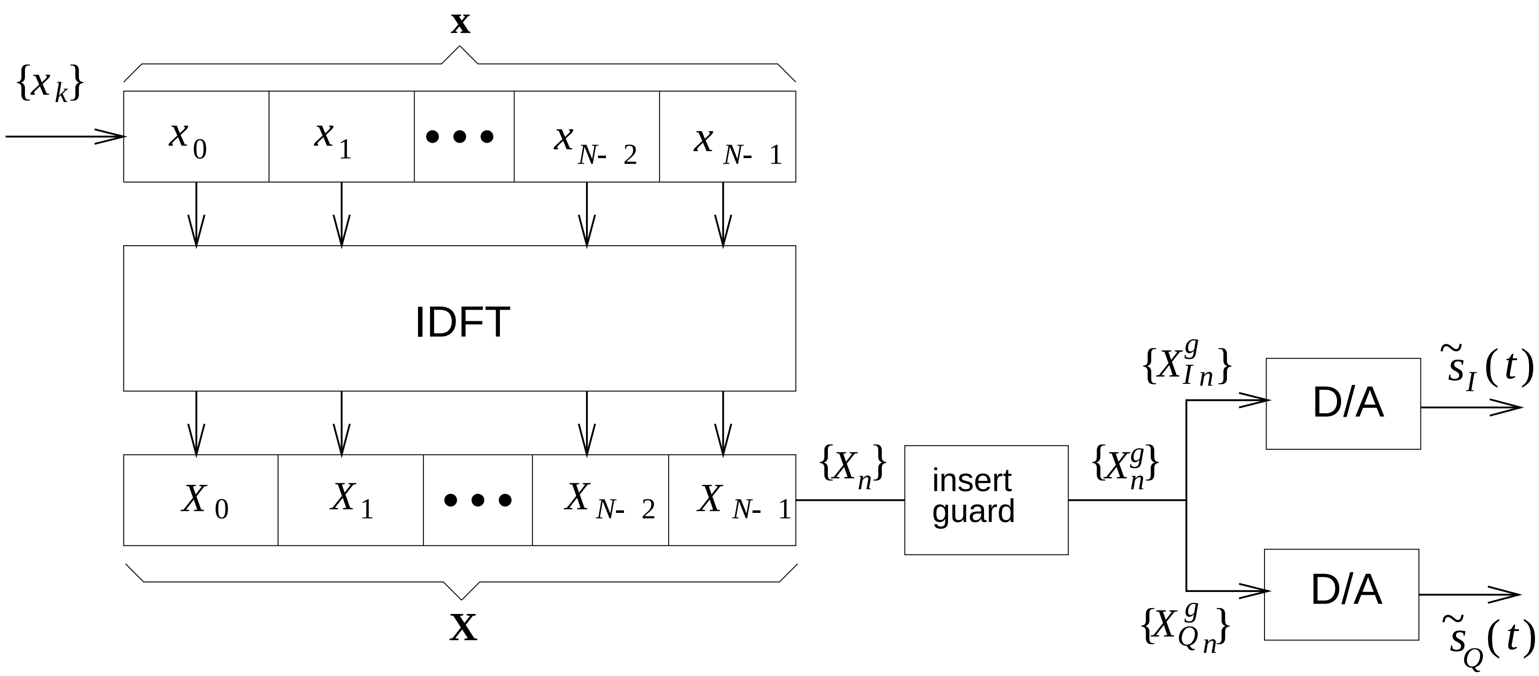
EE6604

Personal & Mobile Communications

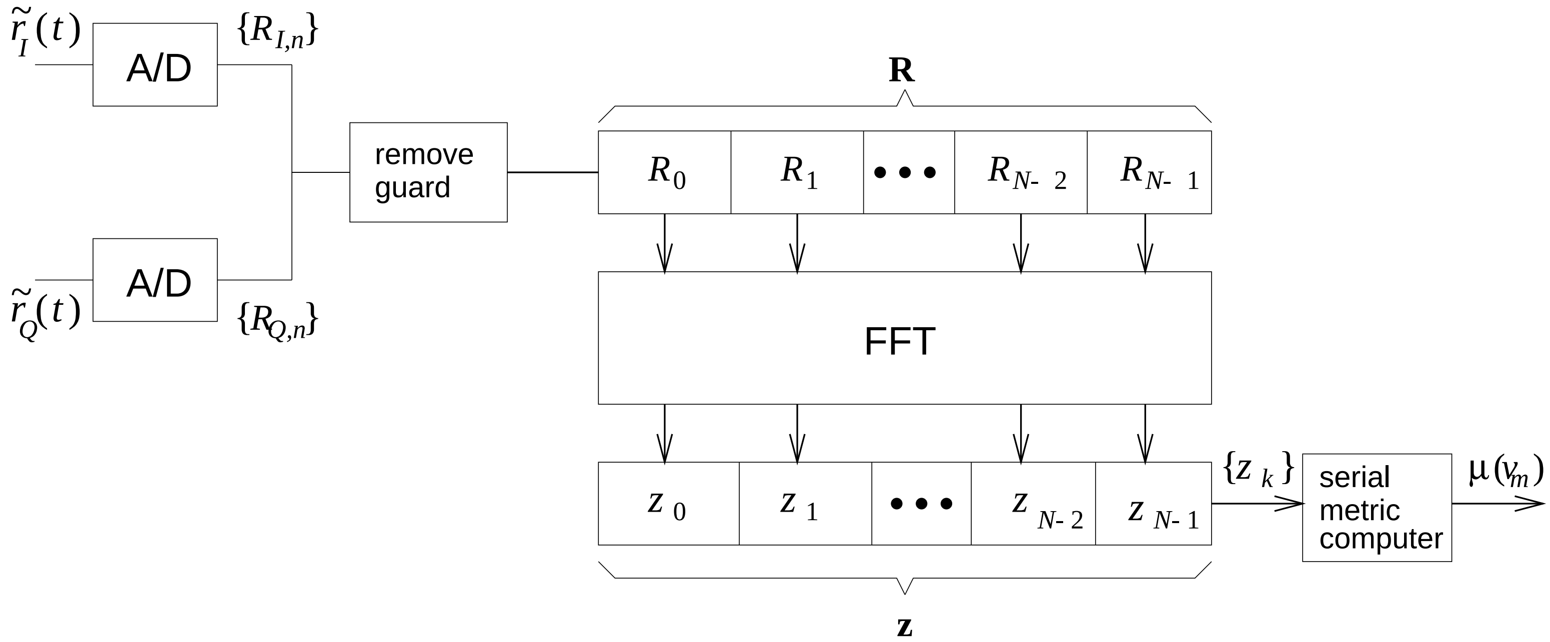
Week 12b

OFDM on AWGN and ISI Channels

Reading: 10.1



FFT-based OFDM Transmitter



FFT-based OFDM Receiver

Performance in AWGN

- Suppose that the discrete-time OFDM time-domain sequence with a cyclic suffix, $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=0}^{N+G-1}$, is passed through a balanced pair of digital-to-analog converters (DACs), and the resulting complex envelope is transmitted over a quasi-static flat fading channel with complex gain g.

- The receiver uses a quadrature demodulator to extract the received complex envelope $\tilde{r}(t) = \tilde{r}_I(t) + j\tilde{r}_Q(t)$.

- Suppose that the quadrature components $\tilde{r}_I(t)$ and $\tilde{r}_Q(t)$ are each passed through an ideal anti-aliasing filter (ideal low-pass filter) having a cutoff frequency $1/(2T_s^g)$ followed by an analog-to-digital converter (ADC)

- This produces the received complex-valued sample sequence $\mathbf{R}_n^g = \{R_{n,m}^g\}_{m=0}^{N+G-1}$, where

$$R_{n,m}^g = gX_{n,m}^g + \tilde{n}_{n,m} \ ,$$

  $g = \alpha \mathrm{e}^{j\phi}$ is the complex channel gain, and the $\tilde{n}_{n,m}$ are the complex-valued Gaussian noise samples.

- For an ideal anti-aliasing filter having a cutoff frequency $1/(2T_s^g)$, the $\tilde{n}_{n,m}$ are independent zero-mean complex Gaussian random variables with variance $\sigma^2 = \frac{1}{2}\mathrm{E}[|\tilde{n}_{n,m}|^2] = N_o/T_s^g$, where $T_s^g = NT_s/(N+G)$.

Performance in AWGN

- Assuming a cyclic suffix, the receiver first removes the guard interval according to

$$R_{n,m} = R^{g}_{n,G+(m-G)_N}, \; 0 \leq m \leq N-1 \; ,$$

where $(m)_N$ is the residue of m modulo N. Demodulation is then performed by computing the FFT on the block $\mathbf{R}_n = \{R_{n,m}\}_{m=0}^{N-1}$ to yield the vector $\mathbf{z}_n = \{z_{n,k}\}_{k=0}^{N-1}$ of N decision variables

$$\begin{aligned} z_{n,k} &= \frac{1}{N}\sum_{m=0}^{N-1} R_{n,m} \mathrm{e}^{-\frac{j2\pi km}{N}} \\ &= gAx_{n,k} + \nu_{n,k} \; , \quad k = 0, \ldots, N-1 \; , \end{aligned}$$

where $A = \sqrt{2E_h/T}$, $T = (N+G)T_s^g$, and the noise terms are given by

$$\nu_{n,k} = \frac{1}{N}\sum_{m=0}^{N-1} \tilde{n}_{n,m} \mathrm{e}^{-\frac{j2\pi km}{N}} \; , \quad k = 0, \ldots, N-1 \; .$$

- It can be shown that the $\nu_{n,k}$ are zero mean complex Gaussian random variables with covariance

$$\phi_{j,k} = \frac{1}{2}\mathrm{E}[\nu_{n,j}\nu^{*}_{n,k}] = \frac{N_o}{NT_s^g}\delta_{jk} \; .$$

Hence, the $z_{n,k}$ are independent Gaussian random variables with mean $g\sqrt{2E_h/T}x_{n,k}$ and variance N_o/NT_s^g.

Performance in AWGN

- To be consistent with our earlier results for PSK and QAM signals, we can multiply the $z_{n,k}$ for convenience by the scalar $\sqrt{NT_s^g}$. Such scaling gives

$$\tilde{z}_{n,k} = g\sqrt{2E_h N/(N+G)}x_{n,k} + \tilde{\nu}_{n,k} \ ,$$

  where the $\tilde{\nu}_{n,k}$ are i.i.d. zero-mean Gaussian random variables with variance N_o.

- Notice that $\sqrt{2E_h N/(N+G)}x_{n,k} = \tilde{s}_{n,k}$ is equal to the complex signal vector that is transmitted on the ith sub-carrier, where the term $N/(N+G)$ represents the loss in effective symbol energy due to the insertion of the cyclic guard interval.

- For *each* of the $\tilde{z}_{n,k}$, the receiver decides in favor of the *signal vector* $\tilde{s}_{n,k}$ that minimizes the squared Euclidean distance

$$\mu(\tilde{s}_{n,k}) = \|\tilde{z}_{n,k} - g\tilde{s}_{n,k}\|^2 \ , \quad k = 0, \ldots, N-1 \ .$$

- It is apparent that the probability of symbol error is identical to that achieved with independent modulation on each of the sub-carriers. This is expected, because the sub-carriers are mutually orthogonal in time.

Combating ISI with OFDM

- Suppose that the IFFT output vector $\mathbf{X}_n = \{X_{n,m}\}_{m=0}^{N-1}$ is appended with a cyclic suffix to yield the vector $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=0}^{N+G-1}$, where

$$\begin{aligned} X_{n,m}^g &= X_{n,(m)_N} \\ &= A \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{\frac{j2\pi km}{N}} \ , \quad m = 0,\ 1,\ \ldots,\ N+G-1 \ , \end{aligned}$$

  G is the length of the guard interval in samples, and $(m)_N$ is the residue of m modulo N. To maintain the data rate $R_s = 1/T_s$, the DAC in the transmitter is clocked with rate $R_s^g = \frac{N+G}{N} R_s$, due to the insertion of the cyclic guard interval.

- Consider a time-invariant ISI channel with impulse response $g(t)$. The combination of the DAC, waveform channel $g(t)$, anti aliasing filter, and DAC yields an overall discrete-time channel with sampled impulse response $\mathbf{g} = \{g_m\}_{m=0}^{L}$, where L is the length of the discrete-time channel impulse response.

- The discrete-time linear convolution of the transmitted sequence $\{\mathbf{X}_n^g\}$ with the discrete-time channel produces the discrete-time received sequence $\{R_{n,m}^g\}$, where

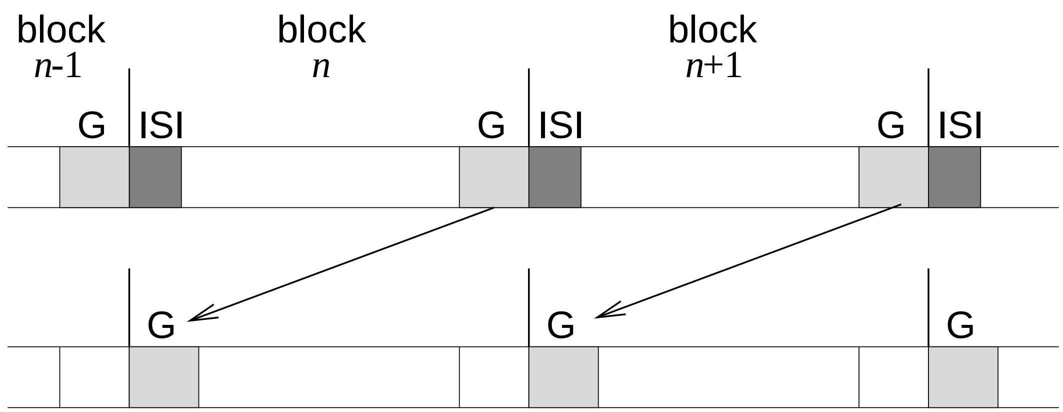
$$R_{n,m}^g = \begin{cases} \sum_{i=0}^{m} g_i X_{n,m-i}^g + \sum_{i=m+1}^{L} g_i X_{n-1,N+G+m-i}^g + \tilde{n}_{n,m} \ , & 0 \leq m < L \\ \sum_{i=0}^{L} g_i X_{n,m-i}^g + \tilde{n}_{n,m} \ , & L \leq m \leq N+G-1 \end{cases} .$$

Removal of Guard Interval

- To remove the ISI introduced by the channel, the first G received samples $\{R^g_{n,m}\}_{m=0}^{G-1}$ are discarded and replaced with the last G received samples $\{R^g_{n,m}\}_{m=N}^{N+G-1}$.
- If the length of the guard interval satisfies $G \geq L$, then we obtain the received sequence

$$
\begin{aligned}
R_{n,m} &= R^g_{n,G+(m-G)_N} \\
&= \sum_{i=0}^{L} g_i X_{n,(m-i)_N} + \tilde{n}_{n,(m-i)_N} \ , \quad 0 \leq m \leq N-1 \ .
\end{aligned}
$$

- Note that the first term represents a circular convolution of the transmitted sequence $\mathbf{X}_n = \{X_{n,m}\}$ with the discrete-time channel $\mathbf{g} = \{g_m\}_{m=0}^{L}$.



Removal of ISI using a cyclic suffix

- The OFDM baseband demodulator computes the DFT of the vector $\mathbf{R}_n$. This yields the output vector

$$\begin{aligned} z_{n,i} &= \frac{1}{N}\sum_{m=0}^{N-1} R_{n,m}\mathrm{e}^{-j\frac{2\pi mi}{N}} \\ &= T_i A x_{n,i} + \nu_{n,i} \ , \quad 0 \leq i \leq N-1 \ , \end{aligned}$$

where

$$T_i = \sum_{m=0}^{L} g_m \mathrm{e}^{-j\frac{2\pi mi}{N}}$$

and the noise samples $\{\nu_{n,i}\}$ are i.i.d with zero-mean and variance $N_o/(NT_s^g)$.

- Note that $\mathbf{T} = \{T_i\}_{i=0}^{N-1}$ is the DFT of the zero padded sequence $\mathbf{g} = \{g_m\}_{m=0}^{N-1}$ and is equal to the sampled frequency response of the channel.

- To be consistent with our earlier results, we can multiply the $z_{n,i}$ for convenience by the scalar $\sqrt{NT_s^g}$, giving

$$\tilde{z}_{n,i} = T_i \hat{A} x_{n,i} + \tilde{\nu}_{n,i} \quad i = 0, \ldots, N-1 \ ,$$

where $\hat{A} = \sqrt{2E_h N/(N+G)}$ and the $\tilde{\nu}_{n,i}$ are i.i.d. zero-mean Gaussian random variables with variance N_o.

- Observe that each $\tilde{z}_{n,i}$ depends only on the corresponding data symbol $x_{n,i}$ and, therefore, the ISI has been completely removed.

- Once again, for *each* of the $\tilde{z}_{n,i}$, the receiver decides in favor of the *signal vector* $\tilde{s}_m$ that minimizes the squared Euclidean distance

$$\mu(\tilde{s}_m) = \|\tilde{z}_{n,i} - T_i \hat{A} x_{n,i}\|^2 \ .$$