Modulation Analysis for Wireless Infrared Communications

Hyuncheol Park and John R. Barry

School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia, 30332-0250

ABSTRACT

The channel model for indoor wireless optical communication links is unusual in that it combines the intensity-modulation constraints of the Poisson photon-counting channel with the multipath dispersion, bandwidth constraints, and additive white Gaussian noise of the conventional radio channel. In this paper we examine the performance of several modulation schemes on the indoor wireless optical channel. Modulation schemes having low duty cycle, such as pulse-position modulation (PPM), offer improved power efficiency at the expense of decreased bandwidth efficiency. We compare the power and bandwidth efficiency of several forms of PPM, including multiple PPM, overlapping PPM, convolutional coded PPM, and trellis coded OPPM. We also examine the differences between the wireless infrared communications channel and the quantum-limited photon-counting channel.

I. INTRODUCTION

The appropriate channel model for wireless optical communications systems using intensity modulation depends on the intensity of the background light. In low background light, it is common to model the received signal as a Poisson process with rate $\lambda_s(t) + \lambda_n$, where $\lambda_s(t)$ is proportional to the instantaneous optical power of the received signal, and λ_n is proportional to the power of the background light; when λ_n is zero, the channel is quantum limited. However, in those applications where λ_n is very large and the receiver employs a wideband photodetector, the photodetector shot-noise is accurately modeled as an additive white Gaussian noise (AWGN) plus a d.c. offset [1][2], and it is often more convenient to use an AWGN model.

Non-directed infrared radiation offers several advantages over radio as a medium for indoor wireless networks, including an immense window of unregulated bandwidth, immunity to multipath fading (but not multipath distortion), and a lack of interference from one room to another [3][4]. But the background light in typical indoor environments is very intense; even after a narrow-band (10 nm) optical filter, λ_n will be between 10^{11} and 10^{14} photons/s, depending on the proximity to a window [3]. Such high rates make the AWGN model extremely accurate. Furthermore, because the multipath propagation destroys spatial coherence, the effects of multipath propagation can be characterized by a baseband linear model [5][6]. This leads to the following equivalent baseband channel model, the AWGN model, for wireless infrared communications using intensity modulation and direct detection:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau + n(t), \tag{1}$$

where $x(t)$ represents the instantaneous optical power of the transmitter, $y(t)$ represents the instantaneous current of the receiving photodetector, $h(t)$ represents the multipath-induced temporal dispersion, and $n(t)$ is white Gaussian noise with two-sided power spectral density N_0.

The same model (1) is used to model conventional radio channels, where $x(t)$ represents amplitude, and it must satisfy $\langle x^2(t)\rangle \le P_o$, where P_o is the average power constraint of the radio transmitter.[1] However, $x(t)$ represents optical power in our application, so it must satisfy:

$$x(t) \ge 0 \text{ and } \langle x(t)\rangle \le P, \tag{2}$$

where P is the average optical power constraint of the transmitter. In this paper we examine the bandwidth efficiency and power efficiency of various modulation schemes under the constraints of (2). To isolate the effects of (2), we neglect multipath dispersion in this paper, so that $h(t) = \delta(t)$ in (1); nevertheless, we note that the bandwidth limitations of both $h(t)$ and the receiver electronics are what motivates us to consider bandwidth efficiency as an important parameter.

The AWGN model is essentially the same as the noisy photon counting channel in the limit as $\lambda_n \to \infty$, so we could use the photon-counting channel as our starting point. However, working directly with the AWGN model is beneficial because of the additional insight it brings to the problem; by starting with a conventional waveform channel model and then introducing the constraints of (2), we can build on intuition developed for conventional channels. Furthermore, as we will see, analysis using the AWGN model is often simpler than that for the Poisson model.

In Sect. II we examine pulse-position modulation (PPM), multiple PPM (MPPM), and overlapping PPM (OPPM). In Sect. III, we consider coded modulation using PPM and OPPM. Our results are summarized in Sect. IV, where we compare the power and bandwidth efficiencies on both the quantum-limited channel ($\lambda_n = 0$) and the AWGN model ($\lambda_n = \infty$).

Supported in part by an IBM Faculty Development Award and by NSF under grant number NCR-9308968.

1. $\langle \cdot \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}(\cdot)\,dt$

To appear in IEEE International Conference on Communications, ICC´95

II. UNCODED MODULATION

We first review the classic problem of determining the error probability for an L-ary modulation scheme in the presence of additive white Gaussian noise, assuming maximum-likelihood (ML) detection, and neglecting intersymbol interference [2][7]. The transmitter conveys information at a rate of R_b bits/s by transmitting one of L non-negative signals $\{x_1(t), x_2(t), \ldots, x_L(t)\}$ every $T = \log_2 L / R_b$ seconds, and the channel adds white Gaussian noise with power spectrum N_0. To prevent intersymbol interference, each signal is confined to the interval $[0, T)$. The signal set satisfies (2) with equality, so that the average signal power is $\frac{1}{L}\Sigma_i\langle x_i(t)\rangle = P$. For example, an on-off-keying (OOK) transmitter emits a rectangular pulse of duration $1/R_b$ and of intensity $2P$ to signify a one bit, and no pulse to signify a zero bit. The bandwidth required by OOK is roughly R_b, the inverse of the pulse width.

To simplify analysis, we make the high-SNR assumption that the bit-error rate (BER) is dominated by the two nearest signals, so that:

$$\mathrm{BER} \approx Q(d_{\min}/2\sqrt{N_0}), \tag{3}$$

where $d_{\min}$ is the minimum Euclidean distance between any pair of valid modulation signals:

$$d_{\min}^2 = \min_{i \neq j} \int (x_i(t) - x_j(t))^2 dt. \tag{4}$$

In fact, (3) is exact for OOK (and any time $L = 2$); the minimum distance between the two signals in the OOK signal set is:

$$d_{OOK} = \frac{2P}{\sqrt{R_b}}, \tag{5}$$

and the BER, assuming ML detection, is $Q(P/\sqrt{N_0 R_b})$.

We will use OOK as a benchmark to compare the power efficiencies of various modulation schemes. The power required by OOK to achieve a given BER is $P_{OOK} \equiv \sqrt{N_0 R_b}\, Q^{-1}(BER)$. The power required by any other modulation scheme to achieve the same BER is approximately $P = (d_{OOK}/d_{\min})P_{OOK}$, assuming the SNR is high enough that (3) is accurate. Therefore, in the remainder of the paper we will use the distance ratio $d_{OOK}/d_{\min}$ to characterize the power requirement of any modulation scheme.

II.A Pulse-Position Modulation

In a pulse-position modulation (PPM) scheme, each symbol interval of duration $T = \log_2 L / R_b$ is partitioned into L sub-intervals, or chips, each of duration T/L, and the transmitter sends an optical pulse during one and only one of these chips. PPM is similar to L-ary FSK, in that all signals are orthogonal and have equal energy. PPM can be viewed as the rate-$\log_2 L / L$ block code consisting of all binary L-tuples having unity Hamming weight. A PPM signal satisfying (2) is:

$$x(t) = LP \sum_{k=0}^{L-1} c_k\, p(t - kT/L), \tag{6}$$

where $[c_0, c_1, \ldots, c_{L-1}]$ is the PPM codeword, and where $p(t)$ is a rectangular pulse of duration T/L and unity height. All of the signals are equidistant, with:

$$d_{\min}^2 = \min_{i \neq j} \int (x_i(t) - x_j(t))^2 dt = 2LP^2 \log_2 L / R_b. \tag{7}$$

Therefore, the average power requirement is approximately:

$$P_{PPM}/P_{OOK} \approx d_{OOK}/d_{\min} = \sqrt{\frac{2}{L \log_2 L}}. \tag{8}$$

From (8) we see that, for any L greater than 2, PPM requires less optical power than OOK. In principle, the optical power requirement can be made arbitrarily small by making L suitably large, at the expense of increased bandwidth; the bandwidth required by PPM to achieve a bit rate of R_b is approximately the inverse of one chip duration, $B = L/T = LR_b/\log_2 L$.

II.B Multiple PPM

In multiple PPM, each symbol interval of duration $T = \log_2 L / R_b$ is partitioned into n chips, each of duration T/n, and the transmitter sends an optical pulse during w of these chips. The transmitted signal is given by:

$$x(t) = a \sum_{k=0}^{n-1} c_k \phi(t - kT/n), \tag{9}$$

where $[c_0, c_1, \ldots, c_{n-1}]$ is a binary n-tuple of weight w, where $\phi(t) = \sqrt{n/T}\, p(t)$ is a unit-energy rectangular pulse of duration T/n, and where the constant a is chosen so that the average optical power is P: $a = (P/w)\sqrt{nT} = d_{OOK}\sqrt{n \log_2 L}/2w$. There are $\binom{n}{w}$ binary n-tuples of weight w, but it may be desirable to use only a fraction L of these; for example, we may choose the codewords to have a large minimum Hamming distance d. That is, we may restrict attention to an (n, d, w) constant weight code [8], which is a set of binary n-tuples having weight w and minimum Hamming distance d.

For a given n, d, and w, let $L \le \binom{n}{w}$ be the number of valid codewords. We must have $d \ge 2$, because it is impossible for two binary n-tuples of weight w to differ in only one position. If we admit all binary n-tuples of weight w, then $L = \binom{n}{w}$ and $d = 2$. Unless noted otherwise, we assume $L = \binom{n}{w}$ in this paper. The bandwidth is roughly n/T, the inverse of the chip duration, so that:

$$B_{MPPM}/R_b = \frac{n}{\log_2 L}. \tag{10}$$

Because $\{\phi(t - kT/n)\}$ is an orthonormal set, (9) implies that the Euclidean distance between any two PPM waveforms $x_i(t)$ and $x_j(t)$ is $a\sqrt{d_{ij}}$, where d_{ij} is the Hamming distance between the corresponding binary n-tuples. Thus, the minimum distance is $d_{\min} = a\sqrt{d}$, where d is the minimum Hamming distance and $a = d_{OOK}\sqrt{n \log_2 L}/2w$. The ratio of d_{OOK} to $d_{\min}$ gives the average power requirement:

$$P_{MPPM}/P_{OOK} = \frac{2w}{\sqrt{nd \log_2 L}}. \tag{11}$$

Note that PPM is a special case of multiple PPM with $n = L$, $d = 2$, and $w = 1$, and that (11) reduces to (8) in this case.

II.C Overlapping PPM

We define $\binom{n}{w}$ *overlapping* PPM (OPPM) as a special case of multiple PPM, where the w ones are constrained to be consecutive. In other words, each symbol interval of duration $T = \log_2 L / R_b$ is divided into n chips, each of duration T/n, and a rectangular pulse spanning w chips is transmitted, beginning at any of the first $L = n - w + 1$ chips. The motivation for constraining the w ones to be consecutive is the decreased bandwidth that results; unfortunately, this benefit is offset by the reduced alphabet size, since L drops from $\binom{n}{w}$ to $n - w + 1$. Note that this definition of OPPM is slightly more general than the usual definition [9], because it allows the possibility that n/w is not an integer. We refer to the ratio $\alpha = w/n$ as the duty cycle. Note also that specifying L does not uniquely specify n and w; for example, 4-OPPM can arise from $\binom{5}{2}$, $\binom{6}{3}$, $\binom{7}{4}$, etc. Thus, it takes two parameters to specify OPPM, either n and w or L and α.

The bandwidth of OPPM is $n/(wT)$ where $T = \log_2 L / R_b$, so that:

$$B_{OPPM}/R_b = \frac{n/w}{\log_2(n - w + 1)}\,, \tag{12}$$

which is clearly smaller than that of PPM, since n/w is less than L. The minimum Hamming distance between OPPM codewords is 2, so that the minimum Euclidean distance between received signals is $d_{min} = \sqrt{2}\,a = (P/w)\sqrt{2nT}$. Dividing d_{OOK} by $d_{\min}$ yields the average power requirement for OPPM:

$$P_{OPPM}/P_{OOK} = \frac{2w}{\sqrt{2n\log_2(n - w + 1)}}\,. \tag{13}$$

With $w = 1$, n becomes L, and this equation reduces to (8).

III. Coded Modulation

III.A Rate $1/n$ convolutional coded 2^n-PPM

One method for combining a convolutional code and PPM is shown in Fig. 1, where a rate $1/n$ convolutional code is followed by a 2^n-PPM encoder [10]. For every information bit coming in, a single pulse is transmitted. Thus, the symbol rate and bit rate are identical and the required bandwidth is roughly $2^n/T$, so the bandwidth requirement is:

$$B_{cc(1/n)}/R_b = 2^n. \tag{14}$$

The bandwidth increases exponentially with n.

A simple rate-1/2 convolutional code and one stage of its trellis diagram are shown in Fig. 2. Associated with each transition on the trellis diagram is a PPM codeword.

Consider a rate $1/n$ convolutional code with constraint length ν (i.e., with $\nu - 1$ memory elements). An information sequence of length K bits will result in a sequence of $K + \nu - 1$ PPM symbols. Because of the encoder memory, two information sequences that differ by only a single bit will result in two trellis paths that disagree in exactly ν consecutive transitions. For example, the trellis in Fig. 3 shows the two paths corresponding to the all-zero sequence and a single one bit. Two paths can differ in more than ν consecutive transitions, but never less. As illustrated in Fig. 2, the Hamming distance between any two branches is either zero or 2. Therefore, an upper bound on the minimum Hamming distance d_H for convolutionally coded PPM is:

$$d_H \le 2\nu. \tag{15}$$

If we design the convolutional encoder so that the distance in each of the ν transitions is always 2, then the Hamming distance due to a one-bit error is 2ν. One way to achieve this is to make one of the generator polynomials all ones, $\boldsymbol{g}^{(i)} = (1\ 1\ 1\ \ldots\ 1)$. However, we must also consider information sequences that differ in more than a single bit. Nevertheless, it seems likely that a rate $1/n$ encoder with constraint length ν can always be found such that the minimum Hamming distance between coded PPM sequences is close to 2ν, at least when ν is small. In fact, the maximal d_{free} codes of Table 11.1(c) in [11] satisfy (15) with equality for $\nu \le 6$. In particular, the minimum Hamming distance for the convolutional encoded PPM system of Fig. 2 is $d_H = 6$, so it satisfies (15) with equality.

The PPM waveform during each symbol period has the form of (9), where the constant a is chosen so that the total average power is P; since there are 2^n chips per symbol, $a = P\sqrt{2^n/R_b}$. The minimum Euclidean distance between coded PPM sequences is given by $d_{\min} = a\sqrt{d_H}$. Therefore, the average power requirement for convolutionally coded PPM is:

$$P_{cc(1/n)}/P_{OOK} = \frac{1}{\sqrt{2^{n-2}d_H}} \approx \frac{1}{\sqrt{2^{n-1}\nu}}\,, \tag{16}$$

the last approximation being valid when (15) is approximately satisfied with equality.

III.B Rate k/n convolutional coded 2^n- PPM

A rate k/n convolutional code can also be combined with PPM. The k bits of information are shifted into the encoder at each symbol period and the n bits of the encoder output are con-

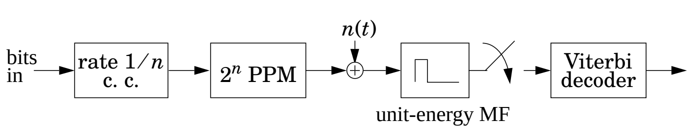

Fig. 1. A rate $1/n$ convolutional coded 2^n-PPM system.

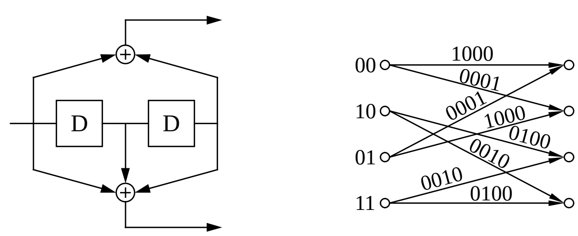

Fig. 2. A simple rate-1/2 convolutional code.

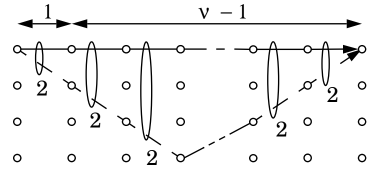

Fig. 3. An error event with Hamming distance 2ν.

verted to a 2^n-PPM symbol. The chip duration is $T/2^n$ and the bit rate is $R_b = k/T$, so the bandwidth expansion factor is:

$$B_{cc(k/n)}/R_b = 2^n/k. \tag{17}$$

Using the same argument as in Sect. III.A, the minimum Hamming distance is bounded by $d_H \le 2\nu_{\min}$, where $\nu_{\min}$ is the minimum constraint length among the k input shift registers. As in (9), the constant a is chosen so that the total average power is P; unlike (9), however, there are 2^n chips per symbol, not n, and the symbol period is $T = k/R_b$, not $1/R_b$, so that $a = P\sqrt{2^n k/R_b}$. Therefore, $d_{OOK}/d_{\min}$ gives the power requirement as:

$$P_{cc(k/n)}/P_{OOK} = \frac{1}{\sqrt{2^{n-2}kd_H}} \approx \frac{1}{\sqrt{2^{n-1}k\nu_{\min}}}, \tag{18}$$

the last approximation being valid when $d_H \approx 2\nu_{\min}$. From (17) and (18) we see that, with respect to bandwidth efficiency and power efficiency, the rate k/n encoders are better than the rate $1/n$ encoders, and that increasing k is always beneficial.

III.C Trellis Coded OPPM

If we use a convolutional code to reduce the probability of error, there is an inevitable increase in bandwidth, as derived in the last two sections. It is well known that trellis-coded modulation (TCM) is a technique that improves performance without increasing the bandwidth. Since PPM has the same Hamming distance between any two codewords, no gains can be made through set partitioning. Overlapping PPM is an attractive alternative since it has a low duty cycle and equal energy signals [9][12][13]. But doubling the number of overlapping PPM symbols without increasing bandwidth, as Ungerboeck suggested [14], requires that the overall duty cycle $\alpha = w/n$ remain fixed, and so the number of slots in each baud interval must increase from $n_u = (L-1)/(1-\alpha)$ to $n_c = (2L-1)/(1-\alpha)$, which, in turn, decreases the minimum distance. Ideally, the coding gain achieved through set partitioning will be large enough to compensate for the decreased minimum distance. Trellis-coded $2L$-OPPM has a bit rate of $R_b = \log_2 L/T$ and a bandwidth expansion factor of:

$$B_{TCM,2L\text{-}OPPM}/R_b = \frac{n/w}{\log_2 L}, \tag{19}$$

which is the same as L-OPPM, but now the required power is:

$$P_{TCM,2L\text{-}OPPM}/P_{OOK} = \sqrt{\frac{4(2L-1)\alpha^2}{d_c(1-\alpha)\log_2 L}}. \tag{20}$$

In contrast, the requirement for uncoded L-OPPM from (13) is:

$$P_{OPPM}/P_{OOK} = \sqrt{\frac{4(L-1)\alpha^2}{d_u(1-\alpha)\log_2 L}}, \tag{21}$$

where d_c and $d_u = 2$ are the minimum Hamming distances for the coded and uncoded systems. The asymptotic coding gain is:

$$\text{Coding gain} = 10\log_{10}\left(\sqrt{\frac{d_c}{2}\left(\frac{L-1}{2L-1}\right)}\right) \approx 10\log_{10}\left(\sqrt{\frac{d_c}{4}}\right). \tag{22}$$

To get improved performance using TCM, the minimum Hamming distance must be greater than about 4.

The signal sets for 8-OPPM ($n_c = 14$, $w_c = 7$) and its set partitioning are shown in Fig. 4. If we use a 4-state TCM, the minimum Hamming distance is 8, which is the distance for a parallel transition resulting from the set partitioning. From (22), therefore, the asymptotic coding gain relative to uncoded 4-OPPM (with $n_u = 6$, $w_u = 3$) is 1.2 dB.

IV. DISCUSSION

The bandwidth and power efficiency for various modulation schemes on the AWGN channel are shown in Fig. 5-a. The benchmark modulation OOK is marked with the symbol '×'. PPM requires less power as L increases, but its bandwidth increases as well. Multiple PPM with weight 2 outperforms PPM both in terms of bandwidth efficiency and power efficiency. Multiple PPM with weight 8 is even more bandwidth efficient, but it requires a large number of chips n to achieve good power efficiency. Overlapping PPM with a duty cycle of $\alpha = 1/2$ is extremely bandwidth efficient. Decreasing the duty cycle to $\alpha = 1/4$ increases the power efficiency at the expense of bandwidth.

Both the rate-$1/2$, 4-PPM and the rate $2/3$, 8-PPM convolutional coded systems require twice the bandwidth of 2-PPM, and neither is able to outperform uncoded PPM, even for large constraint lengths. (The figure assumes that (15) is achieved with equality.) Trellis coded 8-OPPM ($\alpha = 1/2$) does much better; as the coded Hamming distance ranges from 2 to 16, the power requirement decreases from 2.7 dB to –1.8 dB. A distance of $d_c = 16$ can be achieved using the 128-state code from [13], and it offers a a coding gain of about 2.7 dB over 4-OPPM, with no bandwidth expansion.

It is worthwhile to compare the results of Fig. 5-a with those of the quantum-limited (QL) channel ($\lambda_n = 0$), presented in Fig. 5-b. To arrive at this curve, we manipulated the results of [12] to arrive at the following expressions for the average optical power required to achieve an error probability of P_e:

$$P_{MPPM}/P_{OOK} = \frac{2w}{\log_2\binom{n}{w}} \cdot \left[1 - \frac{\log\left(2w - \frac{2w}{n-w+1}\right)}{\log 2P_e}\right] \tag{23}$$

$$P_{OPPM}/P_{OOK} = \frac{2w}{\log_2(n-w+1)} \cdot \left[1 - \frac{\log\left(2 - \frac{2}{n-w+1}\right)}{\log 2P_e}\right]. \tag{24}$$

The second factor in both equations is approximately unity when P_e is small (the figure assumes $P_e = 10^{-6}$).

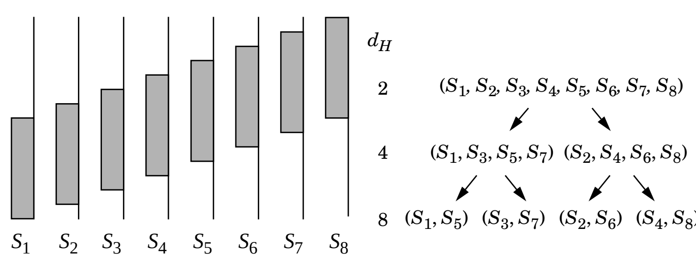

Fig. 4. The 8-OPPM signal set and its set partitioning.

The differences between the two channels are striking. Observe that OPPM is much more power efficient on the AWGN channel than on the QL channel. For example, 32-OPPM with $\alpha = 1/4$ suffers a 6.1 dB penalty relative to OOK on the QL channel, but no penalty at all on the AWGN channel. Relative to 4-PPM, 32-OPPM with $\alpha = 1/4$ suffers a 6.1 dB penalty on the QL channel, but only a 3 dB penalty on the AWGN channel. The $\alpha = 1/2$ and $\alpha = 1/4$ curves intersect on the QL channel, but not on the AWGN channel. On the whole, the power penalties relative to OOK on the QL channel are 2 to 7 dB higher than their counterparts on the AWGN channel. The differences between Fig. 5-a and Fig. 5-b indicate that modulation techniques developed for low background light may not be suitable for high background light.

V. References

[1] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York, 1984.

[2] E. A. Lee, D. G. Messerschmitt, *Digital Communication*, Second Edition, Kluwer Academic Publishers, Boston, 1994.

[3] F. R. Gfeller, U. Bapst, "Wireless In-House Data Communications via Diffuse Infrared Radiation," *Proceeding of IEEE*, vol. 67, no. 11, pp. 1474-1486, November 1979.

[4] J. R. Barry, *Wireless Infrared Communications*, Kluwer Academic Publishers, Boston, 1994.

[5] J. M. Kahn, J. R. Barry, M. D. Audeh, J. B. Carruthers, W. Krause, and G. W. Marsh, "Non-Directed Infrared Links for High-Capacity Wireless LANs," *IEEE Personal Communications Magazine*, vol. 1, no. 2, pp. 12-25, Second Quarter 1994.

[6] J. M. Kahn, W. J. Krause, and J. B. Carruthers, "Experimental Characterization of Non-Directed Indoor Infrared Channels," to appear in *IEEE Transactions on Communications*, 1995.

[7] J. G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill, New York, 1995.

[8] G. E. Atkin, K. S. Fung, "Coded Multipulse Modulation in Optical Communication System," *IEEE Transactions on Communications*, vol. 42, no. 2, pp. 574-582, February 1994.

[9] C. N. Georghiades, "Some Implications of TCM for Optical Direct-Detection Channels," *IEEE Transactions on Communications*, vol. 37, no.5, pp. 481-487, May 1989.

[10] E. Forestieri, R. Gangopadhyay, and G. Prati, "Performance of Convolutional Codes in a Direct-Detection Optical PPM Channel," *IEEE Transactions on Communications*, vol. 37, no. 12, pp. 1303-13, December 1989.

[11] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*, Prentice-Hall, 1983.

[12] C. N. Georghiades, "Modulation and Coding for Throughput-Efficient Optical Systems," *IEEE Transactions on Information Theory*, vol. 40, no.5, pp. 1313-1326, September 1994.

[13] G. J. Pottie, "Trellis Codes for the Optical Direct-Detection Channel," *IEEE Transactions on Communications*, vol. 39, no. 8, pp. 1182-1183, August 1991.

[14] G. Ungerboeck, "Channel Coding with Multilevel/Phase Signals," *IEEE Transactions on Information Theory*, vol. IT-28, no. 1, pp. 55-67, January 1982.

[15] R. D. Gitlin, J. F. Hayes, and S. B. Weinstein, *Data Communication Principles*, Plenum Press, 1992.

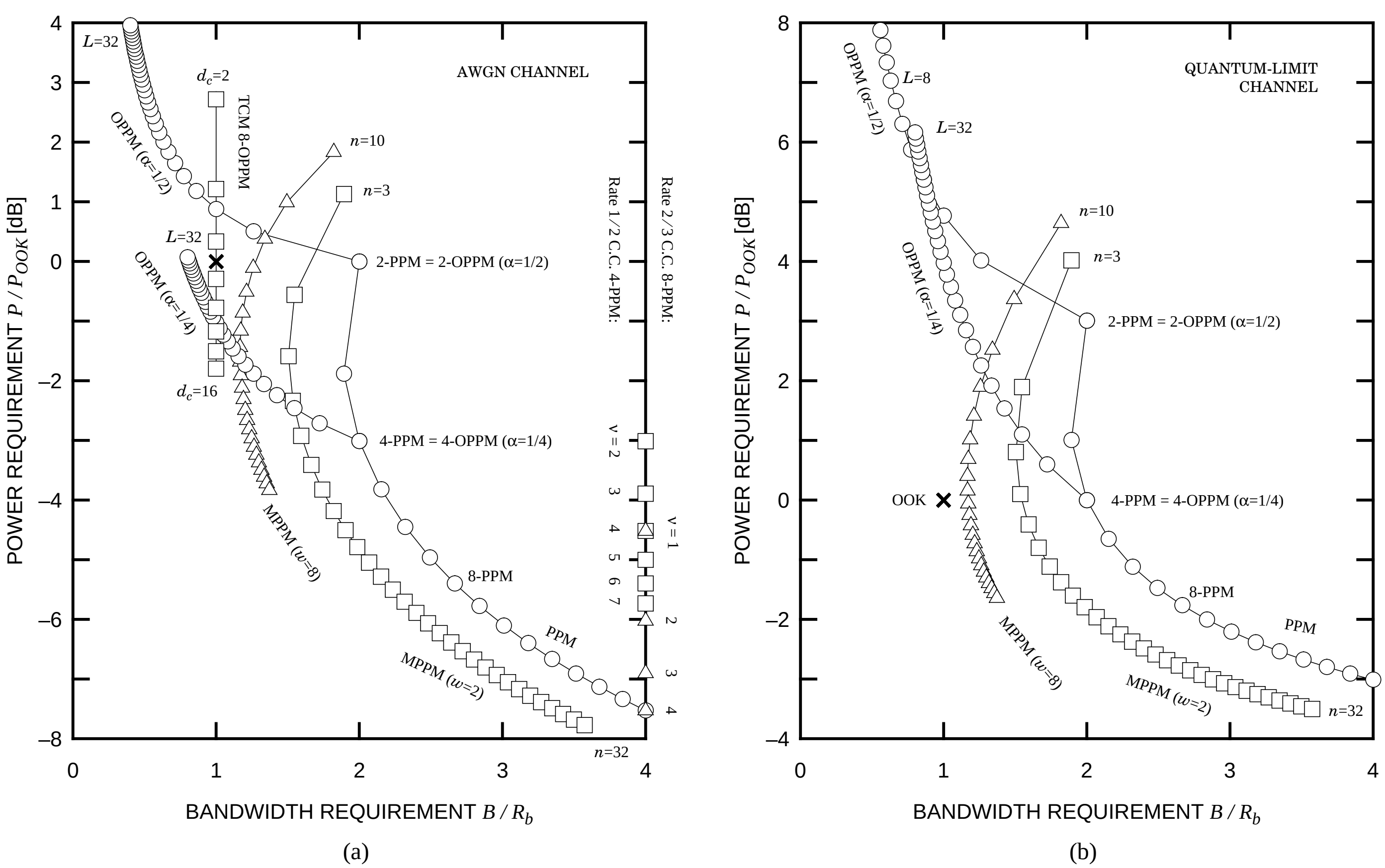

Fig. 5. Power efficiency and bandwidth efficiency on: (a) the AWGN channel; (b) the quantum-limited channel.