# EE6604

# Personal & Mobile Communications

Week 6

Fading Simulators,

Wideband and COST 207 Models,

Baud-Spaced Channel Models

Reading: 2.5

# Method of Exact Doppler Spreads - Deterministic

- Patzold proposed a deterministic simulation model, called the "Method of Exact Doppler Spreads (MEDS)".

- The method is derived by using the integral representation for the zero-order Bessel function of the first kind

$$J_0(x) = \frac{2}{\pi} \int_0^{\pi/2} \cos(x \sin\theta) d\theta$$

and replacing the integral by a series expansion as follows:

$$J_0(x) = \lim_{N_I \to \infty} \frac{2}{\pi} \sum_{n=1}^{N_I} \cos(x \sin\alpha_n) \Delta_\alpha$$

where $\alpha_n \triangleq \pi(2n-1)/(4N_I)$ and $\Delta_\alpha \triangleq \pi/(2N_I)$.

- Hence,

$$\phi_{g_I g_I}(\tau) = \frac{1}{2} J_0(2\pi f_m \tau) = \lim_{N_I \to \infty} \frac{1}{2N_I} \sum_{n=1}^{N_I} \cos(2\pi f_{n,i} \tau)$$

where

$$f_{n,I} = f_m \sin\left[\frac{\pi}{2N_I}\left(n - \frac{1}{2}\right)\right] \quad .$$

- With finite $N_I$, the normalized autocorrelation is

$$\phi^n_{g_I g_I}(\tau) = \frac{1}{N_I} \sum_{n=1}^{N_I} \cos(2\pi f_{n,I} \tau)$$

# Method of Exact Doppler Spreads - Deterministic

- The complex fading envelope is $g(t) = g_I(t) + jg_Q(t)$, where

$$g_{I/Q}(t) = \sqrt{\frac{1}{N_{I/Q}}} \sum_{n=1}^{N_{I/Q}} \cos(2\pi f_{I/Q,n} t + \phi_{I/Q,n}) \ .$$

- The deterministic processes $g_I(t)$ and $g_Q(t)$ are uncorrelated if and only if $f_{I,n} \neq f_{Q,m}$ for all $n = 1, \ldots, N_I$ and $m = 1, \ldots, N_Q$.
  - This can be achieved by setting $N_Q = N_I + 1$.
- If $g_I(t)$ and $g_Q(t)$ are uncorrelated, the phases $\phi_{I/Q,n}$ have no influence on the statistical properties of $g(t)$.
  - Hence, the $\phi_{I/Q,n}$ can be chosen as arbitrary realizations of uniform random variables on $[-\pi, \pi)$.

# Zheng and Xiao's Model - Statistical

- The complex fading envelope is $g(t) = g_I(t) + j g_Q(t)$, where

$$g_I(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \cos(\theta_n) + \phi_{I,n}]$$

$$g_Q(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \sin(\theta_n) + \phi_{Q,n}]$$

  where the angles are

$$\theta_n = \frac{2\pi n - \pi + \theta}{4N}, \qquad n = 1, 2, \ldots, N$$

  where $\theta$, $\phi_{I,n}$, and $\phi_{Q,n}$ are all uniform on $[-\pi, \pi)$, and all values are mutually independent.

- The statistical correlation functions of the quadrature components $\phi_{g_I g_I}(\tau)$, $\phi_{g_I g_Q}(\tau)$, $\phi_{gg}(\tau)$ match the desired functions.

- The autocorrelation function of the squared envelope is

$$\phi_{|g|^2|g|^2}(\tau) = 4 + \frac{J_0(4\pi f_m \tau)}{N} + \frac{4}{N^2} \sum_{n=1}^{N} \sum_{\substack{m=1 \\ n \neq m}}^{N} \mathrm{E}\{\cos[2\pi f_m \tau \cos\theta_n] \cos[2\pi f_m \tau \cos\theta_m]\}$$

# Modified Hoeher Model - Statistical

- Consider the complex faded envelope $g(t) = g_I(t) + jg_Q(t)$, where

$$\begin{aligned} g_I(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N_I} \cos(2\pi f_{I,n} t + \phi_{I,n}) \\ g_Q(t) &= \sqrt{\frac{1}{N}} \sum_{m=1}^{N_Q} \cos(2\pi f_{Q,m} t + \phi_{Q,m}) \end{aligned}$$

  and

$$f_{I/Q,n/m} = f_m \sin\left(\frac{\pi}{2} u_{I/Q,n/m}\right) \quad .$$

- The Doppler frequencies $f_{I/Q,n/m}$ for the $I$ and $Q$ components are determined by $u_{I/Q,n/m}$, where the $u_{I/Q,n/m}$ are uniform on $(0, 1]$ and are mutually independent for all $n$ and $m$. The random phases $\phi_{I/Q,n/m}$ are uniform on $[-\pi, \pi)$, are mutually independent for all $n$ and $m$, and are also independent of the $u_{I/Q,n/m}$.

- For convenience, the number of sinusoids in the quadrature components are usually set equal, i.e., $N_I = N_Q = N$.

- (MEDS) and the modified Hoeher model are similar
  - The set of numbers $\{(n-1/2)/N_I, i = 1, \ldots, N_I\}$ are uniformly spaced on the interval $(0,1]$, while the $u_{I/Q,n/m}$ are uniformly distributed on the interval $(0,1]$.
- The statistical correlation functions for the quadrature components match the desired values.
- The squared envelope correlation is

$$\phi_{|g|^2|g|^2}(\tau) = 4 + 4\frac{N-1}{N} J_0^2(2\pi f_m \tau) + \frac{1}{N} J_0(4\pi f_m \tau)$$

which differs from the ideal value for finite $N$.

# Multiple Faded Envelopes - Zheng & Xiao Model

- The Zheng & Xiao statistical method can be easily extended to generate multiple faded envelopes.

- The $k$th complex envelope, $g_k(t) = g_{I,k}(t) + jg_{Q,k}(t)$, is generated as

$$\begin{aligned} g_{I,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \cos(\theta_{n,k}) + \phi_{I,n,k}] \\ g_{Q,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \sin(\theta_{n,k}) + \phi_{Q,n,k}] \end{aligned}$$

where

$$\theta_{n,k} = \frac{2\pi n - \pi + \theta_k}{4N}, \qquad n = 1, 2.., N$$

and where $\theta_k$, $\phi_{I,n,k}$, and $\phi_{Q,n,k}$ are all uniform on $[-\pi, \pi)$, and all values are mutually independent.

- The $g_k(t)$ are all uncorrelated.

# Multiple Faded Envelopes - Li & Huang Model

- Assume that $P$ uncorrelated fading envelopes are required, each composed of $N$ sinusoids.
- The $k$th faded envelope, $g_k(t) = g_{I,k}(t) + jg_{Q,k}(t)$, is generated as

$$
\begin{aligned}
g_{I,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} \cos(2\pi f_m \cos\theta_{n,k} t + \phi_{n,k}) \\
g_{Q,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} \sin(2\pi f_m \sin\theta_{n,k} t + \phi'_{n,k})
\end{aligned}
$$

- The phases $\phi_{n,k}$ and $\phi'_{n,k}$ arbitrary realizations of independent random variables uniform on $[-\pi, \pi)$, and

$$\theta_{n,k} = \frac{2\pi n}{N} + \frac{2\pi k}{PN} + \theta_{00}, \quad n = 0, \ldots, N, k = 0, \ldots, P-1$$

  where $\theta_{00}$ is an initial arrival angle chosen to satisfy $0 < \theta_{00} < 2\pi/PN$ and $\theta_{00} \neq \pi/PN$.
- Although the Li & Huang model generates uncorrelated faded envelopes, it does not satisfy the correlation functions of the reference model.

# Zajić & Stüber Deterministic Model

- The $k^{th}, 1 \leq k \leq P$ complex faded envelope is defined as $g_k(t) = g_{I,k}(t) + jg_{Q,k}(t)$, where

$$g_{I,k}(t) = \frac{2}{\sqrt{N}} \left[ \sum_{n=0}^{M} a_n \cos(2\pi f_m t \cos \alpha_{nk} + \phi_{nk}) \right],$$

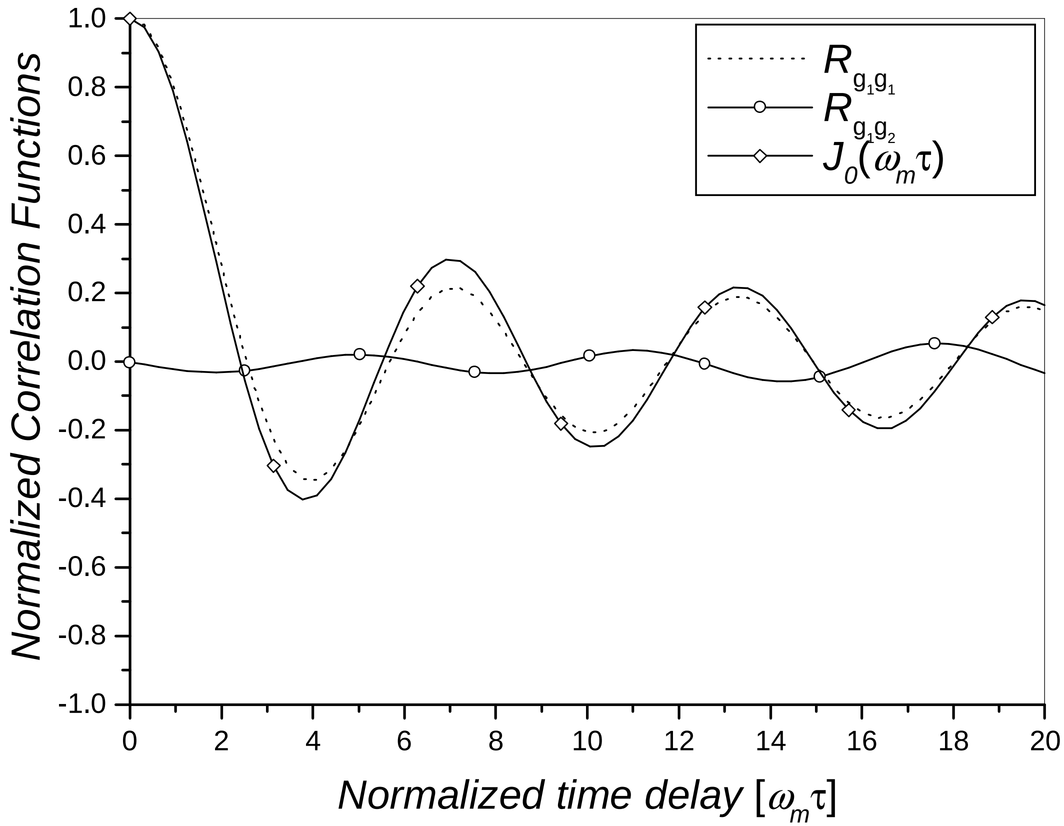
$$g_{Q,k}(t) = \frac{2}{\sqrt{N}} \left[ \sum_{n=0}^{M} b_n \sin(2\pi f_m t \sin \alpha_{nk} + \phi_{nk}) \right].$$

- The phases $\phi_{nk}$ are chosen to be independent random variables uniformly distributed on the interval $[0, 2\pi)$
- Parameters $a_n$ and $b_n$ are defined as follows:

$$a_n = \begin{cases} 2\cos(\beta_n), & n = 1, ..., M \\ \sqrt{2}\cos(\beta_n), & n = 0 \end{cases},$$

$$b_n = \begin{cases} 2\sin(\beta_n), & n = 1, ..., M \\ \sqrt{2}\sin(\beta_n), & n = 0 \end{cases}.$$

- The $\beta_n$ are defined as $\beta_n = \pi n/M$ for $n = 0, \ldots, M$, where $N = 4M + 2$.
- The angles of arrival $\alpha_{nk}$ are defined as $\alpha_{nk} = (2\pi n)/N + (2\pi k)/(PN) + \alpha_{00}$ for $n = 0, \ldots, M$, $k = 0, \ldots, P-1$, where $\alpha_{00} = (0.2\pi)/(PN)$.




Theoretical and simulated auto-correlation functions and the cross-correlation function of the first and the second complex envelope of the deterministic model.

# Zajić & Stüber Statistical Model

- The $k^{th}, 1 \leq k \leq P$ complex faded envelope is defined as $g_k(t) = g_{I,k}(t) + j g_{Q,k}(t)$, where

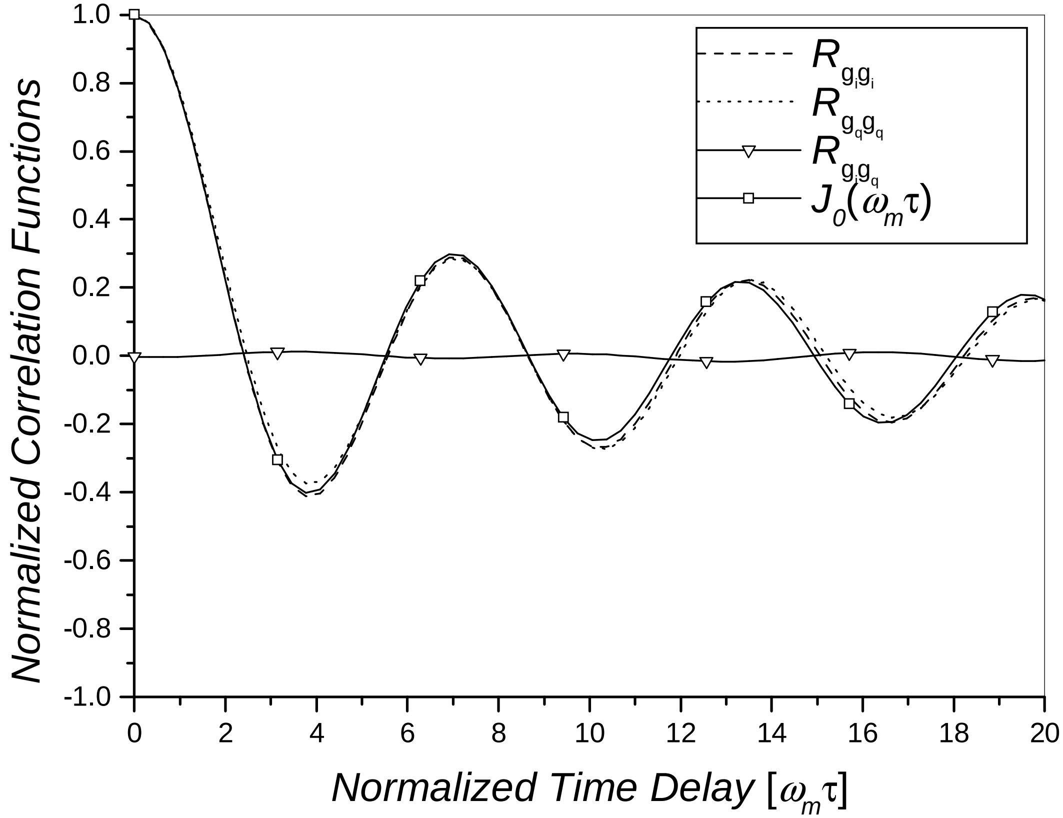
$$g_{I,k}(t) = \frac{2}{\sqrt{N}} \sum_{n=1}^{M} 2\cos(\beta_{nk}) \cos(2\pi f_m t \cos \alpha_{nk} + \phi_{nk}), \qquad (1)$$

$$g_{Q,k}(t) = \frac{2}{\sqrt{N}} \sum_{n=1}^{M} 2\sin(\beta_{nk}) \sin(2\pi f_m t \sin \alpha_{nk} + \phi_{nk}). \qquad (2)$$

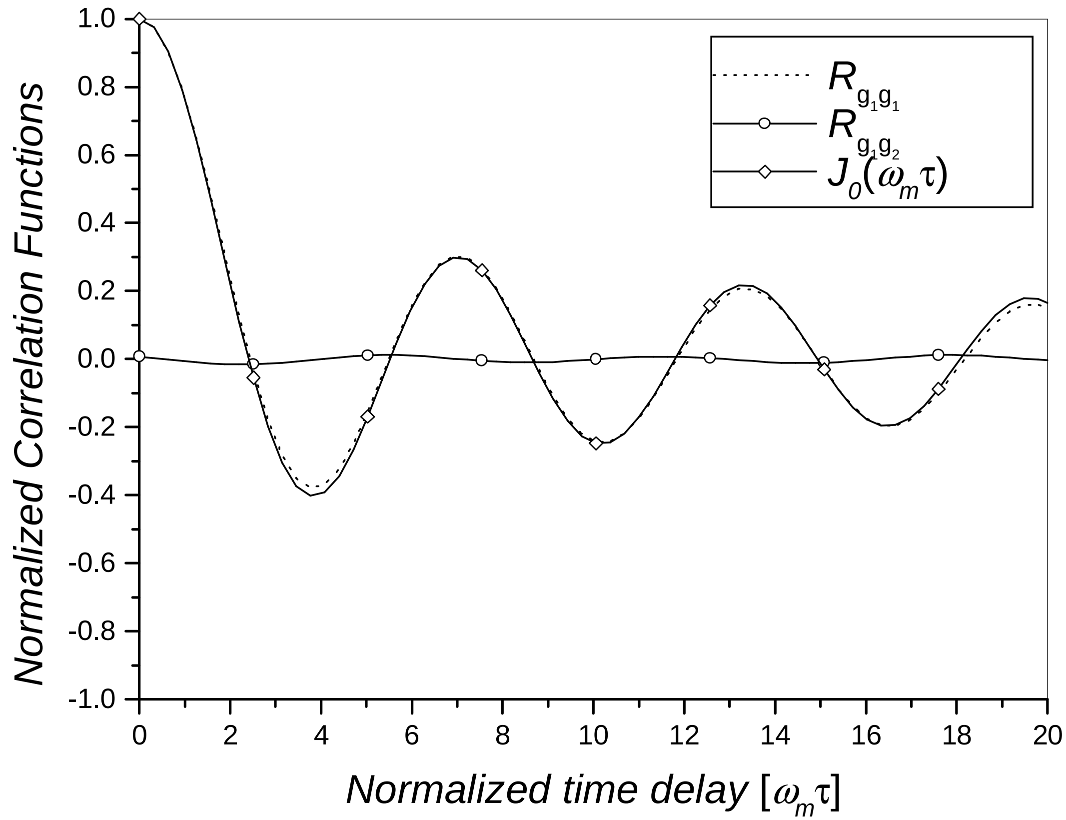
- It is assumed that $P$ independent complex envelopes are desired ($k = 0, \ldots, P-1$), each having $M = N/4$ sinusoidal terms in the I and Q components.

- The angles of arrival are chosen as follows: $\alpha_{nk} = (2\pi n)/N + (2\pi k)/(PN) + (\theta - \pi)/N$, for $n = 1, \ldots, M$, $k = 0, \ldots, P-1$. The angles of arrival in the $k^{th}$ complex faded envelope are obtained by rotating the angles of arrivals in the $(k-1)^{th}$ complex envelope by $(2\pi)/(PN)$.

- The parameters $\phi_{nk}$, $\beta_{nk}$, and $\theta$ are independent random variables uniformly distributed on the interval $[-\pi, \pi)$.




Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the in-phase and the quadrature component of the statistical model.




Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the first and the second fader in the statistical model.

# Wide-band Simulation Models

- A wide-band channel has the time-variant impulse response

$$g(t,\tau) = \sum_{i=1}^{\ell} g_i(t)\delta(\tau - \tau_i) \ ,$$

- The corresponding time-variant transfer function is

$$T(f,t) = \sum_{i=1}^{\ell} g_i(t)\mathrm{e}^{-j2\pi f\tau_i}$$

- Assuming a WSSUS channel, the autocorrelation function of the time-variant channel impulse response is

$$\begin{aligned} \phi_g(t,s;\tau,\eta) &= \mathrm{E}[g(t,\tau)g^*(s,\eta)] \\ &= \sum_{i=1}^{\ell} \phi_{g_ig_i}(\Delta t)\delta(\tau - \tau_i)\delta(\eta - \tau) \end{aligned}$$

- It follows that the channel correlation function is

$$\psi_g(\Delta t;\tau) = \sum_{i=1}^{\ell} \phi_{g_ig_i}(\Delta t)\delta(\tau - \tau_i) \tag{3}$$

- The power-delay profile is

$$\psi_g(\tau) = \psi_g(0;\tau) = \sum_{i=1}^{\ell} \Omega_i \delta(\tau - \tau_i) \ ,$$

where $\Omega_i = \phi_{g_i g_i}(0)$ is the envelope power of the $i$th channel tap, and the total envelope power is

$$\Omega_p = \sum_{k=1}^{\ell} \Omega_k \ .$$

- The channel is described by the power profile

$$\boldsymbol{\Omega} = (\Omega_0,\ \Omega_1, \ldots, \Omega_\ell) \ .$$

and the delay profile

$$\boldsymbol{\tau} = (\tau_1,\ \tau_2, \ldots, \tau_\ell) \ .$$

- Power delay profiles have been specified in the COST 207 and 259 studies. Other studies exist.

# Spaced-time Spaced-Frequency Correlation Function

- Taking the Fourier transform of (3) with respect to the $\tau$ variable yields the spaced-time spaced-frequency correlation function

$$\phi_T(\Delta f;\Delta t) = \sum_{i=1}^{\ell} \phi_{g_i g_i}(\Delta t) \mathrm{e}^{-j2\pi\Delta f \tau_i}$$

- Sometimes the channel taps are assumed to have the same time correlation function, such that

$$\phi_{g_i g_i}(\Delta t) = \Omega_i \phi_t(\Delta t) \ , \quad i = 1,\ldots,\ell \ .$$

- If each tap is characterized by 2-D isotropic scattering then we have

$$\phi_t(\Delta t) = J_0(2\pi f_m \Delta t).$$

- In this case, the spaced-time spaced-frequency correlation function has the separable form

$$\phi_T(\Delta f;\Delta t) = \phi_t(\Delta t)\phi_f(\Delta f) \ ,$$

where

$$\phi_f(\Delta f) = \sum_{i=1}^{\ell} \Omega_i \mathrm{e}^{-j2\pi\Delta f \tau_i} \ .$$

# COST207 Models

- The COST207 models were developed for the GSM system. COST207 specifies four different Doppler spectra, $S_{gg}(f)$. Define

$$G(A, f_1, f_2) = A \exp\left\{-\frac{(f - f_1)^2}{2f_2^2}\right\}$$

The following types are defined;

**a**) CLASS is used for path delays less than 500 ns ($\tau_i \leq 500$ ns);

$$\text{(CLASS)} \quad S_{gg}(f) = \frac{A}{\sqrt{1 - (f/f_m)^2}} \quad |f| \leq f_m$$

**b**) GAUS1 is used for path delays from 500 ns to 2 $\mu$s; (500 ns $\leq \tau_i \leq 2\mu$s)

$$\text{(GAUS1)} \quad S_{gg}(f) = G(A, -0.8f_m, 0.05f_m) + G(A_1, 0.4f_m, 0.1f_m)$$

where $A_1$ is 10 dB below $A$.

**c**) GAUS2 is used for path delays exceeding 2 $\mu$s; ($\tau_i > 2$ $\mu$s)

$$\text{(GAUS2)} \quad S_{gg}(f) = G(B, 0.7f_m, 0.1f_m) + G(B_1, -0.4f_m, 0.15f_m)$$

where $B_1$ is 15 dB below $B$.

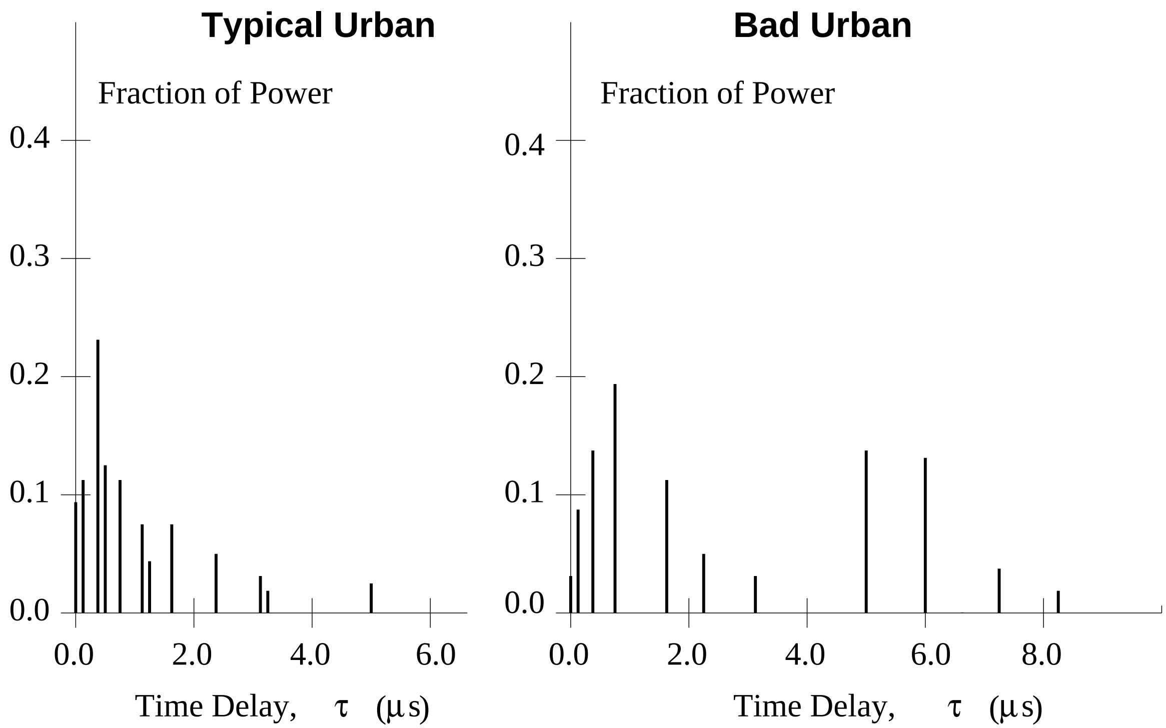
**d**) RICE is a sometimes used for the direct ray;

$$\text{(RICE)} \quad S_{gg}(f) = \frac{0.41}{2\pi f_m \sqrt{1 - (f/f_m)^2}} \; + \; 0.91\delta(f - 0.7f_m) \quad |f| \leq f_m$$

# Typical Urban (TU) and Bad Urban (BU) 12-ray models

| Typical Urban (TU) | | | Bad Urban (BU) | | |
|---|---|---|---|---|---|
| delay $\mu$s | Fractional Power | Doppler Category | delay $\mu$s | Fractional Power | Doppler Category |
| 0.0 | 0.092 | CLASS | 0.0 | 0.033 | CLASS |
| 0.1 | 0.115 | CLASS | 0.1 | 0.089 | CLASS |
| 0.3 | 0.231 | CLASS | 0.3 | 0.141 | CLASS |
| 0.5 | 0.127 | CLASS | 0.7 | 0.194 | GAUS1 |
| 0.8 | 0.115 | GAUS1 | 1.6 | 0.114 | GAUS1 |
| 1.1 | 0.074 | GAUS1 | 2.2 | 0.052 | GAUS2 |
| 1.3 | 0.046 | GAUS1 | 3.1 | 0.035 | GAUS2 |
| 1.7 | 0.074 | GAUS1 | 5.0 | 0.140 | GAUS2 |
| 2.3 | 0.051 | GAUS2 | 6.0 | 0.136 | GAUS2 |
| 3.1 | 0.032 | GAUS2 | 7.2 | 0.041 | GAUS2 |
| 3.2 | 0.018 | GAUS2 | 8.1 | 0.019 | GAUS2 |
| 5.0 | 0.025 | GAUS2 | 10.0 | 0.006 | GAUS2 |

# Typical Urban (TU) and Bad Urban (BU) 12-ray models



# Reduce typical Urban (RTU) and Reduced Bad Urban (RBU) 6-ray models

| Typical Urban (TU) | | | Bad Urban (BU) | | |
|---|---|---|---|---|---|
| delay $\mu$s | Fractional Power | Doppler Category | delay $\mu$s | Fractional Power | Doppler Category |
| 0.0 | 0.189 | CLASS | 0.0 | 0.164 | CLASS |
| 0.2 | 0.379 | CLASS | 0.3 | 0.293 | CLASS |
| 0.5 | 0.239 | CLASS | 1.0 | 0.147 | GAUS1 |
| 1.6 | 0.095 | GAUS1 | 1.6 | 0.094 | GAUS1 |
| 2.3 | 0.061 | GAUS2 | 5.0 | 0.185 | GAUS2 |
| 5.0 | 0.037 | GAUS2 | 6.6 | 0.117 | GAUS2 |

# COST 207 typical case for rural (non-hilly) area (RA)

| delay $\mu$s | Fractional Power | Doppler Category |
|---|---|---|
| 0.0 | 0.602 | RICE |
| 0.1 | 0.241 | CLASS |
| 0.2 | 0.096 | CLASS |
| 0.3 | 0.036 | CLASS |
| 0.4 | 0.018 | CLASS |
| 0.5 | 0.006 | CLASS |

# COST 207 typical case for hilly terrain (HT)

| delay $\mu$s | Fractional Power | Doppler Category |
|---|---|---|
| 0.0 | 0.026 | CLASS |
| 0.1 | 0.042 | CLASS |
| 0.3 | 0.066 | CLASS |
| 0.5 | 0.105 | CLASS |
| 0.7 | 0.263 | GAUS1 |
| 1.0 | 0.263 | GAUS1 |
| 1.3 | 0.105 | GAUS1 |
| 15.0 | 0.042 | GAUS2 |
| 15.2 | 0.034 | GAUS2 |
| 15.7 | 0.026 | GAUS2 |
| 17.2 | 0.016 | GAUS2 |
| 20.0 | 0.011 | GAUS2 |

# Reduced Hilly Terrain (RHT)

| delay $\mu$s | Fractional Power | Doppler Category |
|---|---|---|
| 0.0 | 0.413 | CLASS |
| 0.1 | 0.293 | CLASS |
| 0.3 | 0.145 | CLASS |
| 0.5 | 0.074 | CLASS |
| 15.0 | 0.066 | GAUS2 |
| 17.2 | 0.008 | GAUS2 |

# COST 259 Typical Urban (TUx) channel model; x=3, 50 and 120 km/h

| *delay* $\mu$s | *Fractional Power* | *Doppler Category* |
|---|---|---|
| 0.000 | 0.26915 | CLASS |
| 0.217 | 0.17378 | CLASS |
| 0.512 | 0.09772 | CLASS |
| 0.514 | 0.09550 | CLASS |
| 0.517 | 0.09550 | CLASS |
| 0.674 | 0.07079 | CLASS |
| 0.882 | 0.04571 | CLASS |
| 1.230 | 0.02344 | CLASS |
| 1.287 | 0.02042 | CLASS |
| 1.311 | 0.01950 | CLASS |
| 1.349 | 0.01820 | CLASS |
| 1.533 | 0.01259 | CLASS |
| 1.535 | 0.01259 | CLASS |
| 1.622 | 0.01047 | CLASS |
| 1.818 | 0.00708 | CLASS |
| 1.836 | 0.00692 | CLASS |
| 1.884 | 0.00617 | CLASS |
| 1.943 | 0.00550 | CLASS |
| 2.048 | 0.00447 | CLASS |
| 2.140 | 0.00372 | CLASS |

# COST 259 Rural Area (RAx) channel model; x=120 and 250 km/h

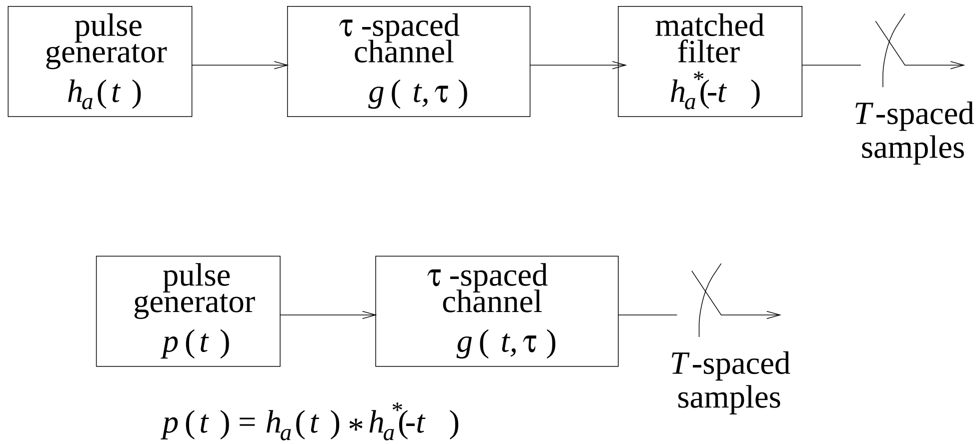
| *delay* $\mu$s | *Fractional Power* | *Doppler Category* |
|---|---|---|
| 0.000 | 0.30200 | Direct Path, $f_0 = 0.7 f_m$ |
| 0.042 | 0.22909 | CLASS |
| 0.101 | 0.14454 | CLASS |
| 0.129 | 0.11749 | CLASS |
| 0.149 | 0.10000 | CLASS |
| 0.245 | 0.04898 | CLASS |
| 0.312 | 0.02951 | CLASS |
| 0.410 | 0.01413 | CLASS |
| 0.469 | 0.00912 | CLASS |
| 0.528 | 0.00575 | CLASS |

# COST 259 Hilly Terrain (HTx) channel model; x=120 km/h

| *delay* $\mu$s | *Fractional Power* | *Doppler Category* |
|---|---|---|
| 0.000 | 0.43652 | CLASS |
| 0.356 | 0.12882 | CLASS |
| 0.441 | 0.09550 | CLASS |
| 0.528 | 0.07079 | CLASS |
| 0.546 | 0.06607 | CLASS |
| 0.609 | 0.05370 | CLASS |
| 0.625 | 0.05012 | CLASS |
| 0.842 | 0.02399 | CLASS |
| 0.916 | 0.01862 | CLASS |
| 0.941 | 0.01698 | CLASS |
| 15.000 | 0.01738 | CLASS |
| 16.172 | 0.00537 | CLASS |
| 16.492 | 0.00389 | CLASS |
| 16.876 | 0.00263 | CLASS |
| 16.882 | 0.00263 | CLASS |
| 16.978 | 0.00240 | CLASS |
| 17.615 | 0.00126 | CLASS |
| 17.827 | 0.00102 | CLASS |
| 17.849 | 0.00100 | CLASS |
| 18.016 | 0.00085 | CLASS |

# Baud-Spaced Channel Models

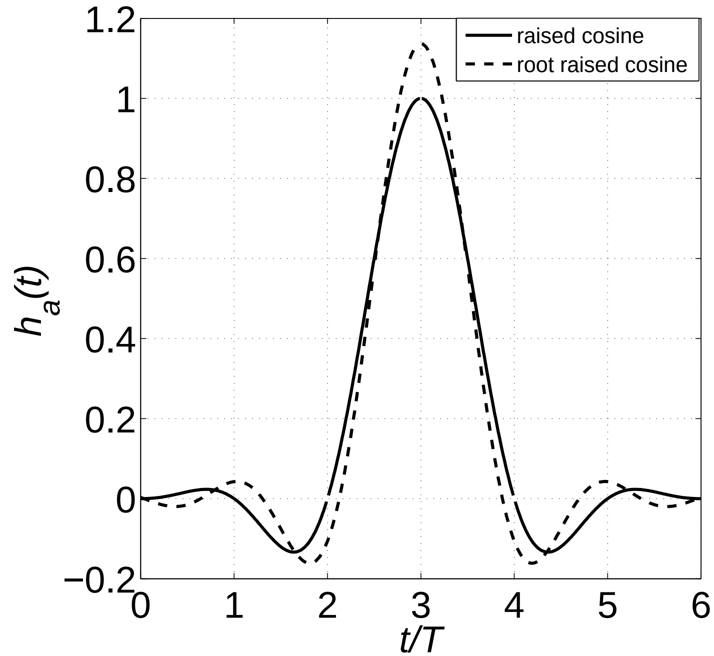
- Wideband channel models usually have a delay profile where the delays, $\tau_i$, not related to the modulated symbol duration $T$. However, time domain simulation models typically have a simulation step size, $T_s$, that is related to the symbol duration, i.e., $T_s = T/2^k$ for some integer $k$. Typically, $k = 4, 8, 16$.
- We need a $T_s$-spaced channel model.
- Consider the method below showing the generation of a $T$-spaced channel model.




*Method for generating correlated tap coefficients in a $T$-spaced channel model.*

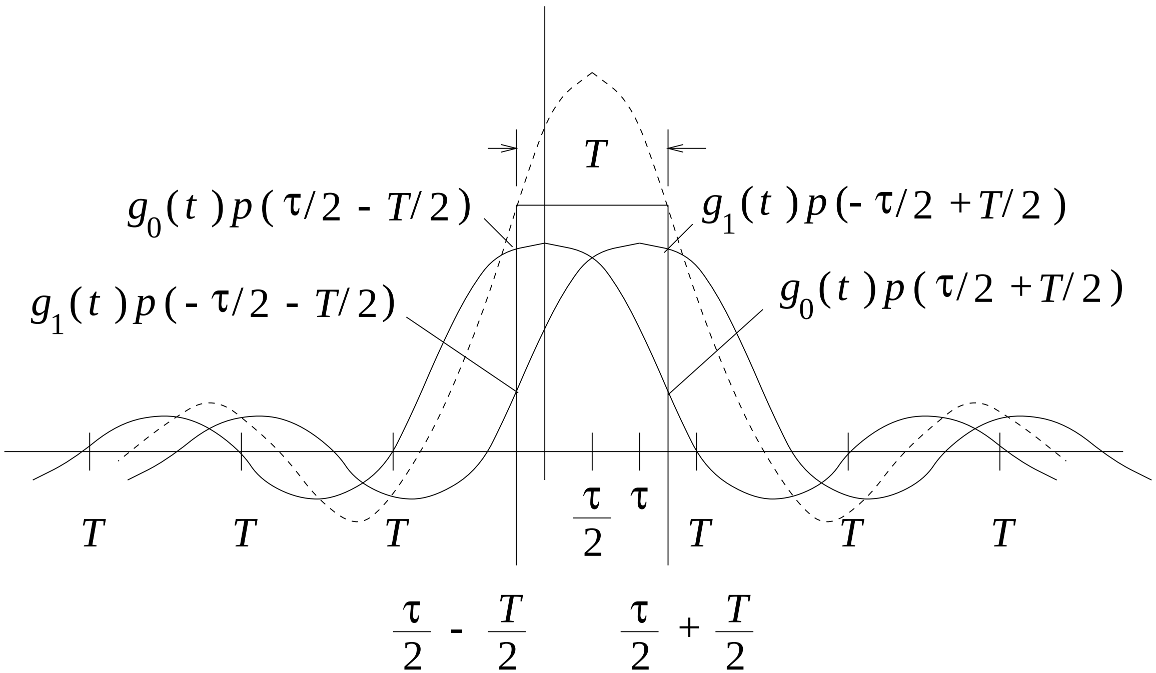
- The pulse generator produces pulses $p(t)$ having a shape that is determined by the combination of the transmitter and receiver filter, e.g., a raised cosine pulse.

- The pulse is passed through a $\tau$-spaced channel having $\ell$ taps, $T$-spaced samples are taken at the output. The $T$-spaced samples are a linear combination of the set of $\ell$ channel gains $\{g_k(t)\}$ in the $\tau$-spaced model.




*Raised cosine and root raised cosine pulses with roll-off factor $\beta = 0.5$. The pulses are truncated to length $6T$ and time shifted by $3T$ to yield causal pulses.*

# Example




*Generation $T$-spaced taps from a $\tau$-spaced model.*

- A vector of $M$, $T$-spaced, time variant tap coefficients can be generated in this manner

$$\mathbf{g}_T(t) = (g_{T_1}(t),\ g_{T_2}(t),\ \ldots,\ g_{T_M}(t))^T$$

  - Even for small $\ell$, the value of $M$ can be surprisingly large.
  - Length $M$ depends on lengths of the channel $g(t, \tau)$ and the pulse $p(t)$.

- We can write

$$\mathbf{g}_T(t) = \mathbf{A}\mathbf{g}(t)$$

where

$$\mathbf{g}(t) = (g_1(t),\ g_2(t),\ \ldots,\ g_\ell(t))^T$$

and $\mathbf{A}$ is an $M \times \ell$ real matrix, and $\mathbf{x}^T$ denotes the transpose of $\mathbf{x}$.

  - Entries of the matrix $\mathbf{A}$ depend on the pulse $p(t)$, and the ($T$-spaced) sampler timing phase.
  - $\mathbf{A}$ needs to be recalculated whenever the sampler timing phase is re-estimated.
  - Sampler timing phase is usually re-estimated for every burst.

- The covariance matrix of the $T$-spaced tap gain vector $\mathbf{g}_T(t)$ is

$$\begin{aligned}\Phi_{\mathbf{g}_T}(\tau) &= \frac{1}{2}\mathrm{E}\left[\mathbf{g}_T(t+\tau)\mathbf{g}_T^H(t)\right] \\ &= \frac{1}{2}\mathrm{E}\left[\mathbf{A}\mathbf{g}(t+\tau)\mathbf{g}^H(t)\mathbf{A}^T\right] \\ &= \mathbf{A}\Phi_{\mathbf{g}}(\tau)\mathbf{A}^T \ .\end{aligned}$$

- For a WSSUS $\tau$-spaced channel with 2-D isotropic scattering

$$\Phi_{\mathbf{g}_T}(\tau) = \frac{1}{2}\mathbf{A}\boldsymbol{\Omega}\mathbf{A}^T J_0(2\pi f_m \tau)$$

where

$$\boldsymbol{\Omega} \triangleq \mathrm{diag}[\Omega_0,\ \Omega_1, \ldots,\ \Omega_\ell]$$

is an $\ell \times \ell$ diagonal matrix, and $\Omega_k = \mathrm{E}[|g_k(t)|^2]$ is the envelope power that is associated with the $k$th tap in the $\tau$-spaced channel.

  - This applies to COST207 and COST259 models where the channel taps have a CLASSICAL Doppler spectrum.

# Example

- Suppose that

$$p(t) = \mathrm{Sa}(\pi t/T) \cdot \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2}$$

  where $\beta = 0.35$.

- The $\tau$-spaced waveform channel is characterized by two equal strength taps ($|g_1(t)|^2 = |g_2(t)|^2$) with a differential delay of $\tau = |\tau_1(t) - \tau_0(t)|$.

- We wish to generate a 2-tap $T$-spaced channel model with taps $g_{T_1}(t)$ and $g_{T_2}(t)$, under the condition that $\tau = T/4$.

- Let

$$\begin{aligned} \mathbf{g}(t) &= (g_1(t), g_2(t))^T \\ \mathbf{g}_T(t) &= (g_{T_1}(t), g_{T_2}(t))^T \end{aligned}$$

  and

$$\mathbf{g}_T(t) = \mathbf{A}\mathbf{g}(t) \ .$$

- The matrix $\mathbf{A}$ depends on the timing phase of the $T$-spaced samples taken at the output of the pulse generator. In a practical system, the sampler timing phase is determined by the synchronization process in the receiver.

# Example

- Sampler timing phase can be performed by maximizing sum energy

$$E_\tau = \sum_{k=1}^{M} |g_{T_k}(t)|^2$$

- Suppose that the sampler timing phase is chosen such that $|g_{T_1}(t)|^2 = |g_{T_2}(t)|^2$.
  - This is optimal since $|g_0(t)|^2 = |g_1(t)|^2$ in the $\tau$-spaced channel for this example.
- The entries of matrix $\mathbf{A}$ can be obtained by writing (refer to following slide)

$$\begin{aligned} g_{0T}(t) &= g_0(t)p(\tau/2 - T/2) + g_1(t)p(-\tau/2 - T/2) \\ g_{1T}(t) &= g_0(t)p(\tau/2 + T/2) + g_1(t)p(-\tau/2 + T/2) \end{aligned}$$

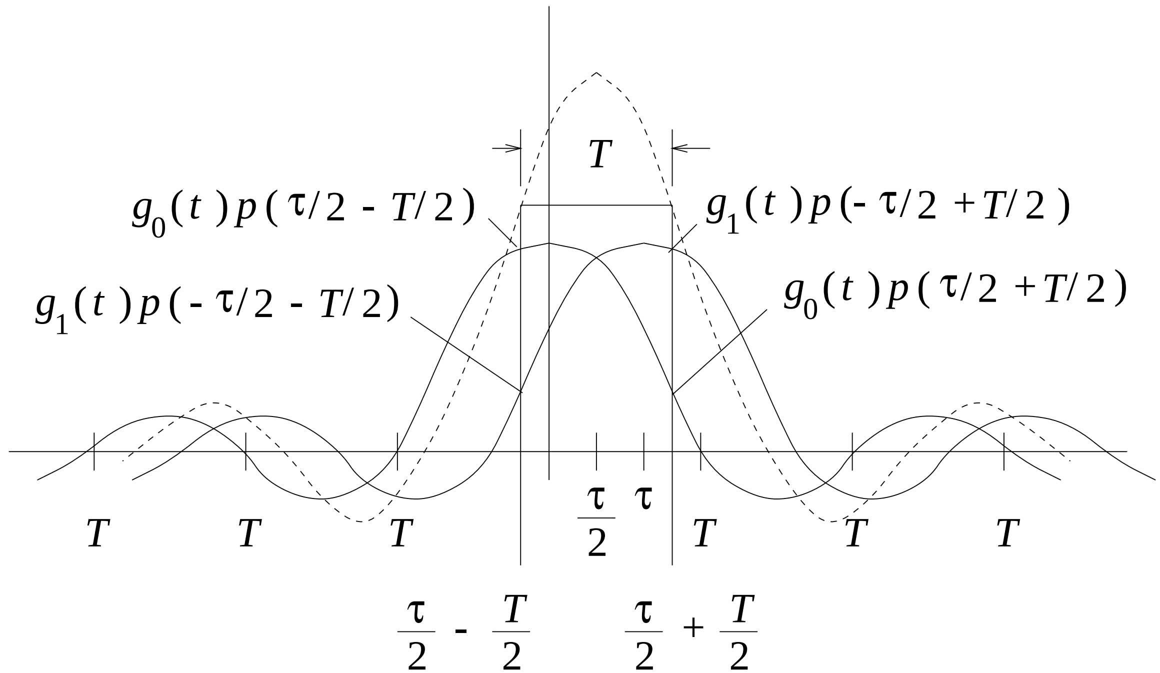
Hence,

$$\mathbf{A} = \begin{bmatrix} p(\tau/2 - T/2) & p(-\tau/2 - T/2) \\ p(\tau/2 + T/2) & p(-\tau/2 + T/2) \end{bmatrix}$$

For $\tau = T/4$ and $\beta = 0.35$

$$\mathbf{A} = \begin{bmatrix} p(-3T/8) & p(-5T/8) \\ p(5T/8) & p(3T/8) \end{bmatrix} = \begin{bmatrix} 0.7717 & 0.4498 \\ 0.4498 & 0.7717 \end{bmatrix}$$

# Example




*Generation $T$-spaced taps from a $\tau$-spaced model.*