Solutions

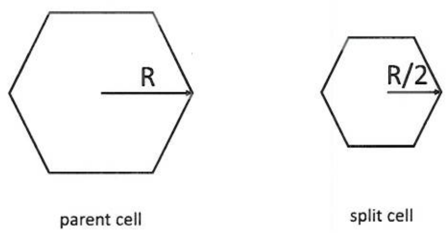
Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Mid Term Test
Fall 2019

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Consider a frequency plan consisting of uniform hexagonal cells of radius R. To accommodate traffic growth, the cells in the frequency plan are all split (made smaller), such that the radii of the split cells are all $R/2$ as shown below



Assume the flat Earth path loss model

$$\mu_{\Omega_p}(d) = k\Omega_t \left(\frac{h_b h_m}{d^2}\right)^2$$

a) **5 marks:** By what factor must the base stations reduce their transmit power, Ω_t, so that the power received by a mobile station located in the corner of a split cell is the same as the power received by a mobile station located in the corner of a parent cell? Express your answer in dB units.

b) **5 marks:** As an alternative to reducing the transmit power, the height of the base station antennas, h_b may be reduced. By what factor must base station antenna heights of the split cells be reduced, so that the power received by a mobile station located in the corner of a split cell is the same as the power received by a mobile station located in the corner of a parent cell?

a) $\Omega_{tp}\left(\frac{h_t h_m}{d_p^2}\right)^2 = \Omega_{ts}\left(\frac{h_t h_m}{d_s^2}\right)^2$

$\Rightarrow \frac{\Omega_{tp}}{\Omega_{ts}} = \left(\frac{d_p}{d_s}\right)^4 = 2^4 = 16$

$= \underline{\underline{12\,dB}}$

b) $\Omega_t\left(\frac{h_{tp} h_m}{d_p^2}\right)^2 = \Omega_t\left(\frac{h_{ts} h_m}{d_s^2}\right)^2$

$\Rightarrow \left(\frac{h_{ts}}{h_{tp}}\right)^2 = \left(\frac{d_s}{d_p}\right)^4$

$\Rightarrow \frac{h_{ts}}{h_{tp}} = \left(\frac{d_s}{d_p}\right)^2 = \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{4}}}$

2) Consider a narrow-band channel with a 700 MHz carrier frequency. The observed Doppler spectrum is such that

$$S_{gg}(f) = \begin{cases} \mathrm{rect}\left(\frac{f}{200}\right), & |f| \leq 100\ Hz \\ 0, & \text{elsewhere} \end{cases}$$

a) 4 marks: What is the maximum possible speed of the mobile station?

b) 3 marks: What is the cross-correlation function $\phi_{g_I g_Q}(\tau)$ of the I and Q components of the faded envelope?

c) 3 marks: The faded envelope $g(t)$ is sampled every T_s seconds. What is the smallest T_s that will yield samples that are uncorrelated?

a) $f_d = f_m \cos\theta = \frac{v}{\lambda_0}\cos\theta = \frac{v f_c}{c}\cos\theta$

$\Rightarrow v = \frac{c f_d}{f_c \cos\theta}$

where θ is the angle of arrival corresponding to the maximum Doppler frequency.

The minimum possible velocity v occurs when $\theta = 0$

$$V_{min} = \frac{c f_d}{f_c}$$

$= 3 \times 10^8 \times 100 / 700 \times 10^6$

$= \underline{\underline{42.86\ m/s}}$

$= \frac{42.86 \times 3600}{1000} = \underline{\underline{154.29\ Km/h}}$

b) $\phi_{gg}(\tau) = 200\,\mathrm{sinc}(200\tau)$ $\underline{\text{from tables}}$

$= \phi_{g_I g_I}(\tau) + j\phi_{g_I g_Q}(\tau)$

$\Rightarrow \phi_{g_I g_Q}(\tau) = 0$

Extra sheet

c) $\mu_{gg}(\tau) = \phi_{gg}(\tau) = 0$

when

$$200k\tau = k\pi \quad , \quad k \text{ integer}, \; k \neq 0$$

$$T_{samp} = \frac{k}{200}$$

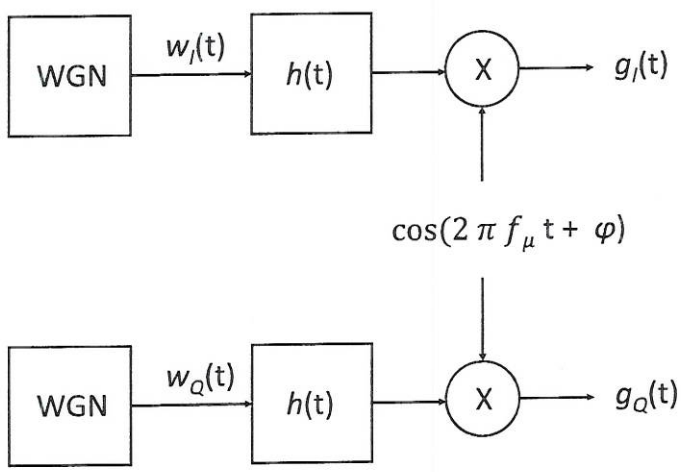
$$\text{Minimum } T_{samp} = \frac{1}{200} \text{ seconds.}$$

$$= 0.05 \text{ seconds.}$$

3) Your friend uses the statistical fading simulator below, where independent white Gaussian noise waveforms $w_I(t)$ and $w_Q(t)$ each having a flat power density spectrum of 2 milliwatts/Hz are input to a pair of filters each having impulse response $h(t)$, followed by multiplication by a cosine waveform having frequency f_μ and a random phase π that is chosen independently and uniformly over the interval $[-\pi, \pi)$ for each simulation run.

The Doppler power spectrum of the faded envelope $g(t) = g_I(t) + jg_Q(t)$ is

$$S_{gg}(f) = \frac{1}{16 + (2\pi(f+100))^2} + \frac{1}{16 + (2\pi(f-100))^2} \quad \text{milliwatts/Hz}$$



a) 4 marks: What is the channel time autocorrelation function $\phi_{gg}(\tau)$?

b) 4 marks: What is the filter impulse response $h(t)$?

c) 1 marks: What is the value of f_μ?

d) 1 marks: Why is the random phase ϕ necessary?

a) From tables $\quad e^{-\alpha|t|} \longleftarrow \dfrac{2a}{a^2 + (2\pi f)^2}$

$$X(t)\cos(2\pi f_c t) \longleftrightarrow \frac{1}{2}\left(X(f-f_c) + X(f+f_c)\right)$$

$$\therefore \quad \phi_{gg}(\tau) = \frac{1}{4} e^{-4|\tau|} \cos(200\pi\tau)$$

Extra sheet

b)

$$\frac{2}{16 + (2\pi f)^2} = |H(f)|^2 \times 2$$

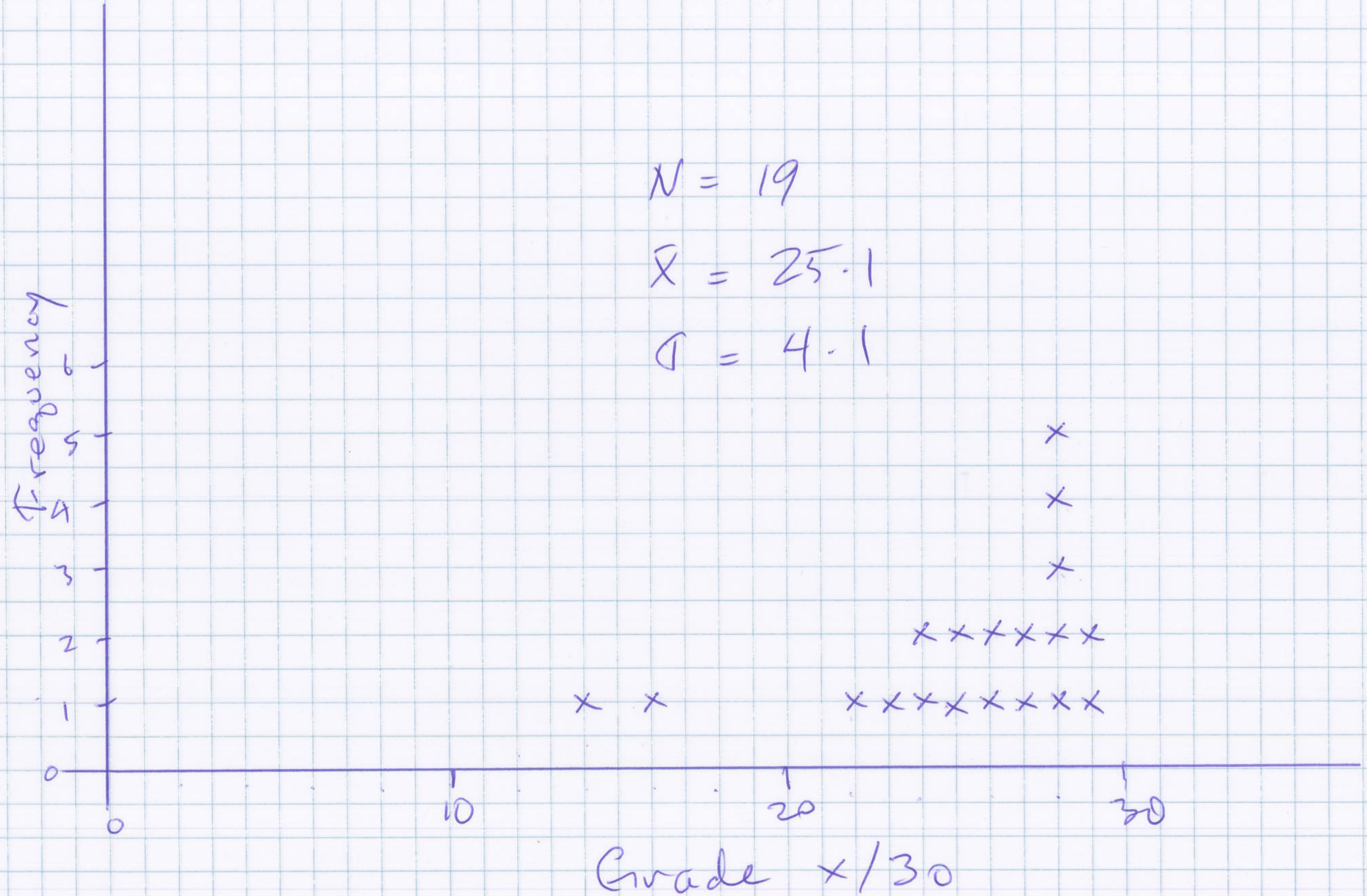
$$\Longrightarrow |H(f)|^2 = \frac{1}{16 + (2\pi f)^2}$$

$$\Longrightarrow H(f) = \frac{1}{4 + j2\pi f}$$

$$\Longrightarrow \underline{\underline{h(t) = e^{-4t}u(t)}}$$

c) $f_u = 100$ Hz

d) ϕ is necessary so that the faded envelope $g(t) = g_I(t) + j g_Q(t)$ is wide-sense stationary.


$N = 19$
$\bar{X} = 25.1$
$\sigma = 4.1$
Frequency
0
1
2
3
4
5
6
0
10
20
30
Grade x/30