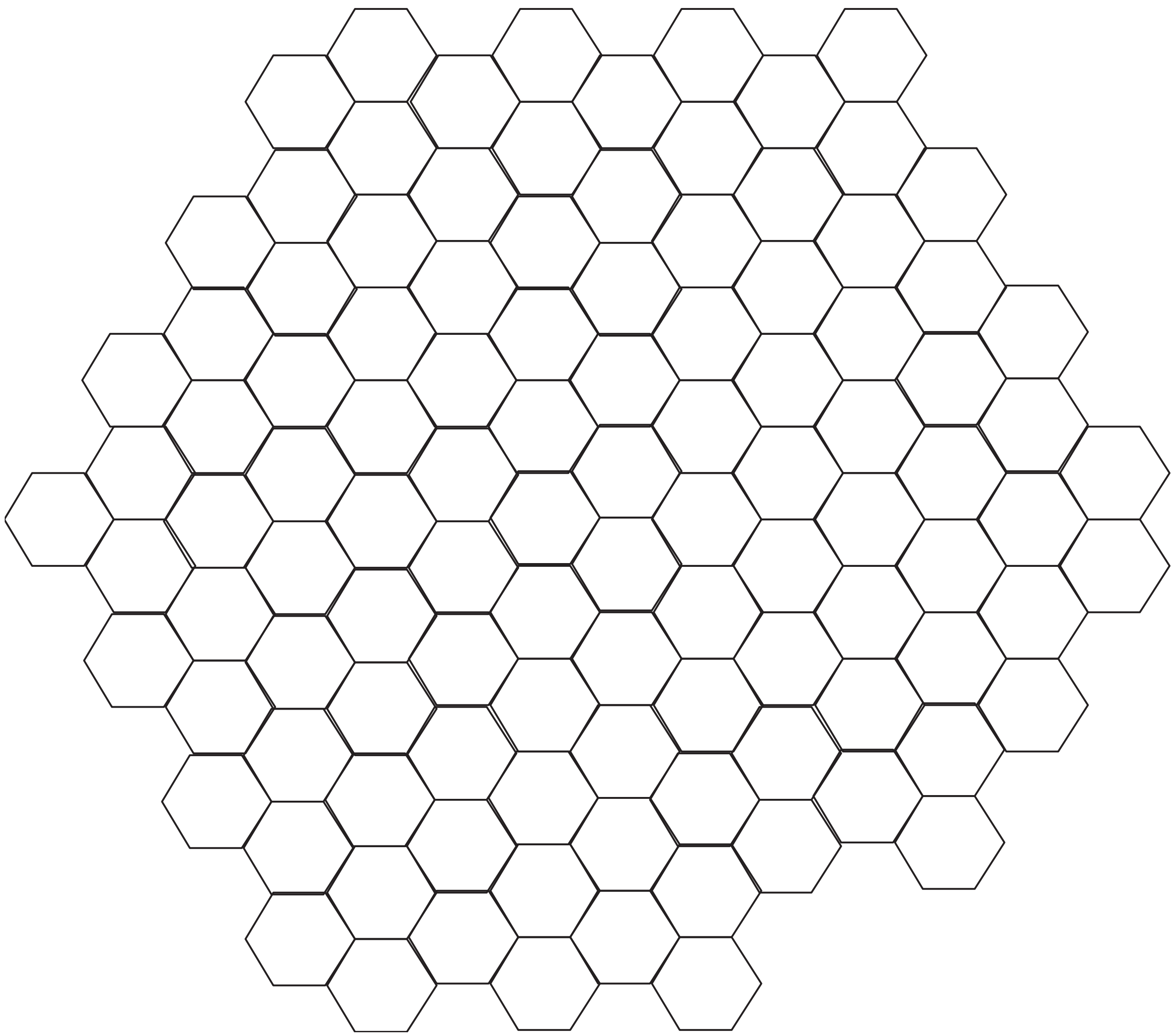
where $\beta = 3.5$, and $\mu_{\Omega_p}(d_o) = 1$ microwatt at $d_o = 1$ km. Assume that each link experiences independent log-normal shadowing with a shadow standard deviation $\sigma_\Omega = 6$ dB.

Consider the forward (base-to-mobile) channel.

a) 2 marks: Using the attached hex paper depict the worst-case co-channel interference situation on the forward channel, accounting only for the first tier of co-channel interferers.

b) 6 marks: Using the Fenton-Wilkinson method, determine the probability density function of the total interfering power in decibel units, $I_{(\mathrm{dBm})}$, again accounting only for the first tier of co-channel interferers.

c) 2 marks: Using the result in part b), determine the probability density function of the carrier-to-interference ratio in decibel units, $(C/I)_{(\mathrm{dB})}$.



Extra sheet

Extra sheet

4) Consider coherent BPSK signaling on an AWGN channel where the channel gain, α, has the following probability density function

$$p_\alpha(x) = 0.2\delta(x) + 0.5\delta(x-1) + 0.3\delta(x-2) \ .$$

Suppose that two-branch antenna diversity is used with *selective combining.* Assume that the diversity branches experience independent fading.

a) 5 marks: Derive an expression for the probability density function of the bit-energy-to-noise ratio at the output of the selective combiner, γ_s^{b}.

b) 2 marks: Express the probability density function obtained in part a) as a function of the average received bit energy-to-noise ratio per diversity branch, $\bar{\gamma}_c$.

c) 2 marks: Derive the probability of bit error a function of the average received bit energy-to-noise ratio per diversity branch $\bar{\gamma}_c$.

d) 1 marks: What value does the probability of bit error approach as $\bar{\gamma}_c$ tends to infinity?

Extra sheet

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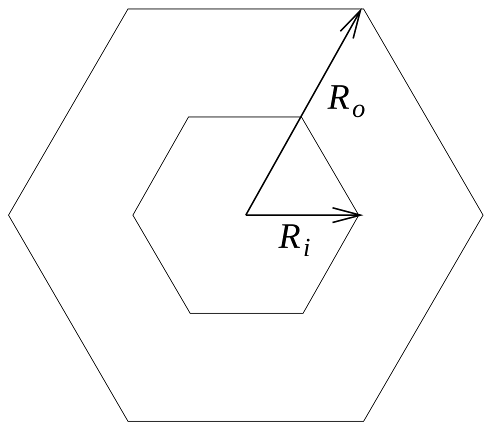
5) One method for improving the capacity of a cellular system is to employ a *two-channel bandwidth* scheme, where each hexagonal cell is divided into two concentric hexagons as shown below. The inner hexagon is serviced by half-rate channels, while the outer hexagon is serviced by full-rate channels. When a mobile station crosses the boundary between the inner and outer portions of a cell a handoff occurs.



It is known that the full-rate channels require $C/I = 7$ dB to maintain an acceptable radio link quality, while the half-rate channels require $C/I = 10$ dB.

Assume a $\beta = 4$ path loss exponent and suppose that the effects of envelope fading and shadowing can be ignored. Consider the reverse link and suppose that there are 6 co-channel interferers at distance D from the serving base station. It follows that the received C/I when a mobile station is located at distance d from its serving base station is $C/I = (D/d)^4/6$.

a) 2 marks: Find the required value of D/R_o so that the worst case $C/I =$ 7 dB in the outer cell, where R_o is the radii of the outer cell.

b) 2 marks: Find the required value of D/R_i so that the worst case $C/I =$ 10 dB in the inner cell, where R_i is the radii of the inner cell.

c) 3 marks: Use the values of D/R_i and D/R_o to determine the ratio of the inner and outer cell areas, A_i/A_o. Use the exact area of a hexagon in terms of its radius.

d) 3 marks: Let N_i and N_o be the number of channels that are allocated to the inner and outer areas of each cell, and assume that the channels are assigned such that $N_i/N_o = A_i/A_o$. Determine the increase in cell capacity (as measured in channels per cell) over a conventional *one-channel bandwidth* system that uses only full-rate channels.

Extra sheet

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TABLE A6.3 *Fourier-transform pairs*

Time Function	*Fourier Transform*
$\text{rect}\left(\frac{t}{T}\right)$	$T\,\text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
$\delta(t)$ = delta function, or unit impulse
$\text{rect}(t)$ = rectangular function of unit amplitude and unit duration centered on the origin
$\text{sgn}(t)$ = signum function
$\text{sinc}(t)$ = sinc function

TABLE A6.2 *Summary of properties of the Fourier transform*

Property	*Mathematical Description*
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$

Formulas for Use in Trigonometry

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n - 1)\alpha\cos\alpha - \sin(n - 2)\alpha$$
$$\cos n\alpha = 2\cos(n - 1)\alpha\cos\alpha - \cos(n - 2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n - 1)\alpha + \tan\alpha}{1 - \tan(n - 1)\alpha\tan\alpha}$$

Formulas for Use in Trigonometry

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\frac{1}{2}(\alpha + \beta)}{\tan\frac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

Power relations

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$