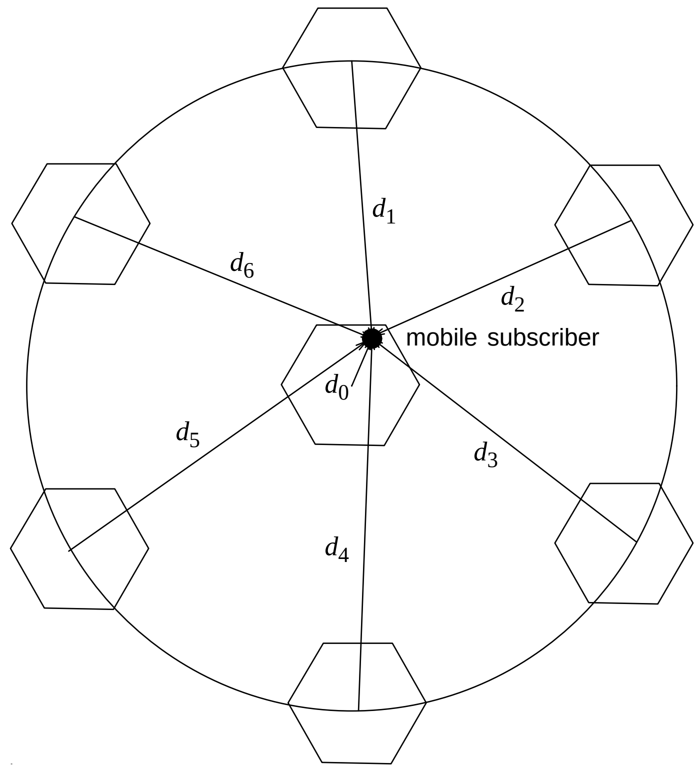
EE6604
Personal & Mobile Communications

Week 9

Co-Channel Interference

Co-channel interference on the forward channel



The mobile station is being served by the center base station.

- **At a particular location, let $\mathbf{d} = (d_0,\ d_1,\ \cdots,\ d_{N_I})$ be the vector of distances between a mobile station and the serving base station BS_0 and N_I co-channel base stations $\mathrm{BS}_k, k = 1, \ldots, N_I$.**

- **The received signal power power at distance d, $\Omega_{p\ (\mathrm{dBm})}(d)$, is a Gaussian random variable that depends on the distance d through the path loss model, i.e.,**

$$\mu_{\Omega_{p\ (\mathrm{dBm})}}(d) = \mathrm{E}[\Omega_{p\ (\mathrm{dBm})}(d)] = \mu_{\Omega_{p\ (\mathrm{dBm})}}(d_o) - 10\beta \log_{10}(d/d_o)$$

- **Experiments have verified that co-channel interferers add noncoherently (power addition) rather than coherently (amplitude addition).**

- **The C/I a function of the vector d is**

$$\Lambda(\mathbf{d}) = \frac{\Omega_p(d_0)}{\sum_{k=1}^{N_I} \Omega_p(d_k)}$$

or in decibel units

$$\Lambda(\mathbf{d})_{(\mathrm{dB})} = \Omega_{p\ (\mathrm{dBm})}(d_0) - 10 \log_{10} \left(\sum_{k=1}^{N_I} \Omega_p(d_k) \right)$$

- **The outage probability given vector d is**

$$O(\mathbf{d}) = \mathrm{P_r} \left(\Lambda(\mathbf{d})_{(\mathrm{dB})} < \Lambda_{\mathrm{th(dB)}} \right)$$

- **Although the $\Omega_p(d_k)$ are log-normal random variables, the sum $\sum_{k=1}^{N_I} \Omega_p(d_k)$ is *not* a log-normal random variable.**

Multiple Log-normal Interferers

- **Consider the sum of N_I log-normal random variables**

$$I = \sum_{k=1}^{N_I} \Omega_k = \sum_{k=1}^{N_I} 10^{\Omega_{k(\text{dBm})}/10}$$

where the $\Omega_{k\ (\text{dBm})}$ are independent Gaussian random variables with mean $\mu_{\Omega_k\ (\text{dBm})}$ and variance $\sigma^2_{\Omega_k}$.

- **The sum I is commonly approximated by another log-normal random variable with appropriately chosen parameters, i.e.,**

$$I = \sum_{k=1}^{N_I} 10^{\Omega_{k(\text{dBm})}/10} \approx 10^{Z_{(\text{dBm})}/10} = \hat{I}$$

where $Z_{(\text{dBm})}$ is a Gaussian random variable with mean $\mu_{Z\ (\text{dBm})}$ and variance σ^2_Z.

- **The task is to find $\mu_{Z\ (\text{dBm})}$ and σ^2_Z.**

Fenton-Wilkinson Method

- **The mean $\mu_{Z\ (\mathrm{dBm})}$ and variance σ_Z^2 of $Z_{(\mathrm{dBm})}$ are obtained by matching the first two moments of I and $\hat{I}$.**
- **Switching from base 10 to base e:**

$$\Omega_k = 10^{\Omega_{k\ (\mathrm{dBm})}/10} = e^{\xi \Omega_{k\ (\mathrm{dBm})}} = e^{\hat{\Omega}_k}$$

where $\hat{\Omega}_k = \xi \Omega_{k\ (\mathrm{dBm})}$ and $\xi = (\ln 10)/10 = 0.23026$.

- **Note that**

$$\begin{aligned} \mu_{\hat{\Omega}_k} &= \xi \mu_{\Omega_{k\ (\mathrm{dBm})}} \\ \sigma^2_{\hat{\Omega}_k} &= \xi^2 \sigma^2_{\Omega_k} \end{aligned}$$

- **The nth moment of the log-normal random variable Ω_k can be obtained from the moment generating function of the Gaussian random variable $\hat{\Omega}_k$ as**

$$\mathrm{E}[\Omega_k^n] = \mathrm{E}[e^{n\hat{\Omega}_k}] = e^{n\mu_{\hat{\Omega}_k} + (1/2)n^2\sigma^2_{\hat{\Omega}}}$$

  - **Here we have assumed identical shadow variances, $\sigma^2_{\hat{\Omega}_k} = \sigma^2_{\hat{\Omega}}$, which is a reasonable assumption.**

- **Suppose that $\hat{\Omega}_1, \ldots, \hat{\Omega}_{N_I}$ are independent with means $\mu_{\hat{\Omega}_1}, \ldots, \mu_{\hat{\Omega}_{N_I}}$ and identical variances $\sigma^2_{\hat{\Omega}}$.**
- **The appropriate moments of the log-normal approximation are obtained by equating the means on both sides of**

$$\mu_I = \mathrm{E}[I] = \sum_{k=1}^{N_I} \mathrm{E}[e^{\hat{\Omega}_k}] \approx \mathrm{E}[e^{\hat{Z}}] = \mathrm{E}[\hat{I}] = \mu_{\hat{I}}$$

where $\hat{Z} = \xi Z_{(\mathrm{dBm})}$.

- **This gives**

$$\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right) e^{(1/2)\sigma^2_{\hat{\Omega}}} = e^{\mu_{\hat{Z}}+(1/2)\sigma^2_{\hat{Z}}} \tag{1}$$

- **Also equate the variances on both sides of**

$$\sigma_I^2 = \mathrm{E}[I^2] - \mu_I^2 \approx \mathrm{E}[\hat{I}^2] - \mu_{\hat{I}}^2 = \sigma_{\hat{I}}^2$$

- **This gives**

$$\left(\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}\right) e^{\sigma^2_{\hat{\Omega}}}(e^{\sigma^2_{\hat{\Omega}}} - 1) = e^{2\mu_{\hat{Z}}} e^{\sigma^2_{\hat{Z}}}(e^{\sigma^2_{\hat{Z}}} - 1) \tag{2}$$

- **To obtain $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$**
  1. **Square Eq. (1) and divide by Eq. (2) to obtain $\sigma^2_{\hat{Z}}$.**
  2. **Obtain $\mu_{\hat{Z}}$ from Eq. (1)**
- **The above procedure yields**

$$\sigma^2_{\hat{Z}} = \ln\left((e^{\sigma^2_{\hat{\Omega}}} - 1) \frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)^2} + 1 \right)$$

$$\mu_{\hat{Z}} = \frac{\sigma^2_{\hat{\Omega}} - \sigma^2_{\hat{Z}}}{2} + \ln\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)$$

- **Given the means $\mu_{\hat{\Omega}_1}, \ldots, \mu_{\hat{\Omega}_{N_I}}$ and variance $\sigma^2_{\hat{\Omega}}$, $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$ are easily obtained.**
- **Finally, we convert back to base 10 by scaling, such that**

$$\mu_{Z\ (\mathrm{dBm})} = \xi^{-1}\mu_{\hat{Z}}$$
$$\sigma^2_Z = \xi^{-2}\sigma^2_{\hat{Z}}$$

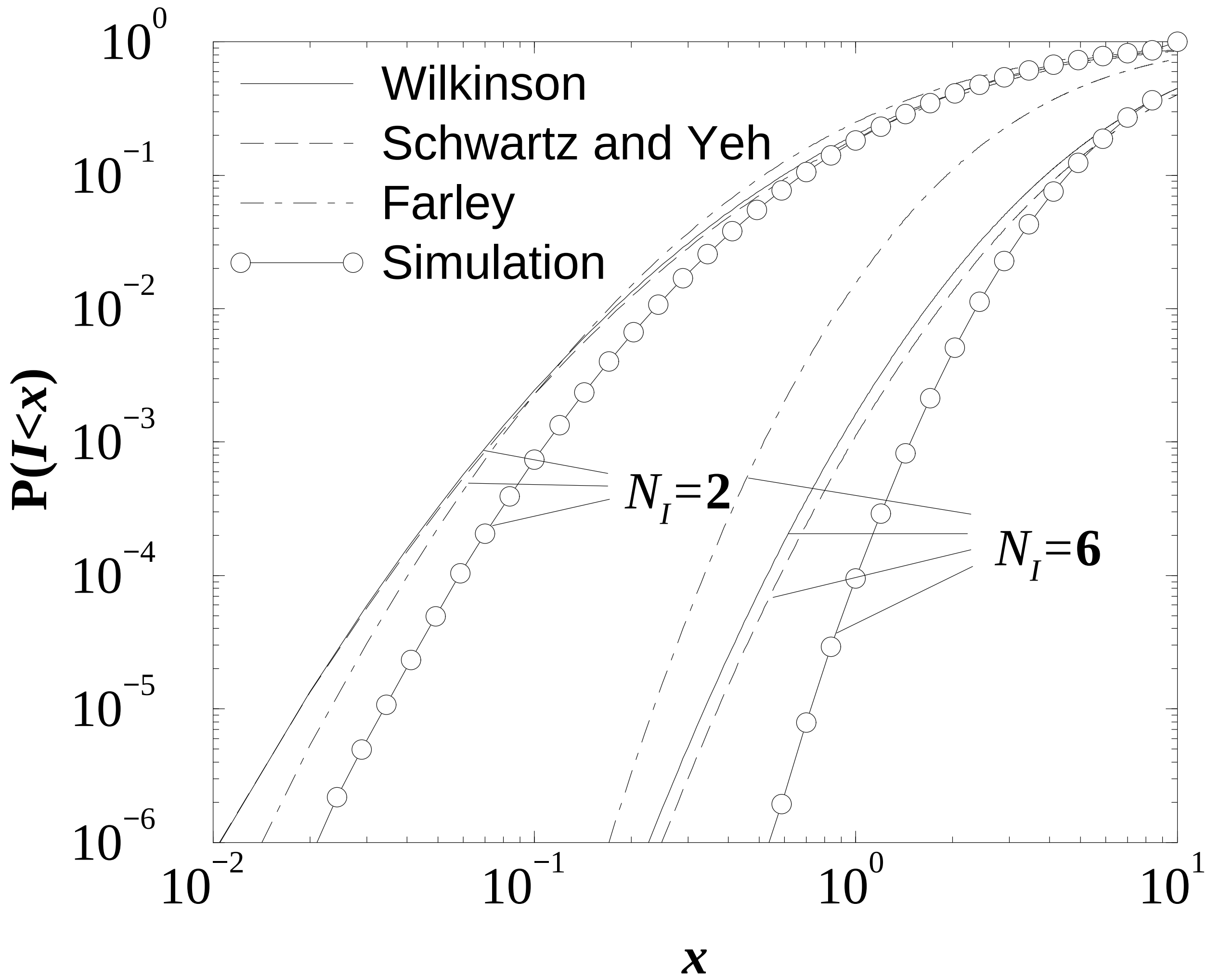
where $\xi = 0.23026$.

- **Fenton's method breaks down in the prediction of the first and second moments for $\sigma_\Omega > 4$ dB.**
  - **Schwartz and Yeh's method yields the exact first and second moments.**
- **However, Fenton's method accurately predicts the tails of the complementary distribution function** *cdfc* $F_I^c(x) = \mathrm{P_r}(I \geq x)$ **and the** *cdf* $F_I(x) = 1 - F_I^c(x) = \mathrm{P_r}(I < x)$**.**
  - **We are interested in the accuracy of the approximations**

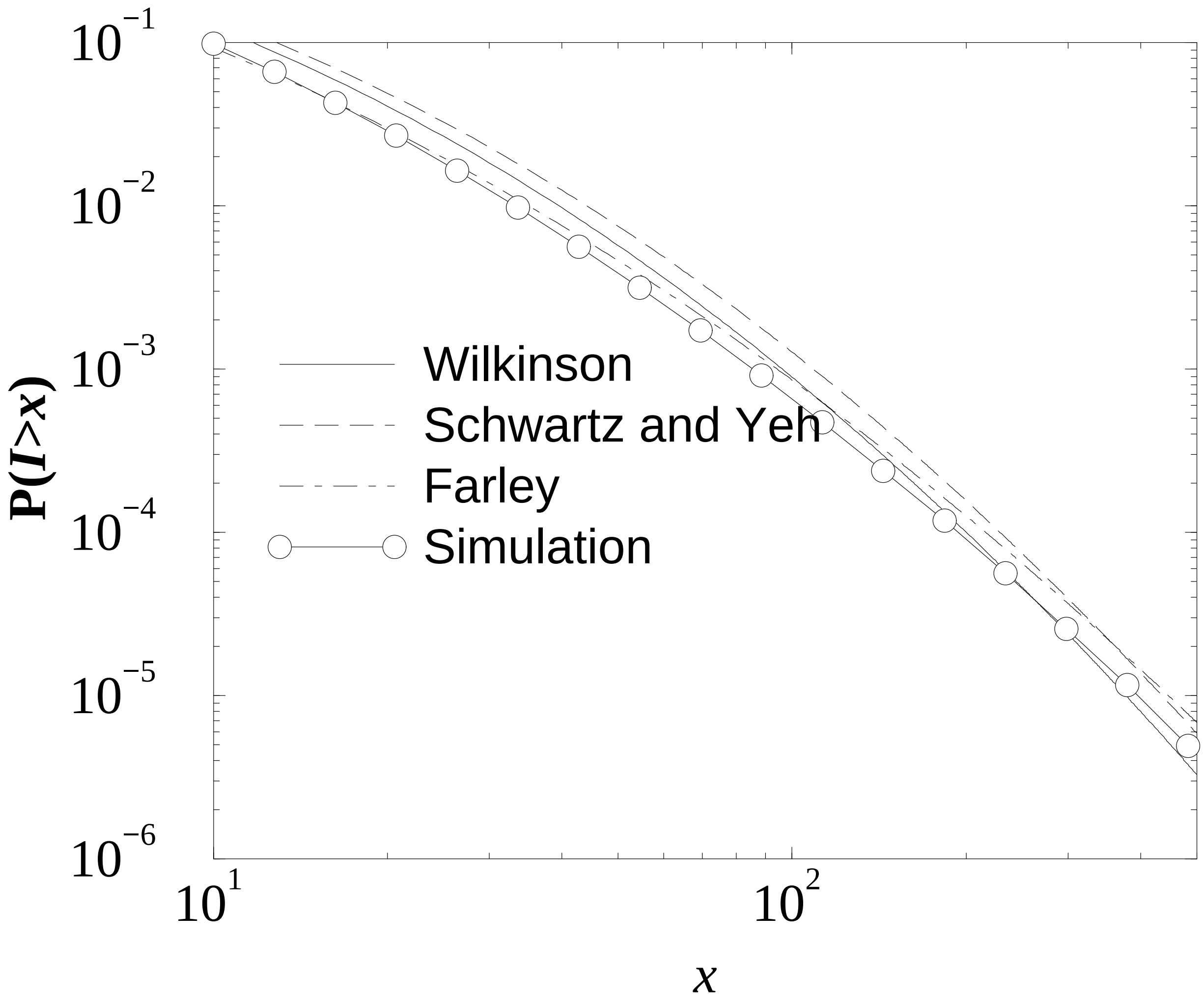
$$F_I^c(x) \approx Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right)$$

$$F_I(x) \approx 1 - Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right)$$

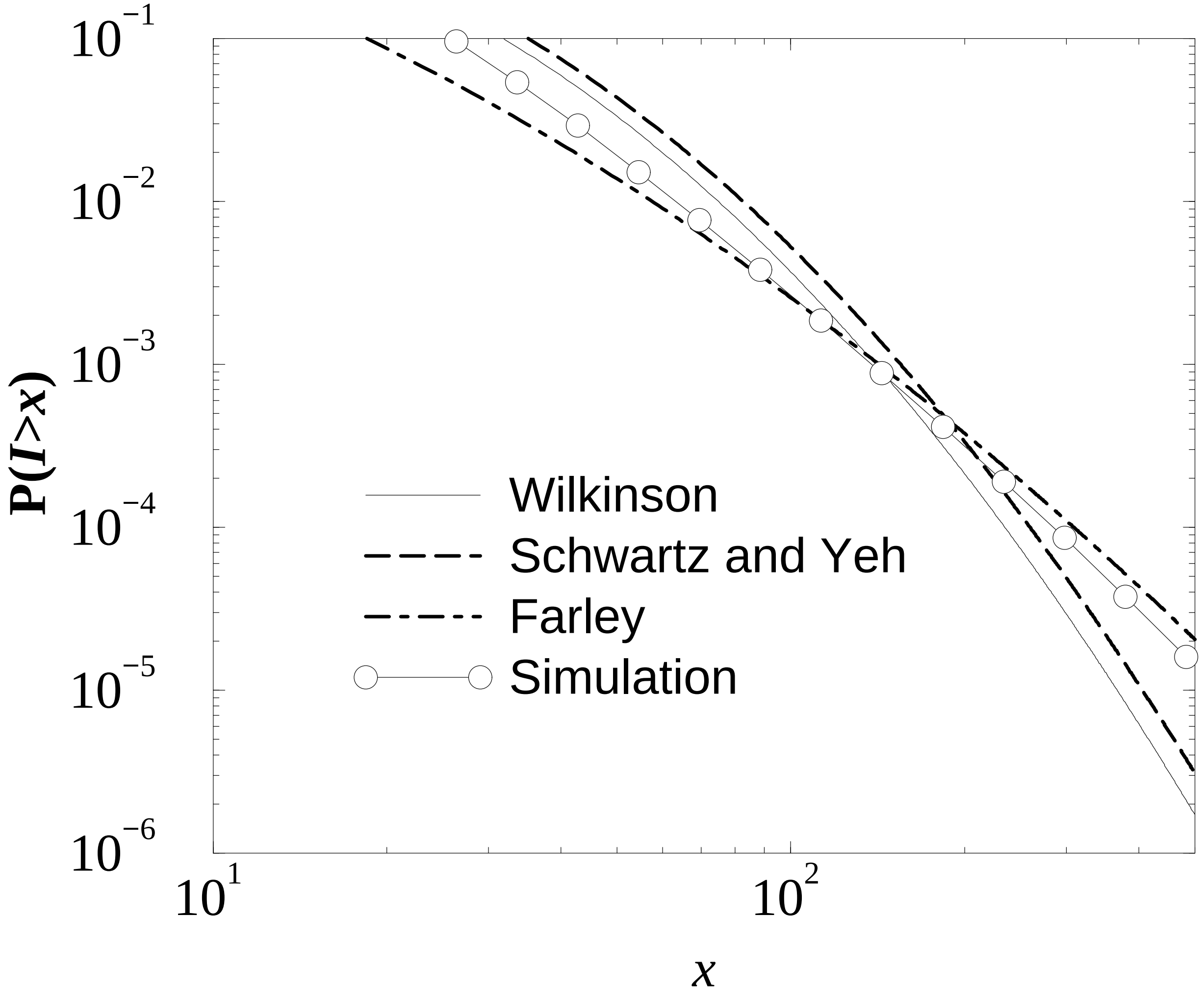
  when x is large and small, respectively.
  - **The cdfc is more important than the cdf for outage calculations and predictions, since outages typically occur when the interference is large.**



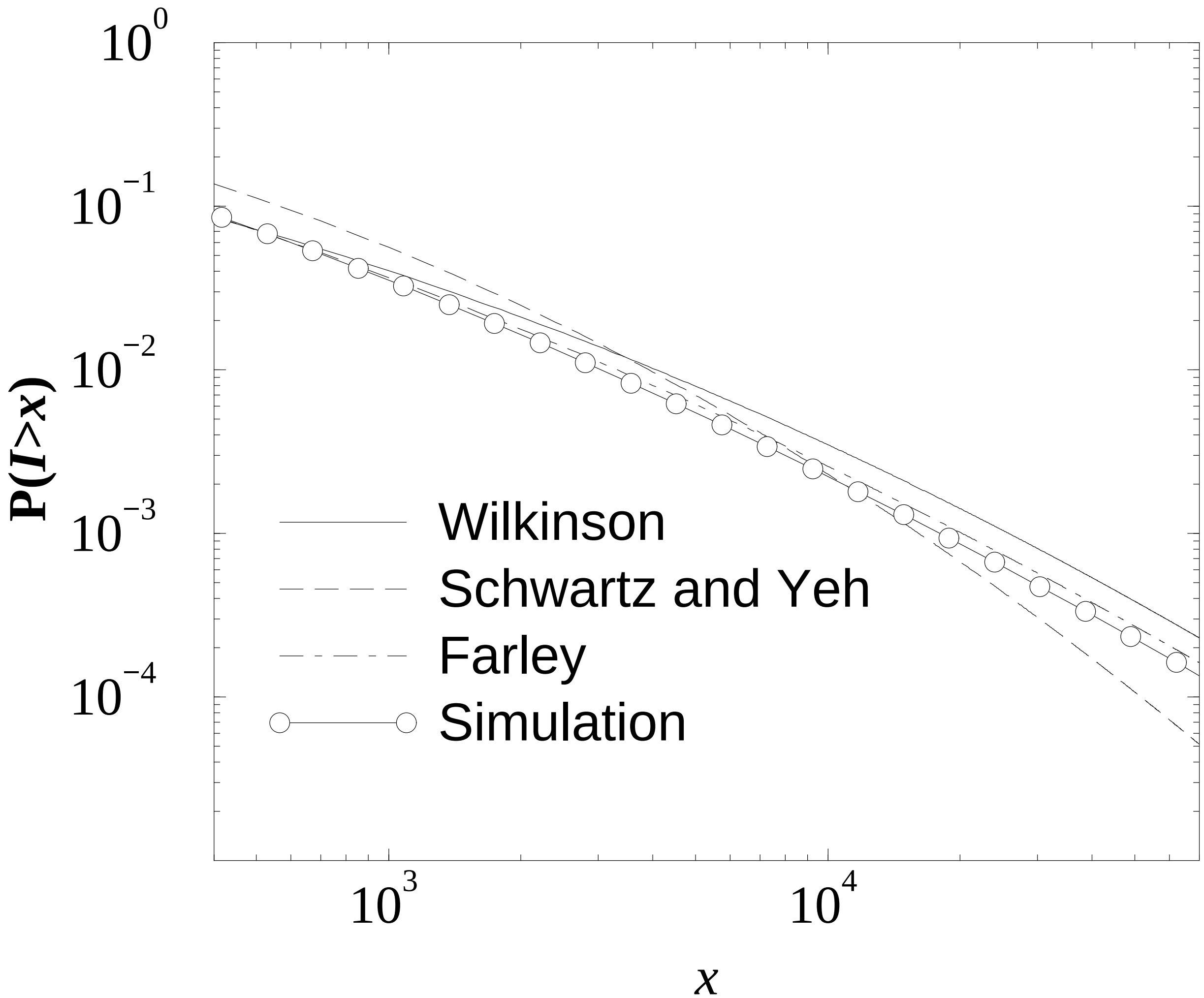
Comparison of the cdf for the sum of two and six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of two log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 12$ dB.

Outage with Multiple Interferers

1. First obtain the mean and variance

$$
\begin{aligned}
\mu_Z &= \mu_{\hat{Z}}/\xi \\
\sigma_Z^2 &= \sigma_{\hat{Z}}^2/\xi^2 \qquad \xi = 0.23026
\end{aligned}
$$

2. Treat the average CIR as Gaussian distributed with mean and variance

$$
\begin{aligned}
\mu_{\Lambda(\mathbf{d})} &= \mu_{\Omega(d_0)} - \mu_Z \ \ (\mathrm{dBm}) \\
\sigma_{\Lambda(\mathbf{d})}^2 &= \sigma_\Omega^2 + \sigma_Z^2 \ \ .
\end{aligned}
$$

3. Compute the outage for a given location, described by d

$$
O(\mathbf{d}) = Q\left(\frac{\mu_{\Omega(d_0)} - \mu_Z - \Lambda_{\mathrm{th(dB)}}}{\sqrt{\sigma_\Omega^2 + \sigma_Z^2}}\right)
$$

4. Average over all locations d by Monte Carlo integration

$$
O = \int_{R^N} O(\mathbf{d}) p_{\mathbf{d}}(\mathbf{d}) d\mathbf{d}
$$

Single Co-channel Interferer

- **For a single co-channel interferer**

$$p_{\Lambda(\mathbf{d})_{(\mathrm{dB})}}(x) = \frac{1}{\sqrt{4\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}})^2}{4\sigma_\Omega^2} \right\}$$

where

$$\mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}} = \mu_{\Omega(d_0)_{(\mathrm{dB})}} - \mu_{\Omega(d_1)_{(\mathrm{dB})}}$$

- **The outage for a given d is**

$$\begin{aligned} O(\mathbf{d}) &= \Pr(\Lambda(\mathbf{d})_{(\mathrm{dB})} < \Lambda_{\mathrm{th(dB)}}) \\ &= \int_{-\infty}^{\Lambda_{\mathrm{th(dB)}}} \frac{1}{\sqrt{4\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}})^2}{4\sigma_\Omega^2} \right\} \, dx \\ &= Q\left(\frac{\mu_{\Lambda(\mathbf{d})_{(\mathrm{dB})}} - \Lambda_{\mathrm{th(dB)}}}{\sqrt{2}\sigma_\Omega} \right) \end{aligned}$$

Worst case interference from a single co-channel base-station.

- **In this case d** $= (R, D - R)$**.**
- **The worst case outage due to a single co-channel interferer is**

$$O(R) = Q\left(\frac{\mu_{\Omega(R)_{(\mathrm{dB})}} - \mu_{\Omega(D-R)_{(\mathrm{dB})}} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}}{\sqrt{2}\sigma_{\Omega}}\right)$$

- **Using a simple inverse-β path loss characteristic**

$$\mu_{\Omega_{(\mathrm{dB})}} = \Omega_{(\mathrm{dB})}(d_o) - 10\beta \log_{10}(d/d_o)$$

gives

$$O(R) = Q\left(\frac{10\log_{10}\left(\frac{D}{R}-1\right)^{\beta} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}}{\sqrt{2}\sigma_\Omega}\right)$$

- **The minimum CIR margin on the cell fringe is**

$$\mathrm{M}_\Lambda = 10\log_{10}\left(\frac{D}{R}-1\right)^{\beta} - \Lambda_{\mathrm{th}\ (\mathrm{dB})}$$

- **For an ideal hexagonal layout $\frac{D}{R} = \sqrt{3N}$, so that**

$$N = \frac{1}{3}\left[10^{\frac{\mathrm{M}_\Lambda + \Lambda_{\mathrm{th}\ (\mathrm{dB})}}{10\beta}} + 1\right]^2$$

  - **A small cluster size is achieved by making the margin M_Λ and receiver threshold Λ_{th} small.**

Rician/Multiple Rayleigh Interferers

- **Sometimes propagation conditions exist such that the received signals experience fading, but not shadowing. In this section, we calculate the outage probability for the case of fading only.**
  - **The received signal may consist of a direct line of sight (LoS) component, or perhaps a specular component, accompanied by a diffuse component. The envelope of the received desired signal experiences Ricean fading.**
  - **The interfering signals are often assumed to be Rayleigh faded, because a direct LoS is unlikely to exist due to the larger physical distances between the co-channel interferers and the receiver.**
- **Let the instantaneous power in the desired signal and the N_I interfering signals be denoted by s_0 and $s_k,\ k = 1,\ \cdots,\ N_I$, respectively. Note that $s_i = \alpha_i^2$, where α_i^2 is the squared-envelope.**
- **The carrier-to-interference ratio is defined as $\lambda = s_0 / \sum_{k=1}^{N_I} s_k$, and for a specified receiver threshold λ_{th}, the outage probability is**

$$O_I = \mathrm{P}\left(\lambda < \lambda_{\text{th}}\right) \quad .$$

Single Interferer

- **For the case of a single interferer, the outage probability reduces to the simple closed form**

$$O_I = \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_1} \exp\left\{-\frac{KA_1}{\lambda_{\rm th} + A_1}\right\} \quad ,$$

where K **is the Rice factor of the desired signal,** $A_1 = \Omega_0/(K+1)\Omega_1$**, and** $\Omega_k = {\rm E}[s_k]$**.**

- **If the desired signal is Rayleigh faded, then the outage probability can be obtained by setting** $K = 0$**.**

Multiple Interferers

- **For the case of multiple interferers, each with mean power Ω_k, the outage probability has the closed form**

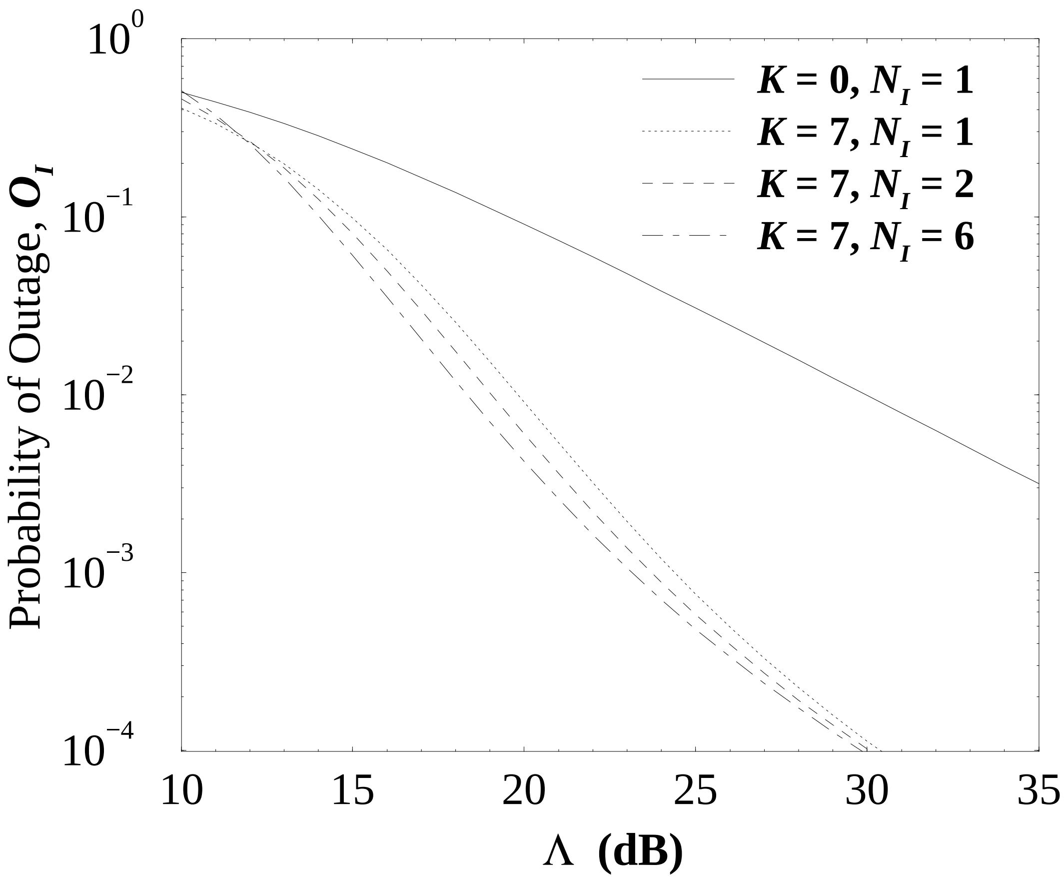
$$O_I = 1 - \sum_{k=1}^{N_I} \left[1 - \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_k} \exp\left\{ - \frac{K A_k}{\lambda_{\rm th} + A_k} \right\} \right] \prod_{\substack{j=1 \\ j \neq k}}^{N_I} \frac{A_j}{A_j - A_k} \quad ,$$

where $A_k = \Omega_0/(K+1)\Omega_k$. This expression is only valid if $\Omega_i \neq \Omega_j$ when $i \neq j$, i.e., the different interferers have different mean power.

- **If all the interferers have the same mean power, then the outage probability can be derived as**

$$\begin{aligned} O_I &= \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_1} \exp\left\{ - \frac{K A_1}{\lambda_{\rm th} + A_1} \right\} \\ &\quad \times \sum_{k=0}^{N_I - 1} \left(\frac{A_1}{(\lambda_{\rm th} + A_1)} \right)^k \sum_{m=0}^{k} \binom{k}{m} \frac{1}{m!} \left(\frac{K \lambda_{\rm th}}{\lambda_{\rm th} + A_1} \right)^m \quad . \end{aligned}$$

- **If the desired signal is Rayleigh faded, then the probability of outage with multiple Rayleigh faded interferers can be obtained by setting $K = 0$.**



Probability of outage with multiple interferers. The desired signal is Ricean faded with various Rice factors, while the interfering signals are Rayleigh faded and of equal power; $\lambda_{\rm th} = 10.0$ dB.

$$\Lambda = \frac{\Omega_0}{N_I \Omega_1}$$