Feedback Control:

1. Consider an open loop controller for a plant of the form: $G_p(s) = \frac{1}{s+1}$



a) Design an open loop controller $G_c(s)$ so that the output has a steady-state value of 10 and the time constant is $\tau=0.5$ sec for a reference input of r(t) = 10u(t).

b) Now suppose that the plant has changed from the nominal that you used for the design. The new plant is $G_p(s) = \frac{1}{s+0.5}$. Apply the controller that you designed in part a) to the new plant. Find the steady-

state value of y(t) and the time constant of the new system.

2. Consider the same plant as given in Problem 1, but this time design a feedback controller for it using a proportional controller, $G_c(s) = K_p$. Let r(t) = 10u(t).



a) Select K_p to satisfy the specification that the closed loop system have a time constant at $\tau = 0.5$ sec.

b) Find the corresponding value of e_{ss} .

c) Sketch the closed loop step response.

d) Sketch the closed loop step response if $K_p = 4$.

e) Verify your plots in parts c) and d) by using MATLAB to compute and plot the step responses of the two closed loop systems (remember to enter the numerator (numcl) and denominator (dencl) of the closed loop system into MATLAB; also use step(10*numcl,dencl) to get response to 10u(t)).

f) Suppose that the plant has now changed from the nominal to $G_p(s) = \frac{1}{s+0.5}$. Use MATLAB to

compute and plot the step responses for the two new closed loop systems: (i) using the new plant and the controller designed in part a); and (ii) the using the new plant and the controller given in part d).

3. Consider the plant of Problem 2 and the proportional controller of $K_{P}=4$. Show how you can modify the reference input so that the output has a steady-state value of $y_{ss}(t) = 10$. What is the resulting steadystate value of y(t) when this modification is applied to the new plant in part f)?

4. Consider the same plant and feedback configuration given in Problem 2, this time use a PI controller:

$$G_{c}(s) = K_{P} + K_{I} \frac{1}{s} = \frac{K_{P}(s + K_{I} / K_{P})}{s}$$
. Let $K_{I}/K_{P} = 1$.

a) Pick K_P such that the closed loop time constant is $\tau=0.5$ sec.

b) What is e_{ss} if r(t) = 10u(t)?

c) Sketch the closed loop response for y(t) if r(t) = 10u(t).

d) Verify your plot in part c) by using MATLAB to compute and plot the closed loop step response.

e) Redo part d), except use the modified plant $G_p(s) = \frac{1}{s+0.5}$ in place of the nominal plant.

5. Use the results of Problems 1-4 to compare the results of open loop control, Proportional control and PI control in terms of accuracy, speed of response and robustness.

6. A feedback controller will be designed for a plant given below. Select a controller $G_c(s) = K_P$ so that the closed loop system has a time constant of no more than 5 sec. and steady-state error to a unit step input of no more than 0.05.



7. Two feedback controllers are designed for a plant given below. Let r(t) be a unit step input. Compare the controllers in terms of speed of response, accuracy and relative stability for the closed loop system. Be as specific as you can in your comparison.



8. Suppose a plant transfer function is given by G_p below. A proportional controller is used in a feedback loop. Determine the range of K_p for stability. Determine the steady-state error to a unit step input if $K_p = 10$.

$$G_{p}(s) = \frac{1}{s^{3} + 5s^{2} + 6s}$$