EE6604

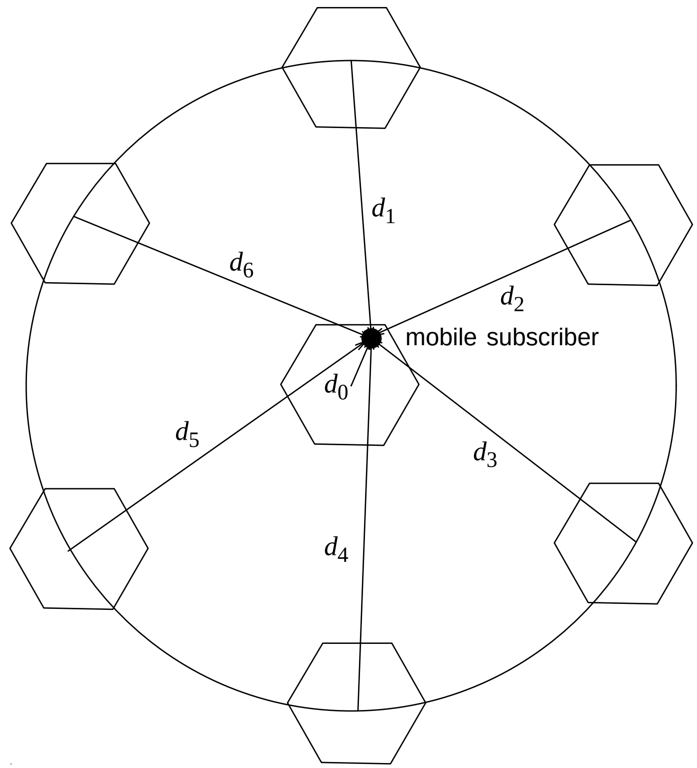
Personal & Mobile Communications

Week 8

Co-Channel Interference

Reading: Chapter 3

Co-channel interference on the forward channel



The mobile station is being served by the center base station.

- At a particular location, let $\mathbf{d} = (d_0,\ d_1,\ \cdots,\ d_{N_I})$ be the vector of distances between a mobile station and the serving base station BS_0 and N_I co-channel base stations BS_k, $k = 1, \ldots, N_I$.
- The received signal power power at distance d, $\Omega_{p\ (\text{dBm})}(d)$, is a Gaussian random variable that depends on the distance d through the path loss model, i.e.,

$$\mu_{\Omega_{p\ (\text{dBm})}}(d) = \text{E}[\Omega_{p\ (\text{dBm})}(d)] = \mu_{\Omega_{p\ (\text{dBm})}}(d_o) - 10\beta \log_{10}(d/d_o)$$

- Experiments have verified that co-channel interferers add **noncoherently** (power addition) rather than **coherently** (amplitude addition).
- The C/I a function of the vector $\mathbf{d}$ is

$$\Lambda(\mathbf{d}) = \frac{\Omega_p(d_0)}{\sum_{k=1}^{N_I} \Omega_p(d_k)}$$

or in decibel units

$$\Lambda(\mathbf{d})_{(\text{dB})} = \Omega_{p\ (\text{dBm})}(d_0) - 10 \log_{10} \left(\sum_{k=1}^{N_I} \Omega_p(d_k) \right)$$

- The outage probability given vector $\mathbf{d}$ is

$$O(\mathbf{d}) = \text{P}_\text{r} \left(\Lambda(\mathbf{d})_{(\text{dB})} < \Lambda_{\text{th}(\text{dB})} \right)$$

- Although the $\Omega_p(d_k)$ are log-normal random variables, the sum $\sum_{k=1}^{N_I} \Omega_p(d_k)$ is *not* a log-normal random variable.

Multiple Log-normal Interferers

- Consider the sum of N_I log-normal random variables

$$I = \sum_{k=1}^{N_I} \Omega_k = \sum_{k=1}^{N_I} 10^{\Omega_{k(\mathrm{dBm})}/10}$$

  where the $\Omega_{k\ (\mathrm{dBm})}$ are independent Gaussian random variables with mean $\mu_{\Omega_k\ (\mathrm{dBm})}$ and variance $\sigma^2_{\Omega_k}$.

- The sum I is commonly approximated by another log-normal random variable with appropriately chosen parameters, i.e.,

$$I = \sum_{k=1}^{N_I} 10^{\Omega_{k(\mathrm{dBm})}/10} \approx 10^{Z_{(\mathrm{dBm})}/10} = \hat{I}$$

  where $Z_{(\mathrm{dBm})}$ is a Gaussian random variable with mean $\mu_{Z\ (\mathrm{dBm})}$ and variance σ^2_Z.

- The task is to find $\mu_{Z\ (\mathrm{dBm})}$ and σ^2_Z.

Fenton-Wilkinson Method

- The mean $\mu_{Z\ (\mathrm{dBm})}$ and variance σ_Z^2 of $Z_{(\mathrm{dBm})}$ are obtained by matching the first two moments of I and $\hat{I}$.
- Switching from base 10 to base e:

$$\Omega_k = 10^{\Omega_{k\ (\mathrm{dBm})}/10} = e^{\xi \Omega_{k\ (\mathrm{dBm})}} = e^{\hat{\Omega}_k}$$

  where $\hat{\Omega}_k = \xi \Omega_{k\ (\mathrm{dBm})}$ and $\xi = (\ln 10)/10 = 0.23026$.
- Note that

$$\begin{aligned} \mu_{\hat{\Omega}_k} &= \xi \mu_{\Omega_{k\ (\mathrm{dBm})}} \\ \sigma^2_{\hat{\Omega}_k} &= \xi^2 \sigma^2_{\Omega_k} \end{aligned}$$

- The nth moment of the log-normal random variable Ω_k can be obtained from the moment generating function of the Gaussian random variable $\hat{\Omega}_k$ as

$$\mathrm{E}[\Omega_k^n] = \mathrm{E}[e^{n\hat{\Omega}_k}] = e^{n\mu_{\hat{\Omega}_k} + (1/2)n^2\sigma^2_{\hat{\Omega}}}$$

  – Here we have assumed identical shadow variances, $\sigma^2_{\hat{\Omega}_k} = \sigma^2_{\hat{\Omega}}$, which is a reasonable assumption.

- Suppose that $\hat{\Omega}_1, \ \ldots, \ \hat{\Omega}_{N_I}$ are independent with means $\mu_{\hat{\Omega}_1}, \ \ldots, \ \mu_{\hat{\Omega}_{N_I}}$ and identical variances $\sigma^2_{\hat{\Omega}}$.
- The appropriate moments of the log-normal approximation are obtained by equating the means on both sides of

$$\mu_I = \mathrm{E}[I] = \sum_{k=1}^{N_I} \mathrm{E}[e^{\hat{\Omega}_k}] \approx \mathrm{E}[e^{\hat{Z}}] = \mathrm{E}[\hat{I}] = \mu_{\hat{I}}$$

where $\hat{Z} = \xi Z_{(\mathrm{dBm})}$.

- This gives

$$\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right) e^{(1/2)\sigma^2_{\hat{\Omega}}} = e^{\mu_{\hat{Z}} + (1/2)\sigma^2_{\hat{Z}}} \tag{1}$$

- Also equate the variances on both sides of

$$\sigma^2_I = \mathrm{E}[I^2] - \mu^2_I \approx \mathrm{E}[\hat{I}^2] - \mu^2_{\hat{I}} = \sigma^2_{\hat{I}}$$

- This gives

$$\left(\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}} \right) e^{\sigma^2_{\hat{\Omega}}} (e^{\sigma^2_{\hat{\Omega}}} - 1) = e^{2\mu_{\hat{Z}}} e^{\sigma^2_{\hat{Z}}} (e^{\sigma^2_{\hat{Z}}} - 1) \tag{2}$$

- To obtain $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$
  1. Square Eq. (1) and divide by Eq. (2) to obtain $\sigma^2_{\hat{Z}}$.
  2. Obtain $\mu_{\hat{Z}}$ from Eq. (1)
- The above procedure yields

$$\begin{aligned}\sigma^2_{\hat{Z}} &= \ln\left((e^{\sigma^2_{\hat{\Omega}}} - 1)\frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)^2} + 1\right) \\ \mu_{\hat{Z}} &= \frac{\sigma^2_{\hat{\Omega}} - \sigma^2_{\hat{Z}}}{2} + \ln\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)\end{aligned}$$

- Given the means $\mu_{\hat{\Omega}_1}, \ \ldots, \ \mu_{\hat{\Omega}_{N_I}}$ and variance $\sigma^2_{\hat{\Omega}}$, $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$ are easily obtained.
- Finally, we convert back to base 10 by scaling, such that

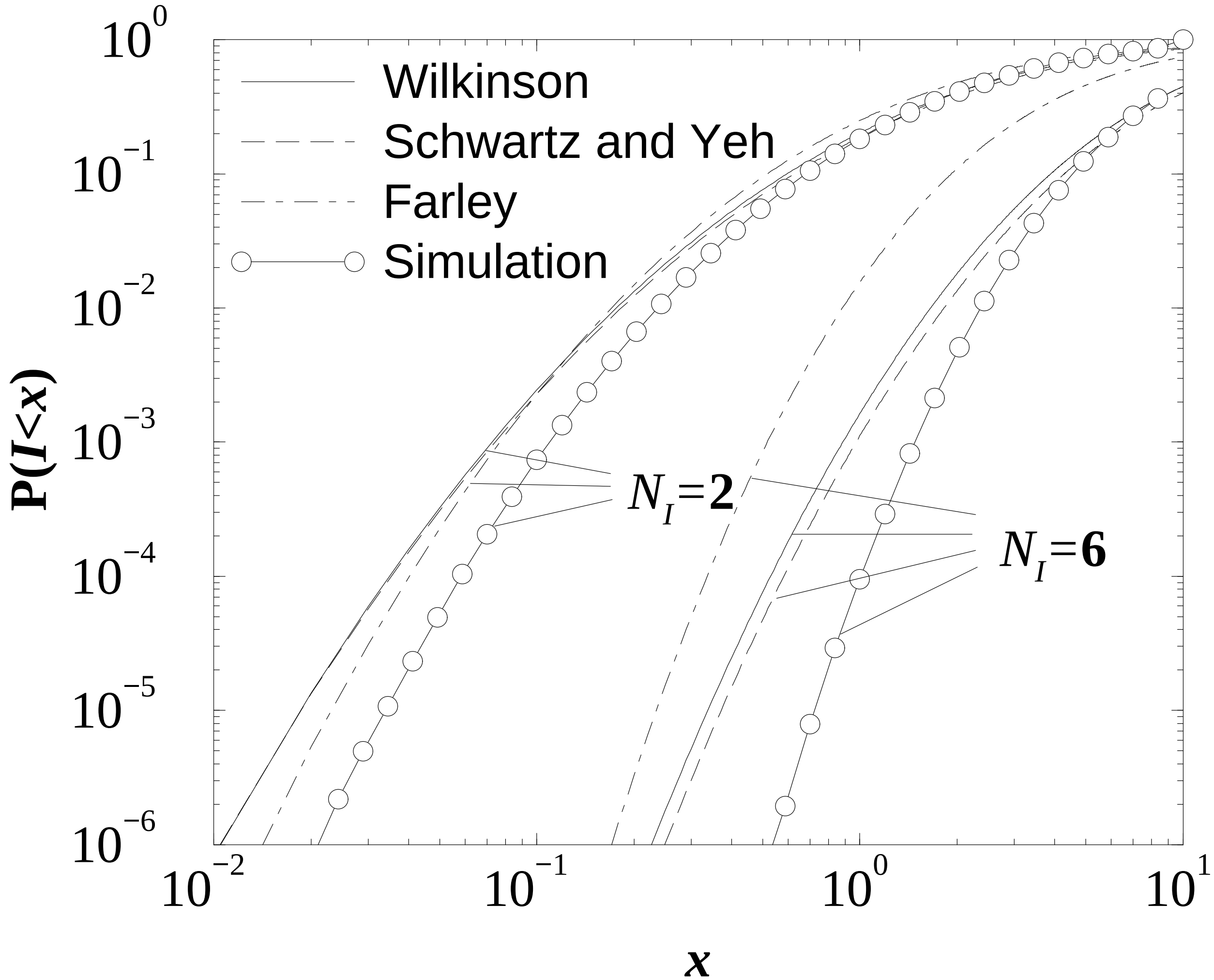
$$\begin{aligned}\mu_{Z \text{ (dBm)}} &= \xi^{-1}\mu_{\hat{Z}} \\ \sigma^2_Z &= \xi^{-2}\sigma^2_{\hat{Z}}\end{aligned}$$

where $\xi = 0.23026$.

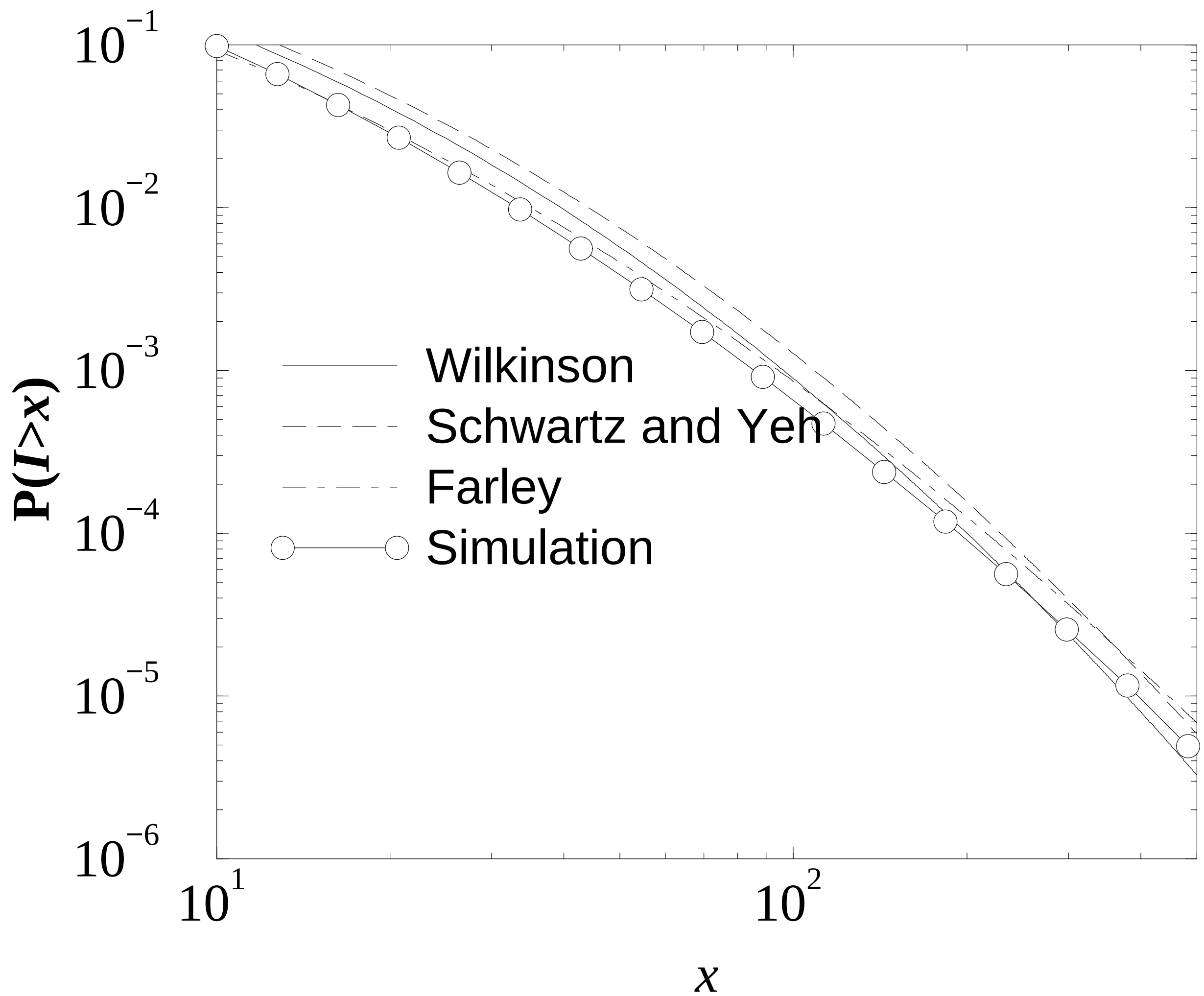
- Fenton's method breaks down in the prediction of the first and second moments for $\sigma_\Omega > 4\,\text{dB}$.
  - Schwartz and Yeh's method yields the exact first and second moments.
- However, Fenton's method accurately predicts the tails of the complementary distribution function *cdfc* $F_I^c(x) = \mathrm{P_r}(I \geq x)$ and the *cdf* $F_I(x) = 1 - F_I^c(x) = \mathrm{P_r}(I < x)$.
  - We are interested in the accuracy of the approximations

$$\begin{aligned} F_I^c(x) &\approx Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right) \\ F_I(x) &\approx 1 - Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right) \end{aligned}$$

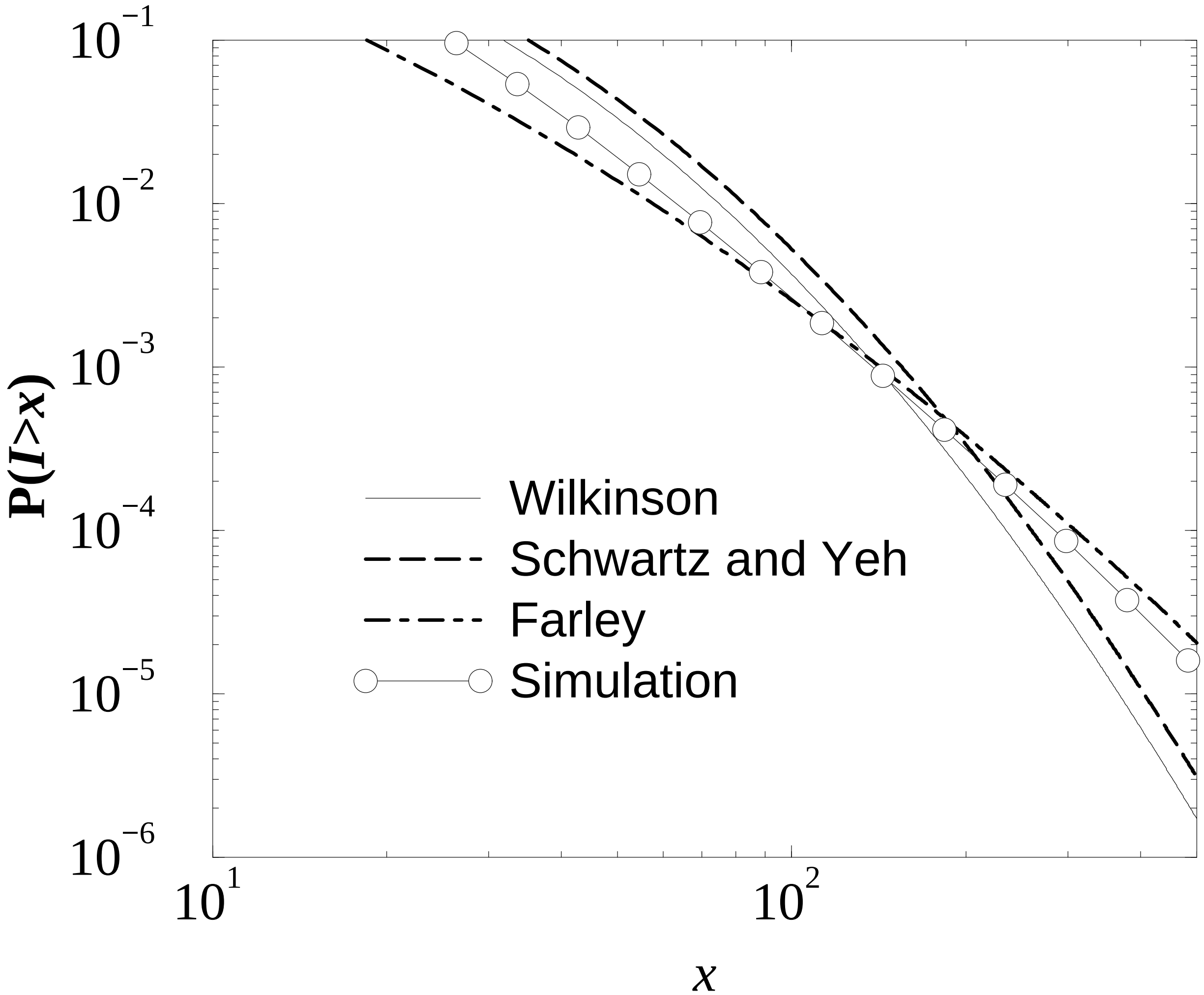
    when x is large and small, respectively.
  - The cdfc is more important than the cdf for outage calculations and predictions, since outages typically occur when the interference is large.



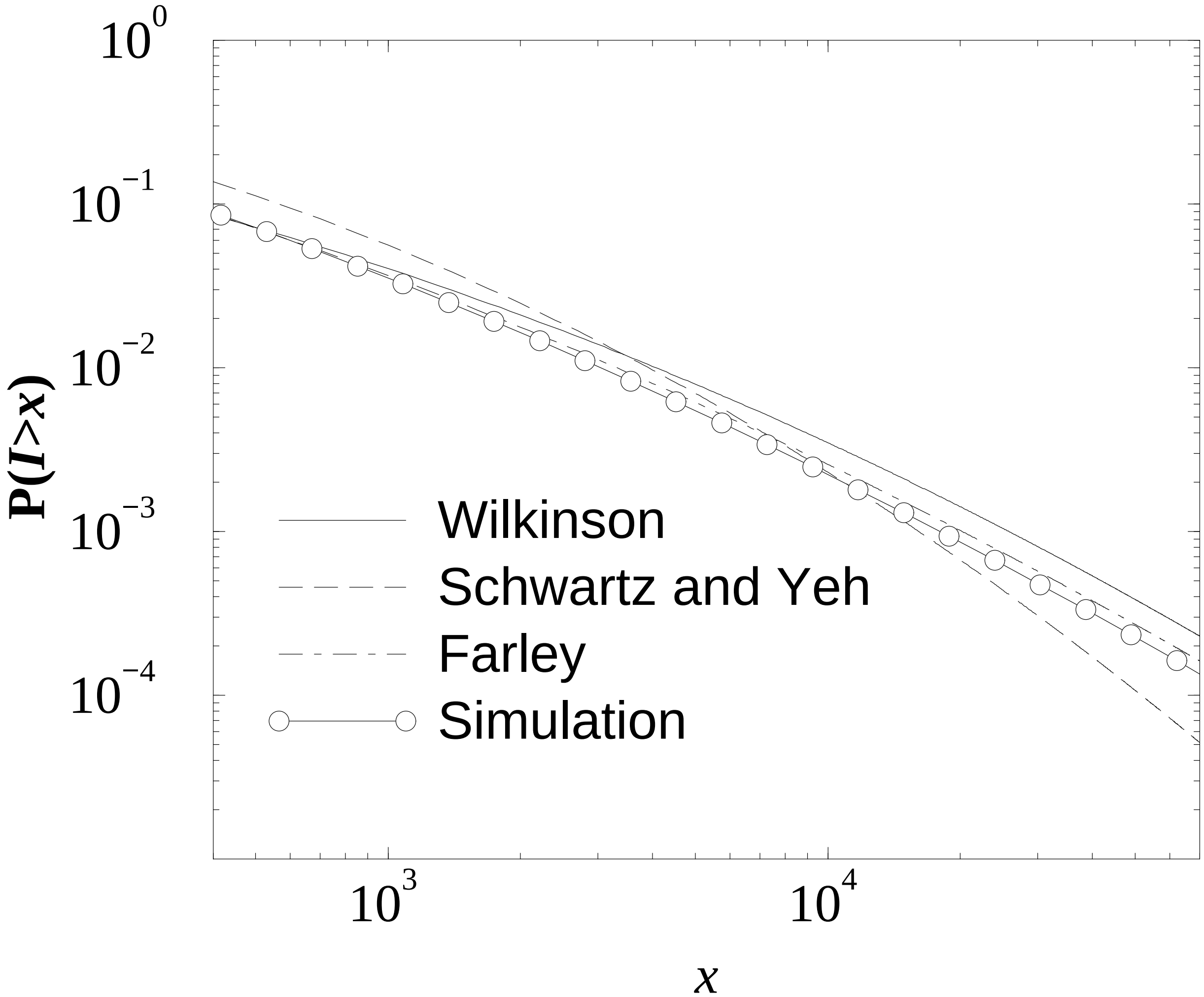
Comparison of the cdf for the sum of two and six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of two log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 12$ dB.

Outage with Multiple Interferers

1. First obtain the mean and variance

$$\begin{aligned} \mu_Z &= \mu_{\hat{Z}}/\xi \\ \sigma_Z^2 &= \sigma_{\hat{Z}}^2/\xi^2 \qquad \xi = 0.23026 \end{aligned}$$

2. Treat the average CIR as Gaussian distributed with mean and variance

$$\begin{aligned} \mu_{\Lambda(\mathbf{d})} &= \mu_{\Omega(d_0)} - \mu_Z \text{ (dBm)} \\ \sigma_{\Lambda(\mathbf{d})}^2 &= \sigma_\Omega^2 + \sigma_Z^2 \ . \end{aligned}$$

3. Compute the outage for a given location, described by **d**

$$O(\mathbf{d}) = Q\left(\frac{\mu_{\Omega(d_0)} - \mu_Z - \Lambda_{\mathrm{th(dB)}}}{\sqrt{\sigma_\Omega^2 + \sigma_Z^2}}\right)$$

4. Average over all locations **d** by Monte Carlo integration

$$O = \int_{R^N} O(\mathbf{d}) p_{\mathbf{d}}(\mathbf{d}) d\mathbf{d}$$

Single Co-channel Interferer

- For a single co-channel interferer

$$p_{\Lambda(\mathbf{d})_{(\mathrm{dB})}}(x) = \frac{1}{\sqrt{4\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}})^2}{4\sigma_\Omega^2} \right\}$$

  where

$$\mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}} = \mu_{\Omega(d_0)_{(\mathrm{dB})}} - \mu_{\Omega(d_1)_{(\mathrm{dB})}}$$

- The outage for a given $\mathbf{d}$ is

$$\begin{aligned} O(\mathbf{d}) &= \Pr(\Lambda(\mathbf{d})_{(\mathrm{dB})} < \Lambda_{\mathrm{th}(\mathrm{dB})}) \\ &= \int_{-\infty}^{\Lambda_{\mathrm{th}(\mathrm{dB})}} \frac{1}{\sqrt{4\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}})^2}{4\sigma_\Omega^2} \right\} \, dx \\ &= Q\left(\frac{\mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}} - \Lambda_{\mathrm{th}(\mathrm{dB})}}{\sqrt{2}\sigma_\Omega} \right) \end{aligned}$$

Worst case interference from a single co-channel base-station.

- In this case $\mathbf{d} = (R, D - R)$.
- The worst case outage due to a single co-channel interferer is

$$O(R) = Q\left(\frac{\mu_{\Omega(R)_{(\mathrm{dB})}} - \mu_{\Omega(D-R)_{(\mathrm{dB})}} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}}{\sqrt{2}\sigma_\Omega}\right)$$

- Using a simple inverse-β path loss characteristic

$$\mu_{\Omega_{(\mathrm{dB})}} = \Omega_{(\mathrm{dB})}(d_o) - 10\beta \log_{10}(d/d_o)$$

gives

$$O(R) = Q\left(\frac{10\log_{10}\left(\frac{D}{R}-1\right)^{\beta} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}}{\sqrt{2}\sigma_{\Omega}}\right)$$

- The minimum CIR margin on the cell fringe is

$$\mathrm{M}_{\Lambda} = 10\log_{10}\left(\frac{D}{R}-1\right)^{\beta} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}$$

- For an ideal hexagonal layout $\frac{D}{R} = \sqrt{3N}$, so that

$$N = \frac{1}{3}\left[10^{\frac{\mathrm{M}_{\Lambda}+\Lambda_{\mathrm{th}\ (\mathrm{dB})}}{10\beta}} + 1\right]^{2}$$

  - A small cluster size is achieved by making the margin M_{Λ} and receiver threshold Λ_{th} small.

Rician/Multiple Rayleigh Interferers

- Sometimes propagation conditions exist such that the received signals experience fading, but not shadowing. In this section, we calculate the outage probability for the case of fading only.
  - The received signal may consist of a direct line of sight (LoS) component, or perhaps a specular component, accompanied by a diffuse component. The envelope of the received desired signal experiences Ricean fading.
  - The interfering signals are often assumed to be Rayleigh faded, because a direct LoS is unlikely to exist due to the larger physical distances between the co-channel interferers and the receiver.
- Let the instantaneous power in the desired signal and the N_I interfering signals be denoted by s_0 and $s_k,\ k = 1, \cdots, N_I$, respectively. Note that $s_i = \alpha_i^2$, where α_i^2 is the squared-envelope.
- The carrier-to-interference ratio is defined as $\lambda = s_0 / \sum_{k=1}^{N_I} s_k$, and for a specified receiver threshold λ_{th}, the outage probability is

$$O_I = \mathrm{P}\left(\lambda < \lambda_{\text{th}}\right) \quad .$$

Single Interferer

- For the case of a single interferer, the outage probability reduces to the simple closed form

$$O_I = \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_1} \exp\left\{ -\frac{K A_1}{\lambda_{\rm th} + A_1} \right\} \ ,$$

  where K is the Rice factor of the desired signal, $A_1 = \Omega_0/(K+1)\Omega_1$, and $\Omega_k = {\rm E}[s_k]$.

- If the desired signal is Rayleigh faded, then the outage probability can be obtained by setting $K = 0$.

Multiple Interferers

- For the case of multiple interferers, each with mean power Ω_k, the outage probability has the closed form

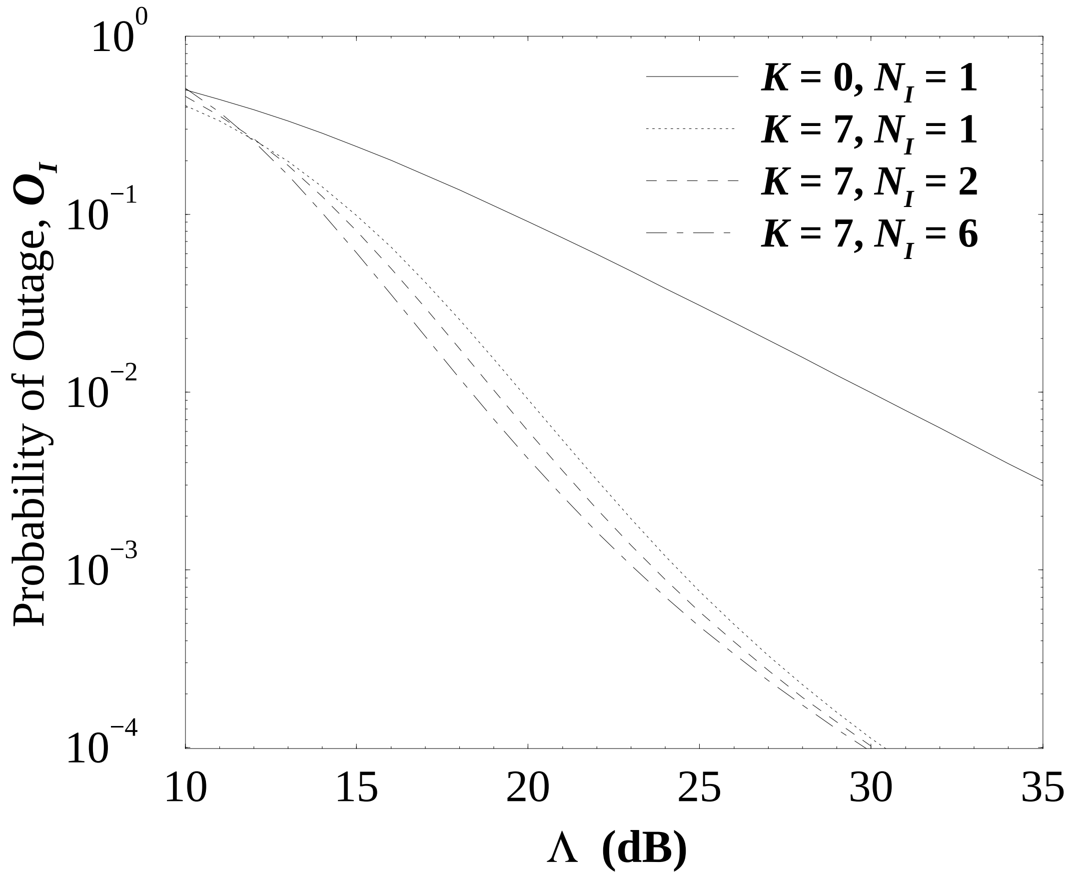
$$O_I = 1 - \sum_{k=1}^{N_I} \left[1 - \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_k} \exp\left\{ - \frac{K A_k}{\lambda_{\rm th} + A_k} \right\} \right] \prod_{\substack{j=1 \\ j \neq k}}^{N_I} \frac{A_j}{A_j - A_k} \quad ,$$

  where $A_k = \Omega_0/(K+1)\Omega_k$. This expression is only valid if $\Omega_i \neq \Omega_j$ when $i \neq j$, i.e., the different interferers have different mean power.

- If all the interferers have the same mean power, then the outage probability can be derived as

$$\begin{aligned} O_I &= \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_1} \exp\left\{ - \frac{K A_1}{\lambda_{\rm th} + A_1} \right\} \\ &\quad \times \sum_{k=0}^{N_I - 1} \left(\frac{A_1}{(\lambda_{\rm th} + A_1} \right)^k \sum_{m=0}^{k} \binom{k}{m} \frac{1}{m!} \left(\frac{K \lambda_{\rm th}}{\lambda_{\rm th} + A_1} \right)^m \quad . \end{aligned}$$

- If the desired signal is Rayleigh faded, then the probability of outage with multiple Rayleigh faded interferers can be obtained by setting $K = 0$.



Probability of outage with multiple interferers. The desired signal is Ricean faded with various Rice factors, while the interfering signals are Rayleigh faded and of equal power; $\lambda_{\text{th}} = 10.0$ dB.

$$\Lambda = \frac{\Omega_0}{N_I \Omega_1}$$