ECE6604
PERSONAL & MOBILE COMMUNICATIONS

Week 4

Envelope Correlation
Space-time Correlation

Reading: Chapter 2, 2.1.1, 2.1.2, 2.1.6

Autocorrelation of a Bandpass Random Process

- Consider again the received band-pass random process

$$r(t) = g_I(t)\cos 2\pi f_c t \;-\; g_Q(t)\sin 2\pi f_c t$$

where

$$\begin{aligned} g_I(t) &= \sum_{n=1}^{N} C_n \cos\phi_n(t) \\ g_Q(t) &= \sum_{n=1}^{N} C_n \sin\phi_n(t) \end{aligned}$$

- Assuming that $r(t)$ is wide-sense stationary, the autocorrelation of $r(t)$ is

$$\begin{aligned} \phi_{rr}(\tau) &= \mathrm{E}[r(t)r(t+\tau)] \\ &= \mathrm{E}[g_I(t)g_I(t+\tau)]\cos 2\pi f_c\tau + \mathrm{E}[g_Q(t)g_I(t+\tau)]\sin 2\pi f_c\tau \\ &= \phi_{g_Ig_I}(\tau)\cos 2\pi f_c\tau - \phi_{g_Ig_Q}(\tau)\sin 2\pi f_c\tau \end{aligned}$$

where $\mathrm{E}[\;\cdot\;]$ is the ensemble average operator, and

$$\begin{aligned} \phi_{g_Ig_I}(\tau) &\stackrel{\triangle}{=} \mathrm{E}[g_I(t)g_I(t+\tau)] \\ \phi_{g_Ig_Q}(\tau) &\stackrel{\triangle}{=} \mathrm{E}[g_I(t)g_Q(t+\tau)] \;. \end{aligned}$$

Note that the wide-sense stationarity of $r(t)$ imposes the condition

$$\begin{aligned} \phi_{g_Ig_I}(\tau) &= \phi_{g_Qg_Q}(\tau) \\ \phi_{g_Ig_Q}(\tau) &= -\phi_{g_Qg_I}(\tau) \;. \end{aligned}$$

Auto- and Cross-correlation of Quadrature Components

- The phases $\phi_n(t)$ are statistically independent random variables at any time t, uniformly distributed over the interval $[-\pi, \pi)$.
- The azimuth angles of arrival, θ_n are all independent due to the random placement of scatterers. Also, in the limit $N \to \infty$, the discrete azimuth angles of arrival θ_n can be replaced by a continuous random variable θ having the probability density function $p(\theta)$.
- By using the above properties, the auto- and cross-correlation functions can be obtained as follows:

$$\phi_{g_I g_I}(\tau) = \phi_{g_Q g_Q}(\tau) = \lim_{N\to\infty} \mathrm{E}_{\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\phi}}[g_I(t)g_I(t+\tau)] = \frac{\Omega_p}{2}\mathrm{E}_\theta[\cos(2\pi f_m \tau \cos\theta)]$$

$$\phi_{g_I g_Q}(\tau) = -\phi_{g_Q g_I}(\tau) = \lim_{N\to\infty} \mathrm{E}_{\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\phi}}[g_I(t)g_Q(t+\tau)] = \frac{\Omega_p}{2}\mathrm{E}_\theta[\sin(2\pi f_m \tau \cos\theta)]$$

$$\begin{aligned}
\boldsymbol{\tau} &= (\tau_1, \tau_2, \ldots, \tau_N) \\
\boldsymbol{\theta} &= (\theta_1, \theta_2, \ldots, \theta_N) \\
\boldsymbol{\phi} &= (\phi_1, \phi_2, \ldots, \phi_N) \\
\Omega_p &= \mathrm{E}[g_I^2(t)] + \mathrm{E}[g_Q^2(t)] = \sum_{n=1}^{N} C_n^2
\end{aligned}$$

and Ω_p is the total received envelope power.

2-D Isotropic Scattering

- Evaluation of the expectations for the auto- and cross-correlation functions requires the azimuth distribution of arriving plane waves $p(\theta)$, and the receiver antenna gain pattern $G(\theta)$, as a function of the azimuth angle θ.
- With 2-D isotropic scattering, the plane waves are confined to the $x-y$ plane and arrive uniformly distributed angle of incidence, i.e.,

$$p(\theta) = \frac{1}{2\pi}\ , \quad -\pi \leq \theta \leq \pi$$

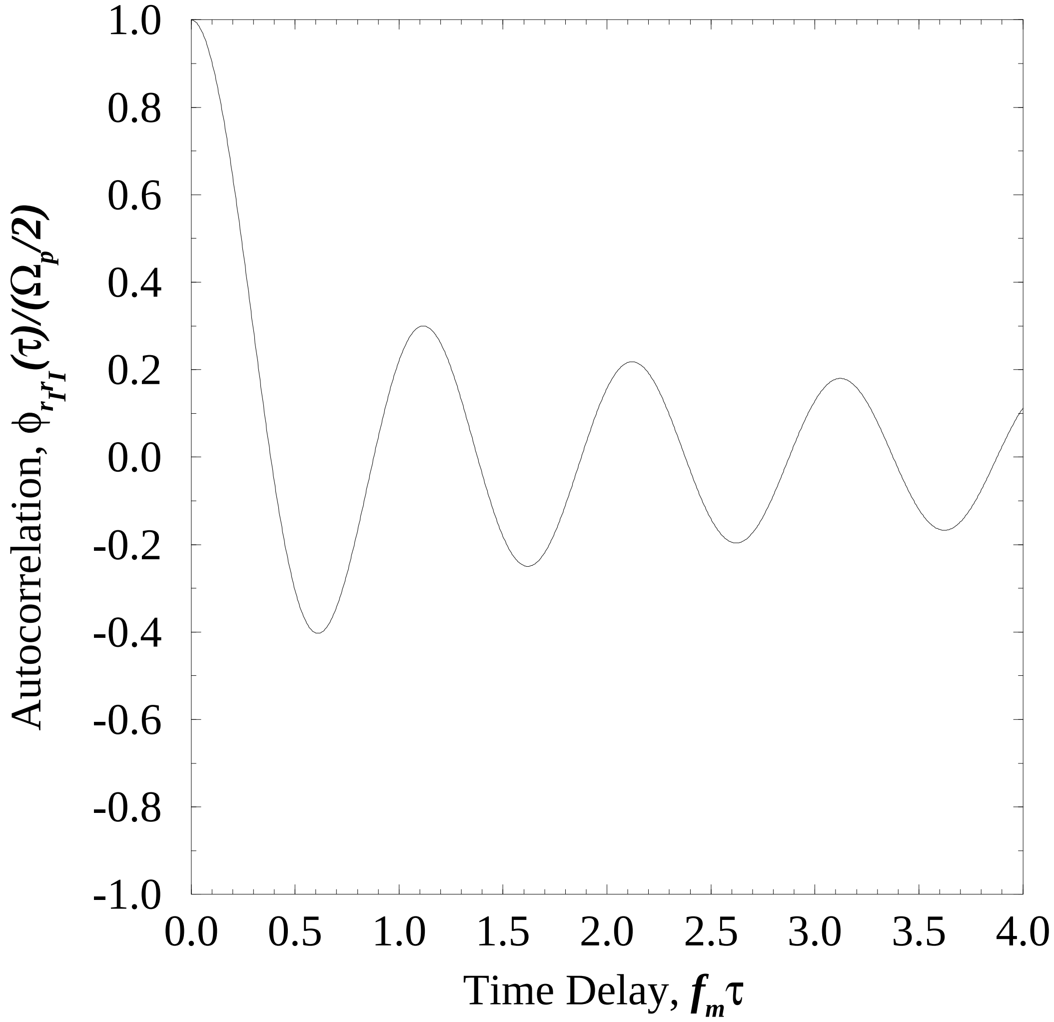
- With 2-D isotropic scattering and an isotropic receiver antenna with gain $G(\theta) = 1, \theta \in [-\pi, \pi)$, the auto- and cross-correlation functions become

$$\begin{aligned} \phi_{g_I g_I}(\tau) &= = \frac{\Omega_p}{2} J_0(2\pi f_m \tau) \\ \phi_{g_I g_Q}(\tau) &= 0 \end{aligned}$$

where

$$J_0(x) = \frac{1}{\pi}\int_0^\pi \cos(x\cos\theta)\mathrm{d}\theta$$

is the zero-order Bessel function of the first kind.



Normalized autocorrelation function of the quadrature components of the received complex envelope with 2-D isotropic scattering and an isotropic receiver antenna.

Doppler Spectrum

- The autocorrelation function and power spectral density (psd) are Fourier transform pairs.

$$\begin{aligned} S_{gg}(f) &= \int_{-\infty}^{\infty} \phi_{gg}(\tau) e^{-j2\pi f\tau} d\tau \\ \phi_{gg}(\tau) &= \int_{-\infty}^{\infty} S_{gg}(f) e^{j2\pi f\tau} df \end{aligned}$$

- The autocorrelation of the received complex envelope $g(t) = g_I(t) + jg_Q(t)$ is

$$\begin{aligned} \phi_{gg}(\tau) &= \frac{1}{2}\mathrm{E}[g^*(t)g(t+\tau)] \\ &= \phi_{g_Ig_I}(\tau) + j\phi_{g_Ig_Q}(\tau) \end{aligned}$$

- The Fourier transform of $\phi_{gg}(\tau)$ gives the Doppler psd

$$S_{gg}(f) = S_{g_Ig_I}(f) + jS_{g_Ig_Q}(f) \ .$$

Sometimes $S_{gg}(f)$ is just called the "**Doppler spectrum**."

Bandpass Doppler Spectrum

- We can also relate the power spectrum of the complex envelope $g(t)$ to that of the band-pass process $r(t)$. We have

$$\phi_{rr}(\tau) = \mathrm{Re}\left[\phi_{gg}(\tau)e^{j2\pi f_c \tau}\right] \quad .$$

- By using the identity

$$\mathrm{Re}\left[z\right] = \frac{z + z^*}{2}$$

and the property $\phi_{gg}(\tau) = \phi^*_{gg}(-\tau)$, it follows that the band-pass Doppler psd is

$$S_{rr}(f) = \frac{1}{2}\left[S_{gg}(f - f_c) + S_{gg}(-f - f_c)\right] \quad .$$

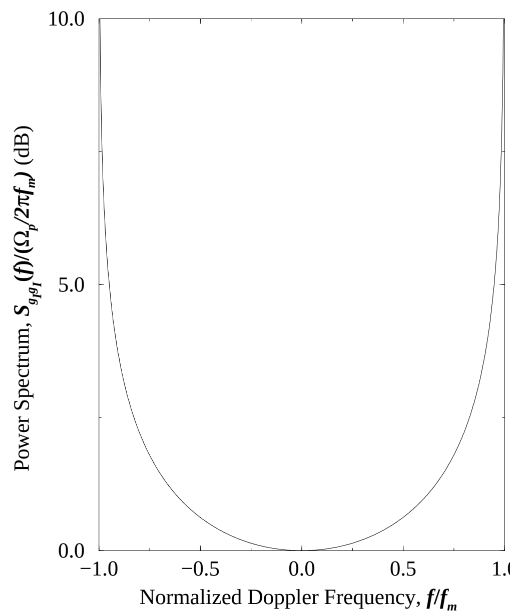
- Since $\phi_{gg}(\tau) = \phi^*_{gg}(-\tau)$, the Doppler spectrum $S_{gg}(f)$ is always a real-valued function of frequency, but not necessarily even. However, the band-pass Doppler spectrum $S_{rr}(f)$ is always real-valued and even.

Isotropic Scattering

- For 2-D isotropic scattering, the psd and cross psd of $g_I(t)$ and $g_Q(t)$ are

$$S_{g_Ig_I}(f) = \mathcal{F}[\phi_{g_Ig_I}(\tau)] = \begin{cases} \frac{\Omega_p}{2\pi f_m}\frac{1}{\sqrt{1-\left(\frac{f}{f_m}\right)^2}} & , \quad |f| \leq f_m \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$S_{g_Ig_Q}(f) = 0$$



Normalized psd of the quadrature components of the received complex envelope with 2-D isotropic scattering and an isotropic receiver antenna. Sometimes this is called the CLASSICAL Doppler power spectrum.

Non-isotropic Scattering – Rician Fading

- Suppose that the propagation environment consisting of a strong specular component plus a scatter component. The azimuth distribution $p(\theta)$ might have the form

$$p(\theta) = \frac{1}{K+1}\hat{p}(\theta) + \frac{K}{K+1}\delta(\theta - \theta_0)$$

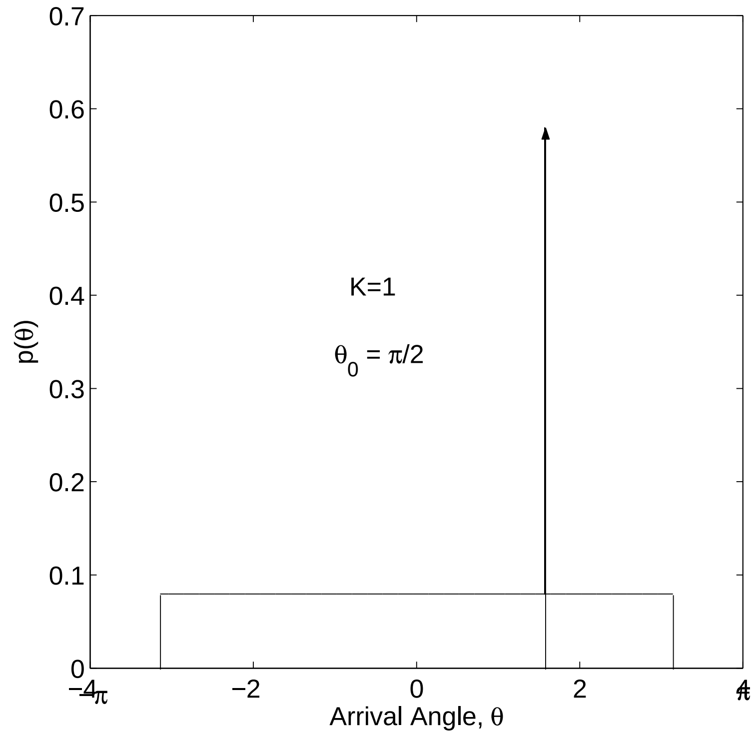
where $\hat{p}(\theta)$ is the continuous AoA distribution of the *scatter* component, θ_0 is the AoA of the specular component, and K is the ratio of the received specular to scattered power.

- One such scattering environment, assumes that the scatter component exhibits 2-D isotropic scattering, i.e., $\hat{p}(\theta) = 1/(2\pi), \theta \in [-\pi, \pi)$.

- The correlation functions $\phi_{g_I g_I}(\tau)$ and $\phi_{g_I g_Q}(\tau)$ are

$$\phi_{g_I g_I}(\tau) = \frac{1}{K+1}\frac{\Omega_p}{2} J_0(2\pi f_m \tau) + \frac{K}{K+1}\frac{\Omega_p}{2}\cos(2\pi f_m \tau \cos\theta_0)$$

$$\phi_{g_I g_Q}(\tau) = \frac{K}{K+1}\frac{\Omega_p}{2}\sin(2\pi f_m \tau \cos\theta_0) \quad .$$



Plot of $p(\theta)$ vs. θ with 2-D isotropic scattering plus a LoS or specular component arriving at angle $\theta_0 = \pi/2$.

- The azimuth distribution

$$p(\theta) = \frac{1}{K+1}\hat{p}(\theta) + \frac{K}{K+1}\delta(\theta - \theta_0)$$

yields a complex envelope having a Doppler spectrum of the form

$$S_{gg}(f) = \frac{1}{K+1}S_{gg}^{c}(f) + \frac{K}{K+1}S_{gg}^{d}(f) \tag{1}$$

where $S_{gg}^{d}(f)$ is the discrete portion of the Doppler spectrum due to the specular component and $S_{gg}^{c}(f)$ is the continuous portion of the Doppler spectrum due to the scatter component.

- For the case when $\hat{p}(\theta) = 1/(2\pi), \theta \in [-\pi, \pi]$, the power spectrum of $g(t) = g_I(t) + jg_Q(t)$ is

$$S_{gg}(f) = \begin{cases} \frac{1}{K+1} \cdot \frac{\Omega_p}{2\pi f_m}\frac{1}{\sqrt{1-(f/f_m)^2}} \\ \quad + \frac{K}{K+1}\frac{\Omega_p}{2}\delta(f - f_m\cos\theta_0) & 0 \leq |f| \leq f_m \\ 0 & \text{otherwise} \end{cases} .$$

- Note the discrete tone at frequency $f_c + f_m \cos\theta_0$ due to the line-of-sight or specular component arriving from angle θ_0.

Non-isotropic scattering – Other Cases

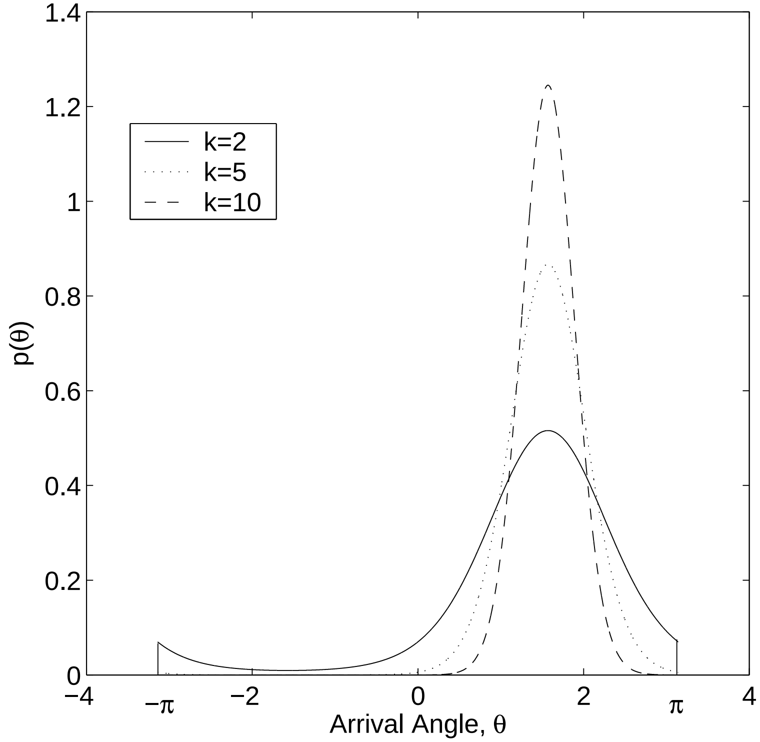
- Sometimes the azimuth distribution $p(\theta)$ may not be uniform, a condition commonly called non-isotropic scattering. Several distributions have been suggested to model non-isotropic scattering.
- Once possibility is the Gaussian distribution

$$p(\theta) = \frac{1}{\sqrt{2\pi}\sigma_S} \exp\left\{ -\frac{(\theta-\mu)^2}{2\sigma_S^2} \right\}$$

  where μ is the mean AoA, and σ_S is the rms AoA spread.
- Another possibility is the von Mises distribution

$$p(\theta) = \frac{1}{2\pi I_0(k)} \exp\left[k\cos(\theta-\mu) \right] \ ,$$

  where $\theta \in [-\pi, \pi)$, $I_0(\ \cdot\ )$ is the zeroth-order modified Bessel function of the first kind, $\mu \in [-\pi, \pi)$ is the mean AoA, and k controls the spread of scatterers around the mean.



Plot of $p(\theta)$ vs. θ for the von Mises distribution with a mean angle-to-arrival $\mu = \pi/2$.

Calculating the Doppler Spectrum

- The Doppler spectrum can be derived by using a different approach that is sometimes very useful because it can avoid the need to evaluate integrals. The Doppler spectrum can be expressed as

$$S_{gg}(f)|df| = \frac{\Omega_p}{2}(G(\theta)p(\theta) \ + \ G(-\theta)p(-\theta))|d\theta| \ .$$

- The Doppler frequency associated with the incident plane wave arriving at angle θ is

$$f = f_m \cos(\theta) \ ,$$

and, hence,

$$|df| = f_m| - \sin(\theta)d\theta| = \sqrt{f_m^2 - f^2} \ |d\theta| \ .$$

- Therefore,

$$S_{gg}(f) = \frac{\Omega_p/2}{\sqrt{f_m^2 - f^2}}(G(\theta)p(\theta) \ + \ G(-\theta)p(-\theta)) \ ,$$

where

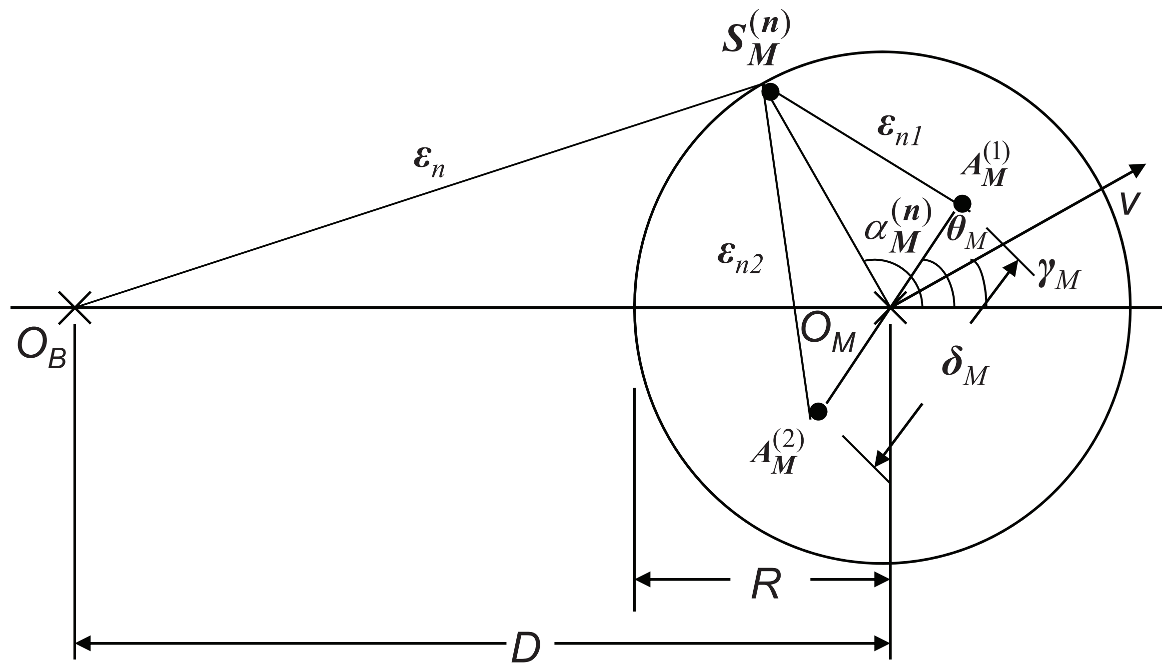
$$\theta = \cos^{-1}(f/f_m) \ .$$

- Hence, if $p(\theta)$ and $G(\theta)$ are known, the Doppler spectrum can be easily calculated. For example, with 2-D isotropic scattering and an isotropic antenna $G(\theta)p(\theta) = 1/(2\pi)$.

Space-time Correlation Functions

- Many mobile radio systems use antenna diversity, where spatially separated receiver antennas provide multiple faded replicas of the same information bearing signal.
- The spatial decorrelation of the channel tell us the required spatial separation between antenna elements so that they will be "sufficiently" decorrelated.
- Sometimes it is desirable to simultaneously characterize both the spatial and temporal correlation characteristics of the channel, e,g, when using space-time coding. This can be described by the space-time correlation function.
- To obtain the spatial or space-time correlation functions, we must specify some kind of radio scattering geometry.

Spatial Correlation at the Mobile Station



Single-ring scattering model for NLoS propagation on the forward link of a cellular system. The MS is surrounded by a scattering ring of radius R and is at distance D from the BS, where $R \ll D$.

Model Parameters

- O_B: base station location
- O_M: mobile station location
- D: LoS distance from base station to mobile station
- R: scattering radius
- γ_M: mobile station moving direction w.r.t x-axis
- v: mobile station speed
- θ_M mobile station array orientation w.r.t. x-axis
- $A_M^{(i)}$: location of ith mobile station antenna element
- δ_M: distance between mobile station antenna elements
- $S_M^{(n)}$: location of nth scatterer.
- $\alpha_M^{(n)}$: angle of arrival from the nth scatterer.
- ϵ_n: distance $O_B - S_M^{(n)}$.
- ϵ_{ni}: distance $S_M^{(n)} - A_M^{(i)}$.

Received Complex Envelope

- The channel from O_B to $A_M^{(q)}$ has the complex envelope

$$g_q(t) = \sum_{n=1}^{N} C_n e^{j\phi_n - j2\pi(\epsilon_n + \epsilon_{nq})/\lambda_c} e^{j2\pi f_m t \cos(\alpha_M^{(n)} - \gamma_M)}, \ q = 1, 2$$

  where ϵ_n and ϵ_{nq} denote the distances $O_B - S_M^{(n)}$ and $S_M^{(n)} - A_M^{(q)}$, $q = 1, 2$, respectively, and ϕ_n is a uniform random phase on the interval $[-\pi, \pi)$.

- From the Law of Cosines, the distances ϵ_n and ϵ_{nq} can be expressed as a function of the angle-of-arrival $\alpha_M^{(n)}$ as follows:

$$\begin{aligned} \epsilon_n^2 &= D^2 + R^2 + 2DR\cos\alpha_M^{(n)} \text{ Note sign change since the angle is } \pi - \alpha_M^{(n)} \\ \epsilon_{nq}^2 &= [(1.5 - q)\delta_M]^2 + R^2 - 2(1.5 - q)\delta_M R \cos(\alpha_M^{(n)} - \theta_M) \ , q = 1, 2 \ . \end{aligned}$$

- Assuming that $R/D \ll 1$ (local scattering), $\delta_M \ll R$ and $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small x, we have

$$\begin{aligned} \epsilon_n &\approx D + R\cos\alpha_M^{(n)} \\ \epsilon_{nq} &\approx R - (1.5 - q)\delta_M \cos(\alpha_M^{(n)} - \theta_M) \ , q = 1, 2 \ . \end{aligned}$$

- Hence,

$$\begin{aligned} g_q(t) &= \sum_{n=1}^{N} C_n e^{j\phi_n - j2\pi\left(D + R\cos\alpha_M^{(n)} + R - (1.5-q)\delta_M \cos(\alpha_M^{(n)} - \theta_M)\right)/\lambda_c} \\ &\quad \times e^{j2\pi f_m t \cos(\alpha_M^{(n)} - \gamma_M)} \ , q = 1, 2 \ . \end{aligned}$$

Space-time Correlation Function

- The space-time correlation function between the two complex faded envelopes $g_1(t)$ and $g_2(t)$ is

$$\phi_{g_1,g_2}(\delta_M,\tau) = \frac{1}{2}\mathrm{E}\left[g_1^*(t)g_2(t+\tau)\right]$$

- The space-time correlation function between $g_1(t)$ and $g_2(t)$ can be written as

$$\phi_{g_1,g_2}(\delta_M,\tau) = \frac{\Omega_p}{2N}\sum_{n=1}^{N}\mathrm{E}\Big[e^{j2\pi(\delta_M/\lambda_c)\cos(\alpha_M^{(n)}-\theta_M)}e^{-j2\pi f_m\tau\cos(\alpha_M^{(n)}-\gamma_M)}\Big] \ .$$

- Since the number of scatters is infinite, the discrete angles-of-arrival $\alpha_M^{(n)}$ can be replaced with a continuous random variable α_M with probability density function $p(\alpha_M)$.
- Hence, the space-time correlation function becomes

$$\phi_{g_1,g_2}(\delta_M,\tau) = \frac{\Omega_p}{2}\int_0^{2\pi} e^{jb\cos(\alpha_M-\theta_M)}e^{-ja\cos(\alpha_M-\gamma_M)}p(\alpha_M)d\alpha_M \ .$$

where $a = 2\pi f_m\tau$ and $b = 2\pi\delta_M/\lambda_c$.

2-D Isotropic Scattering

- For the case of 2-D isotropic scattering with an isotropic receive antennas, $p(\alpha_M) = 1/(2\pi)$, $-\pi \leq \alpha_M \leq \pi$, and the space-time correlation function becomes

$$\phi_{g_1,g_2}(\delta_M, \tau) = \frac{\Omega_p}{2} J_0\left(\sqrt{a^2 + b^2 - 2ab\cos(\theta_M - \gamma_M)}\right) \quad .$$

- The spatial and temporal correlation functions can be obtained by setting $\tau = 0$ and $\delta_M = 0$, respectively. This gives

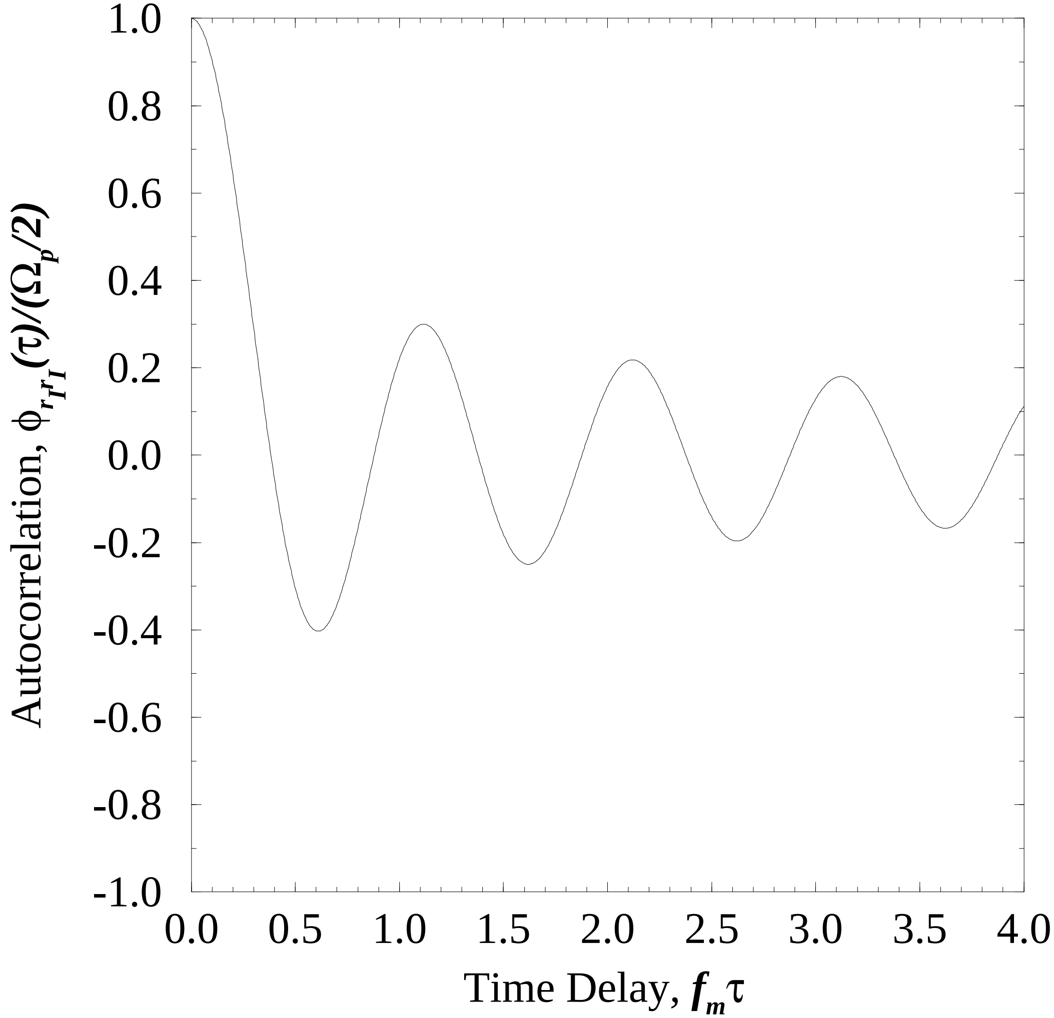
$$\begin{aligned} \phi_{g_1,g_2}(\delta_M) &= \phi_{g_1,g_2}(\delta_M, 0) = \frac{\Omega_p}{2} J_0(2\pi\delta_M/\lambda_c) \\ \phi_{gg}(\tau) &= \phi_{g_1,g_2}(0, \tau) = \frac{\Omega_p}{2} J_0(2\pi f_m \tau) \end{aligned}$$

- Finally, we note that

$$f_m\tau = \frac{v \cdot \tau}{\lambda_c} = \frac{\delta_M}{\lambda_c}$$

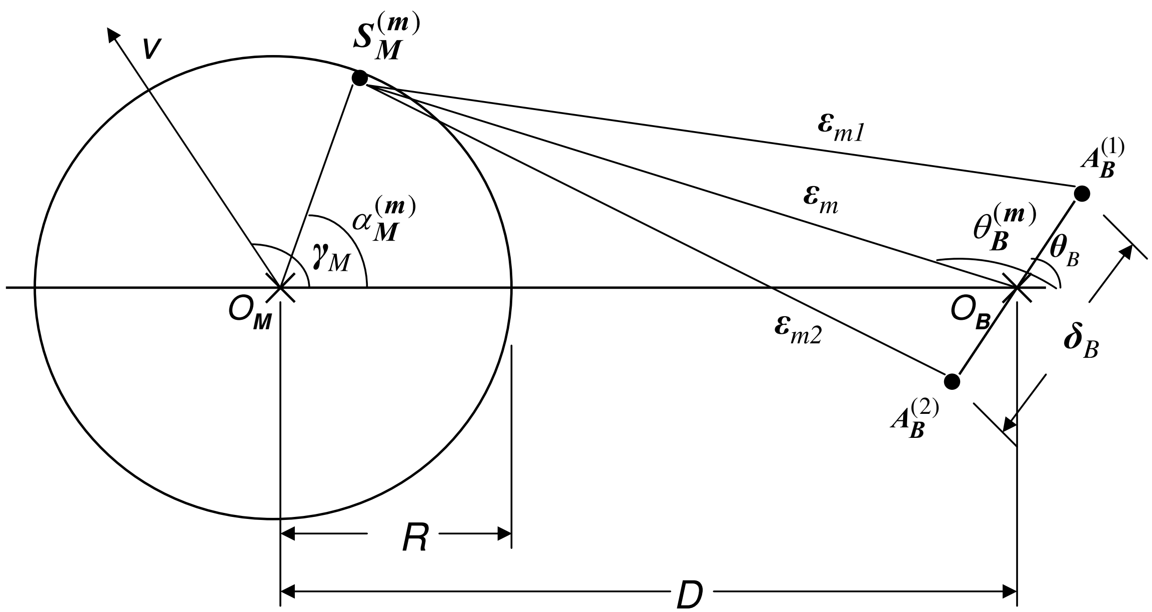
  For this scattering environment, the normalized time $f_m\tau$ is equivalent to the normalized distance δ_M/λ_c.

- The antenna branches are uncorrelated if they are separated by $\delta_M \approx 0.5\lambda_c$.



Temporal and spatial correlation functions at the MS with 2-D isotropic scattering and an isotropic receiver antenna. Note that $f_m\tau = \delta_M/\lambda_c$.

Spatial Correlation at the Base Station



Single-ring scattering model for NLoS propagation on the reverse link of a cellular system. The MS is surrounded by a scattering ring of radius R and is at distance D from the BS, where $R \ll D$.

Model Parameters

- O_B: base station location
- O_M: mobile station location
- D: LoS distance from base station to mobile station
- R: scattering radius
- γ_M: mobile station moving direction w.r.t x-axis
- v: mobile station speed
- θ_B base station array orientation w.r.t. x-axis
- $A_B^{(i)}$: location of ith base station antenna element
- δ_B: distance between mobile station antenna elements
- $S_M^{(m)}$: location of mth scatterer.
- $\alpha_M^{(m)}$: angle of departure to the nth scatterer.
- ϵ_m: distance $S_M^{(m)} - O_B$.
- ϵ_{mi}: distance $S_M^{(m)} - A_B^{(i)}$.

Received Complex Envelope

- The channel from O_M to $A_B^{(q)}$ has the complex envelope

$$g_q(t) = \sum_{m=1}^{N} C_m e^{j\phi_m - j2\pi(R+\epsilon_{mq})/\lambda_c} e^{j2\pi f_m t \cos(\alpha_M^{(m)} - \gamma_M)}, \ q = 1, 2 \tag{2}$$

  where ϵ_{mq} denote the distance $S_M^{(m)} - A_B^{(q)}$, $q = 1, 2$, and ϕ_m is a uniform random phase on $(-\pi, \pi]$. To proceed further, we need to express ϵ_{mq} as a function of $\alpha_M^{(m)}$.

- Applying the Law of Cosines to the triangle $\triangle S_M^{(m)} O_B A_B^{(q)}$, the distance ϵ_{mq} can be expressed as a function of the angle $\theta_B^{(m)} - \theta_B$ as follows:

$$\epsilon_{mq}^2 = [(1.5 - q)\delta_B]^2 + \epsilon_m^2 - 2(1.5 - q)\delta_B \epsilon_m \cos(\theta_B^{(m)} - \theta_B) \ , q = 1, 2 \ . \tag{3}$$

  where ϵ_m is the distance $S_M^{(m)} - O_B$.

- By applying the Law of Sines to the triangle $\triangle O_M S_M^{(m)} O_B$ we obtain following identity

$$\frac{\epsilon_m}{\sin \alpha_M^{(m)}} = \frac{R}{\sin\left(\pi - \theta_B^{(m)}\right)} = \frac{D}{\sin\left(\pi - \alpha_M^{(m)} - \left(\pi - \theta_B^{(m)}\right)\right)} \ .$$

Received Complex Envelope

- Since the angle $\pi - \theta_B^{(m)}$ is small, we can apply the small angle approximations $\sin x \approx x$ and $\cos x \approx 1$ for small x, to the second equality in the above identity. This gives

$$\frac{R}{(\pi - \theta_B^{(m)})} \approx \frac{D}{\sin\left(\pi - \alpha_M^{(m)}\right)}$$

or

$$(\pi - \theta_B^{(m)}) \approx (R/D)\sin(\pi - \alpha_M^{(m)}) \ .$$

- It follows that the cosine term in (2) becomes

$$\begin{aligned} \cos(\theta_B^{(m)} - \theta_B) &= \cos(\pi - \theta_B - (\pi - \theta_B^{(m)})) \\ &= \cos(\pi - \theta_B)\cos(\pi - \theta_B^{(m)}) + \sin(\pi - \theta_B)\sin(\pi - \theta_B^{(m)}) \\ &\approx \cos(\pi - \theta_B) + \sin(\pi - \theta_B)(R/D)\sin(\pi - \alpha_M^{(m)}) \\ &= -\cos(\theta_B) + (R/D)\sin(\theta_B)\sin(\alpha_M^{(m)}) \end{aligned} \tag{4}$$

- Using the approximation in (4) in (2), along with $\delta_B/\epsilon_m \ll 1$, gives

$$\epsilon_{mq}^2 \approx \epsilon_m^2 \left[1 - 2(1.5 - q)\frac{\delta_B}{\epsilon_m}\left[(R/D)\sin(\theta_B)\sin(\alpha_M^{(m)}) - \cos(\theta_B)\right]\right] \ .$$

Received Complex Envelope

- Applying the approximation $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small x, we have

$$\epsilon_{mq} \approx \epsilon_m - (1.5 - q)\delta_B \left[(R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] \quad . \tag{5}$$

- Applying the Law of Cosines to the triangle $\triangle O_M S_M^{(m)} O_B$ we have

$$\begin{aligned} \epsilon_m^2 &= D^2 + R^2 - 2DR\cos(\alpha_M^{(m)}) \\ &\approx D^2 \left[1 - 2(R/D)\cos(\alpha_M^{(m)}) \right] \quad , \end{aligned}$$

  and again using the approximation $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small x, we have

$$\epsilon_m \approx D - R\cos(\alpha_M^{(m)}) \tag{6}$$

- Finally, using (5) in (4) gives

$$\epsilon_{mq} \approx D - R\cos(\alpha_M^{(m)}) - (1.5 - q)\delta_B \left[(R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] \quad . \tag{7}$$

- Substituting (7) into (2) gives the result

$$\begin{aligned} g_q(t) &= \sum_{m=1}^{N} C_m e^{j\phi_m + j2\pi f_m t \cos(\alpha_M^{(m)} - \gamma_M)} \\ &\quad \times e^{-j2\pi \left(R + D - R\cos(\alpha_M^{(m)}) - (1.5-q)\delta_B \left[(R/D)\sin(\theta_B)\sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] \right) / \lambda_c}, \end{aligned} \tag{8}$$

  which no longer depends on the ϵ_{mq} and is a function of the angle of departure $\alpha_M^{(m)}$.

Space-time Correlation Function

- The space-time correlation function between the two complex faded envelopes $g_1(t)$ and $g_2(t)$ at the BS is once again given by

$$\phi_{g_1,g_2}(\delta_B,\tau) = \frac{1}{2}\mathrm{E}\left[g_1^*(t)g_2(t+\tau)\right]$$

Using (8), the space-time correlation function between $g_1(t)$ and $g_2(t)$ can be written as

$$\begin{aligned}\phi_{g_1,g_2}(\delta_B,\tau) &= \frac{\Omega_p}{2N}\sum_{m=1}^{N}\mathrm{E}\Big[e^{j2\pi(\delta_B/\lambda_c)\left[(R/D)\sin(\theta_B)\sin(\alpha_M^{(m)})-\cos(\theta_B)\right]} \\ &\quad \times e^{-j2\pi f_m\tau\cos(\alpha_M^{(m)}-\gamma_M)}\Big] \ .\end{aligned}$$

- Since the number of scatters around the MS is infinite, the discrete angles-of-departure $\alpha_M^{(m)}$ can be replaced with a continuous random variable α_M with probability density function $p(\alpha_M)$.

- Hence, the space-time correlation function becomes.

$$\phi_{g_1,g_2}(\delta_B,\tau) = \frac{\Omega_p}{2}\int_{-\pi}^{\pi} e^{-ja\cos(\alpha_M-\gamma_M)}e^{jb[(R/D)\sin(\theta_B)\sin(\alpha_M)-\cos(\theta_B)]}p(\alpha_M)d\alpha_M \ ,$$

where $a = 2\pi f_m\tau$ and $b = 2\pi\delta_B/\lambda_c$.

2-D Isotropic Scattering

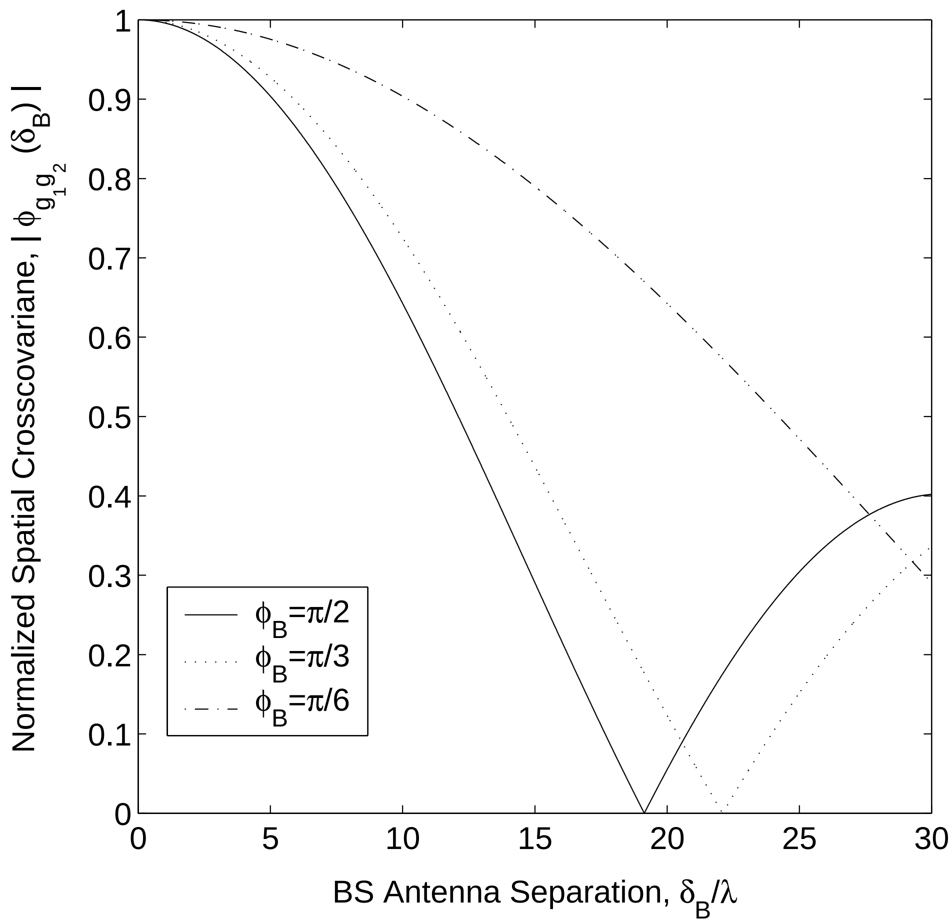
- For the case of 2-D isotropic scattering with an isotropic MS transmit antenna, $p(\alpha_M) = 1/(2\pi), -\pi \leq \alpha_M \leq \pi$, and the space-time correlation function becomes

$$\begin{aligned}\phi_{g_1,g_2}(\delta_B,\tau) &= \frac{\Omega_p}{2}e^{-jb\cos(\theta_B)} \\ &\times J_0\Big(\sqrt{a^2+b^2(R/D)^2\sin^2(\theta_B)-2ab(R/D)\sin(\theta_B)\sin(\gamma_M)}\Big) \ .\end{aligned}$$

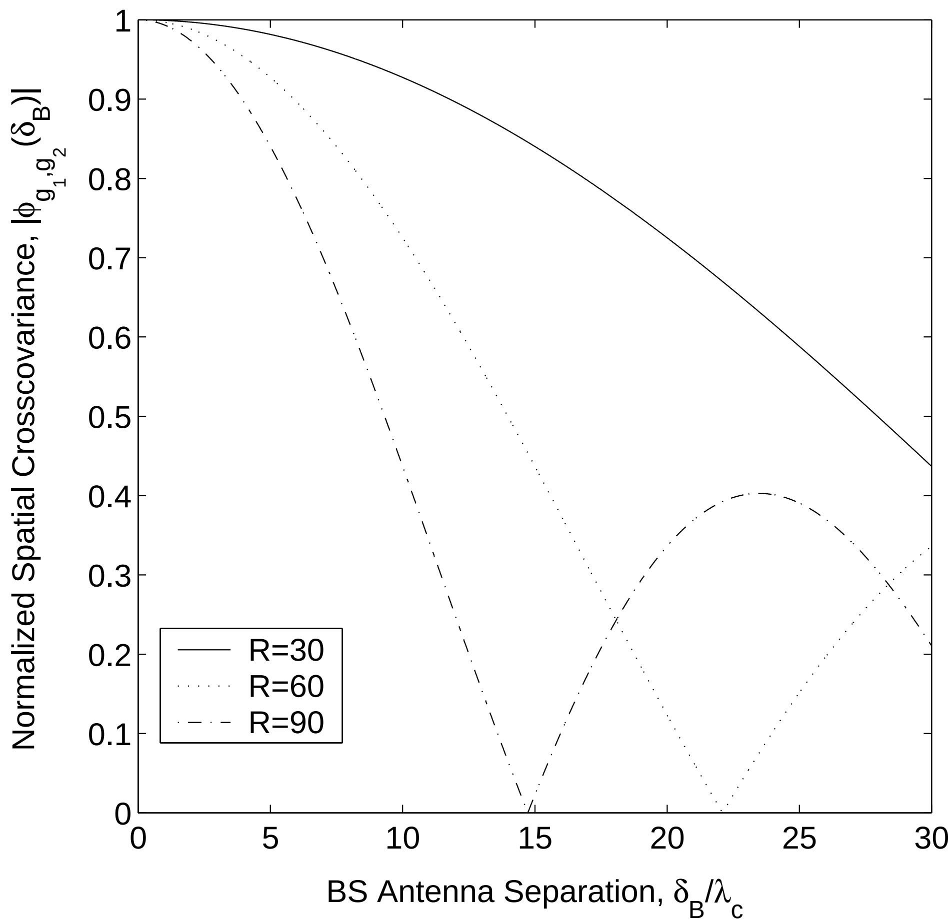
- The spatial and temporal correlation functions can be obtained by setting $\tau = 0$ and and $\delta_B = 0$, respectively.
- The temporal correlation function $\phi_{gg}(\tau) = \phi_{g_1,g_2}(0,\tau) = \frac{\Omega_p}{2}J_0(2\pi f_m \tau)$ which matches our result for the received signal at a mobile station.
- The spatial correlation function is

$$\begin{aligned}\phi_{g_1,g_2}(\delta_B) &= \phi_{g_1,g_2}(\delta_B,0) = \frac{\Omega_p}{2}e^{-jb\cos(\theta_B)}J_0(b(R/D)\sin(\theta_B)) \\ &= \frac{\Omega_p}{2}e^{-jb\cos(\theta_B)}J_0\Big((2\pi\delta_B/\lambda_c)(R/D)\sin(\theta_B)\Big)\end{aligned}$$

- Observe that a much greater spatial separation is required to achieve a given degree of envelope decorrelation at the BS as compared to the MS. This can be readily seen by the term $R/D \ll 1$ in the argument of the Bessel function.



Envelope crosscorrelation magnitude at the base station for different base station antenna orientation angles, θ_B; $D = 3000$ m, $R = 60$ m. Broadside base station antennas have the lowest crosscorrelation.



Envelope crosscorrelation magnitude at the base station for $\theta_B = \pi/3$ and various scattering radii, R; $D = 3000$ m. Smaller scattering radii will result in larger a crosscorrelations.