On the Application of Superposition to Dependent Sources in Circuit Analysis

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Abstract—**Many introductory circuits texts state or imply that superposition of dependent sources cannot be used in linear circuit analysis. Although the use of superposition of only independent sources leads to the correct solution, it does not make use of the full power of superposition. The use of superposition of dependent sources often leads to a simpler solution than other techniques of circuit analysis. A formal proof is presented that superposition of dependent sources is valid provided the controlling variable is not set to zero when the source is deactivated. Several examples are given which illustrate the technique.**

Index Terms— **Circuit analysis, superposition, dependent sources, controlled sources**

I. PREFACE

When he was a sophomore in college, the author was required to take two semesters of circuit theory. The text was a reproduction of a set of typewritten notes written by Ronald E. Scott which were published later that year as a hard bound text [18]. The chapter that covered superposition had a number of problems where the student was instructed to write by inspection the solution for a voltage or a current by using only superposition, Ohm's law, voltage division, and current division. The author found these problems to be fascinating and he spent hours mastering them.

In describing the principle of superposition, Scott only had independent sources in his circuits. However, the author found that he could also solve circuits containing controlled sources using superposition. Later, after becoming a teacher, he was challenged by students when he used these techniques in teaching electronics courses. The students said that they had been taught that superposition with controlled sources was not allowed. The students were easily satisfied when they were shown that a node voltage analysis yields the same solution.

After encountering so many challenges by students, the author researched circuits books in the library at his school and at a store of a large bookstore chain. None of the books said that superposition can be used with controlled sources. Indeed, the majority stated clearly that it could not.

Thinking that other educators might find the topic to be interesting and useful, the author submitted a paper on the topic to the *IEEE Trans. on Education* in early 1994. It was rejected after the reviewer conceived a circuit that it could not be applied to. The reviewer's circuit contained a floating node at which

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the voltage was indeterminate. The circuit could not be analyzed by any conventional technique. A request to the editor to have it reviewed again by another reviewer went unanswered.

The paper was then submitted to the *IEEE Trans. on Circuits and Systems* in 1995. The reviewer recommended several changes. The paper was revised and submitted a second time in 1996. The reviewer then recommended that it be submitted instead to the *IEEE Trans. on Education*. It was submitted a second time to the latter journal. The editor rejected it without having it peer reviewed. The author concluded that the paper was of no interest to educators, so he published it on the internet in late 1996. A copy of the first review is included at the end of this document.

II. INTRODUCTION

The author has investigated the presentation of superposition in circuits texts by surveying twenty introductory books on circuit analysis [1]-[20]. Fourteen explicitly state that if a dependent source is present, it is never deactivated and must remain active (unaltered) during the superposition process. The remaining six specifically refer to the sources as being independent in stating the principle of superposition. Three of these present an example circuit containing a dependent source which is never deactivated. The other three do not present an example in which dependent sources are present. From this limited survey, it is clear that circuits texts either state or imply that superposition of dependent sources is not allowed. The author contends that this is a misconception.

A simple argument can be used to justify this premise. Suppose a linear circuit containing independent and dependent sources is analyzed by any means other than superposition. Both the output of the circuit and the value of each dependent source are solved for. Solve the circuit a second time using superposition of the independent and dependent sources, treating the dependent sources as independent sources having the values found in the first solution. It is clear that the same result must be obtained. Thus superposition must hold with the dependent sources if their values are known. It can then be argued that superposition must hold even if the values of the dependent sources are not known, provided they are treated as independent sources. Of course, the output of the circuit cannot be determined until their values are known, but these values can be calculated as part of the superposition.

To apply superposition to dependent sources, the controlling variables must not be set to zero when a source is deactivated. This is illustrated in the following with several examples. When

all sources but one are deactivated, the circuit must contain no nodes at which the voltage is indeterminate and no branches in which the current is indeterminate. This specifically rules out cases where current sources are connected in series or voltage sources are connected in parallel. In such cases, superposition cannot be used even if all the sources are independent. For example, if two current sources having the same current are connected in series, the voltage at the common node between the sources is indeterminate. If two voltage sources having the same voltage are connected in parallel, the current in each source is indeterminate.

Because the techniques that are described in this paper are counter to those presented in all of the texts that the author has examined, the question arises as to which technique is the most effective as a teaching tool. The author has no quantitative way of evaluating this. However, he has used the methods successfully over a period of many years in teaching junior and senior level electronics courses at the Georgia Institute of Technology. He has received nothing but positive responses from students.

III. THE PRINCIPLE OF SUPERPOSITION

A proof of the principle of superposition is presented in [18], where all sources are considered to be independent. With some modifications, the proof presented here follows the one in [18], but it assumes that dependent sources are present. The starting point is to assume a general set of mesh or node equations for any given linear circuit. Node equations are assumed here. If the circuit contains voltage sources, they must first be converted into equivalent current sources by making Norton equivalent circuits. Such a transformation does not change the response of the network external to the source. In the case of a voltage source with no series resistance, a transformation which in [18] is called "pushing a voltage source through a node" must first be performed. This replaces a voltage source having no series resistance with several voltage sources, one in each of the branches radiating from the node to which the original source connects.

The general node equations for a linear circuit containing n nodes can be written

$$
\begin{aligned}
+y_{11}v_1 - y_{12}v_2 \cdots - y_{1n}v_n &= \Sigma i_1 \\
-y_{12}v_1 + y_{22}v_2 \cdots - y_{2n}v_n &= \Sigma i_2 \\
&\vdots \\
-y_{1n}v_1 - y_{2n}v_2 \cdots + y_{nn}v_n &= \Sigma i_3
\end{aligned}
\tag{1}
$$

where Σi_j is the sum of the currents delivered to node j by both independent and dependent sources, v_j is the voltage at node j, y_{jj} is the total admittance radiating from node j, and y_{ij} is the admittance between nodes i and j. The equations can be written in the matrix form $\mathbf{Y} \times \mathbf{v} = \mathbf{i}$, where $\mathbf{Y}$ is an $n \times n$ admittance matrix. Because the dependent sources are contained in the current vector $\mathbf{i}$, the $\mathbf{Y}$ matrix corresponds to what is called the branch admittance matrix. This is the admittance matrix with all dependent sources deactivated. It is symmetrical about the main diagonal, the main diagonal terms are all positive, and all off diagonal terms are negative.

A determinant solution for v_1 can be written

$$
v_1 = \frac{1}{\Delta}
\begin{vmatrix}
\Sigma i_1 & -y_{12} & \cdots & -y_{1n} \\
\Sigma i_2 & y_{22} & \cdots & -y_{2n} \\
\vdots & \vdots & & \vdots \\
\Sigma i_n & -y_{2n} & \cdots & y_{nn}
\end{vmatrix}
\tag{2}
$$

where Δ is the determinant

$$
\Delta =
\begin{vmatrix}
y_{11} & -y_{12} & \cdots & -y_{1n} \\
-y_{12} & y_{22} & \cdots & -y_{2n} \\
\vdots & \vdots & & \vdots \\
-y_{1n} & -y_{2n} & \cdots & y_{nn}
\end{vmatrix}
\tag{3}
$$

A cofactor expansion of (2) yields

$$
v_1 = \frac{\Delta_{11}}{\Delta}\Sigma i_1 - \frac{\Delta_{12}}{\Delta}\Sigma i_2 + \frac{\Delta_{13}}{\Delta}\Sigma i_3 - \frac{\Delta_{14}}{\Delta}\Sigma i_4 + \cdots
\tag{4}
$$

where Δ_{ij} is the determinant formed by deleting row i and column j in Δ. Similar solutions follow for the other node voltages.

Each term in (4) is identical to the term which would be written if only the current Σi_j is active, thus proving the principle of superposition. But each Σi_j can be a sum of both independent and dependent current sources. It follows that superposition applies to each of these sources. If a source connects from node j to the datum node, its current appears in the superposition at only one node. If a source connects between two nodes, neither of which is the datum node, its current appears in the superposition at two nodes. In this case, superposition can be applied to each side of the source by treating the two sides as separate current sources, each of which can be turned on and off independently.

The above proof does not imply the controlling variables of a dependent source are deactivated when applying superposition to the source. Only the output of the source is set to zero. This procedure makes it is possible to write circuit equations by considering only one source at a time or to one side of a source at a time. This considerably simplifies the use of superposition with dependent sources compared to the way that it is presented in most circuits texts.

IV. EXAMPLES

The following examples illustrate the proper use of superposition of dependent sources. All superposition equations are written by inspection using voltage division, current division, series-parallel combinations, and Ohm's law. In each case, it is simpler not to use superposition if the dependent sources remain active. Some of the examples are taken from texts cited in the references.

A. Example 1

This example comes from [7]. The object is to solve for the current i in the circuit of Fig. 1. By superposition, one can write

$$
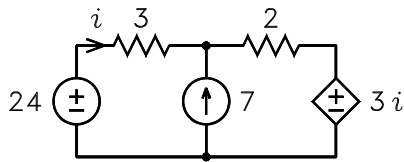i = \frac{24}{3+2} - 7\frac{2}{3+2} - \frac{3i}{3+2} = 2 - \frac{3}{5}i
$$

Fig. 1.   Circuit for example 1.

Solution for i yields

$$i = \frac{2}{1+3/5} = \frac{5}{4}\,\text{A}$$

If the controlled source is not zeroed in the superposition, two solutions must be found. Let $i = i_a + i_b$, where i_a is the solution with the $7\,\text{A}$ source zeroed and i_b is the solution with the $24\,\text{V}$ source zeroed. We can write two loop equations.

$$-24 + i_a\,(3+2) + 3i_a = 0 \quad \text{which yields} \quad i_a = \frac{24}{8} = 3\,\text{A}$$

$$3i_b + 2\,(i_b + 7) + 3i_b = 0 \quad \text{which yields} \quad i_b = -\frac{7}{4}\,\text{A}$$

The total solution is

$$i = i_a + i_b = \frac{5}{4}\,\text{A}$$

This is the same answer obtained by using superposition of the controlled source.

B. Example 2

This example comes from [9]. The object is to solve for the voltages v_1 and v_2 across the current sources in Fig. 2, where the datum node is the lower branch. By superposition, the current i is given by

$$i = 2\frac{7}{7+15+5} + \frac{3}{7+15+5} + 4i\frac{7+15}{7+15+5} = \frac{17}{27} + \frac{88}{27}i$$

Solution for i yields

$$i = \frac{17/27}{1-88/27} = -\frac{17}{61}\,\text{A}$$

Although superposition can be used to solve for v_1 and v_2, it is simpler to write

$$v_2 = 5i = -1.393\,\text{V} \qquad v_1 = v_2 - (4i - i)\,15 = 11.148\,\text{V}$$

If superposition is not used at all, the circuit equations are

$$2 - \frac{v_1 - 3}{7} - \frac{v_1 - v_2}{15} = 0 \quad 4i - \frac{v_{2b}}{5} - \frac{v_2 - v_1}{15} = 0 \quad i = \frac{v_2}{5}$$

These three equations can be solved simultaneously for v_1, v_2, and i. If superposition of the independent sources is used, two solutions must be found. Let $v_1 = v_{1a} + v_{1b}$, $v_2 = v_{2a} + v_{2b}$, and $i = i_a + i_b$, where the a subscript denotes the solution with the $3\,\text{V}$ source zeroed and the b subscript denotes the solution with the $2\,\text{A}$ source zeroed. The node voltage equations are

$$2 - \frac{v_{1a}}{7} - \frac{v_{1a} - v_{2a}}{15} = 0 \quad 4i_a - \frac{v_{2a}}{5} - \frac{v_{2a} - v_{1a}}{15} = 0 \quad i_a = \frac{v_{2a}}{5}$$
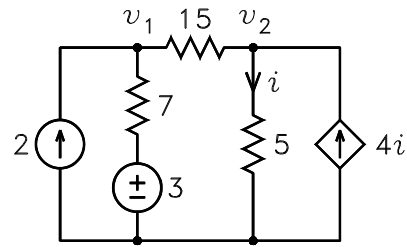

Fig. 2.   Circuit for example 2.

$$-\frac{v_{1b} - 3}{7} - \frac{v_{1b} - v_{2b}}{15} = 0 \quad 4i_b - \frac{v_{2b}}{5} - \frac{v_{2b} - v_{1b}}{15} = 0 \quad i_b = \frac{v_{2b}}{5}$$

It should be obvious without going further that not using superposition at all leads to a solution with less work. However, the first solution that included the dependent source in the superposition is simpler.

C. Example 3

This example comes from [4]. The object is to solve for the current i_1 in the circuit of Fig. 3. By superposition, one can write

$$i_1 = \frac{30}{6+4+2} + 3\frac{4}{6+4+2} - 8i_1\frac{6}{6+4+2} = \frac{42}{12} - 4i_1$$

Solution for i_1 yields

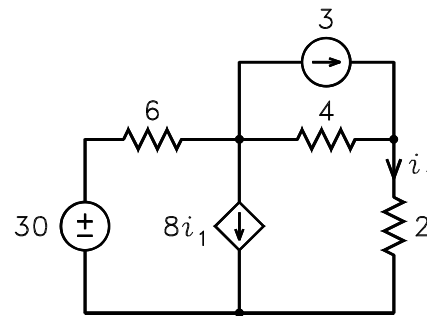$$i_1 = \frac{42/12}{1+4} = 0.7\,\text{A}$$

Fig. 3.   Circuit for example 3.

D. Example 4

This example comes from [4]. The object to solve for the Thévenin equivalent circuit seen looking into the terminals $A - A'$ in the circuit of Fig. 4. By superposition, the voltage v_x is given by

$$v_x = (3 - i_o)\,(2\|40) + 5v_x\frac{2}{40+2} = \frac{80}{42}\,(3 - i_o) + \frac{10}{42}v_x$$

where i_o is the current drawn by any external load and the symbol "$\|$" denotes a parallel combination. Solution for v_x yields

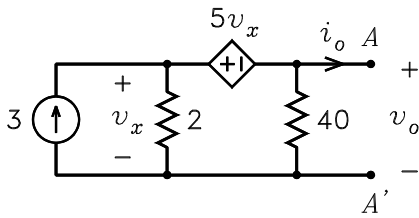$$v_x = \frac{80/42}{1-10/42}\,(3 - i_o) = 2.5\,(3 - i_o)$$

Fig. 4. Circuit for example 4.

Although superposition can be used to solve for v_o, it is simpler to write

$$v_o = v_x - 5v_x = -30 + 10i_o$$

It follows that the Thévenin equivalent circuit consists of a $-30\,\text{V}$ source in series with a $-10\,\Omega$ resistor. The circuit is shown in Fig. 5.
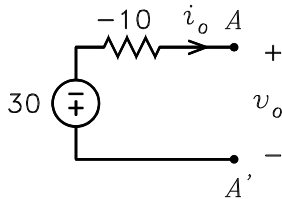

Fig. 5. Thévenin equivalent circuit.

E. Example 5

This example comes from [15]. The object is to solve for the voltage v_o in the circuit of Fig. 6. By superposition, the current i_b is given by

$$
\begin{aligned}
i_b &= \frac{70}{4\|20 + 2\|10} \frac{20}{4+20} + \frac{50}{10 + 4\|20\|2} \frac{20\|2}{4+20\|2} \\
&\quad - \frac{2i_b}{20\|2 + 4\|10} \frac{10}{4+10} \\
&= \frac{35}{3} + \frac{25}{18} - \frac{11}{36}i_b
\end{aligned}
$$

Solution for i_b yields

$$i_b = \frac{35/3 + 25/18}{1 + 11/36} = 10\,\text{A}$$

Although superposition can be used to solve for v_o, it is simpler to write

$$v_o = 70 - 4i_b = 30\,\text{V}$$

F. Example 6

This example comes from [15]. The object is to solve for the voltage v_o in the circuit of Fig. 7. By superposition, the voltage v_Δ is given by

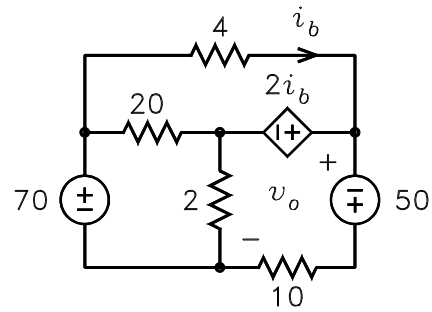$$v_\Delta = -0.4v_\Delta \times 10 + 5 \times 10$$

Fig. 6. Circuit for example 5.

This can be solved for v_Δ to obtain

$$v_\Delta = \frac{5 \times 10}{1 + 0.4 \times 10} = 10\,\text{V}$$

By superposition, i_Δ is given by

$$i_\Delta = \frac{10}{5+20} - 0.4v_\Delta\frac{20}{20+5} = \frac{10}{25} - 0.4v_\Delta\frac{20}{25} = -\frac{70}{25}\,\text{A}$$

Thus v_o is given by
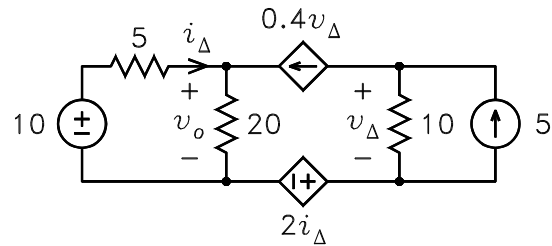
$$v_o = 10 - 5i_\Delta = 24\,\text{V}$$

Fig. 7. Circuit for example 6.

G. Example 7

This example comes from [13] in which it is stated in bold, red letters, "Source superposition cannot be used for dependent sources." The object is to solve for the voltage v as a function of v_s and i_s in the circuit in Fig. 8. By superposition, the current i is given by

$$i = \frac{v_s}{5} - \frac{2}{5}i_s - \frac{3}{5} \times 3i$$

This can be solved for i to obtain

$$i = \frac{v_s}{14} - \frac{i_s}{7}$$

By superposition, the voltage v is given by

$$
\begin{aligned}
v &= \frac{v_s}{5} - \frac{2}{5}i_s + \frac{2}{5} \times 3i \\
&= \frac{v_s}{5} - \frac{2}{5}i_s + \frac{2}{5} \times 3\left(\frac{v_s}{14} - \frac{i_s}{7}\right) \\
&= \frac{2}{7}v_s - \frac{4}{7}i_s
\end{aligned}
$$

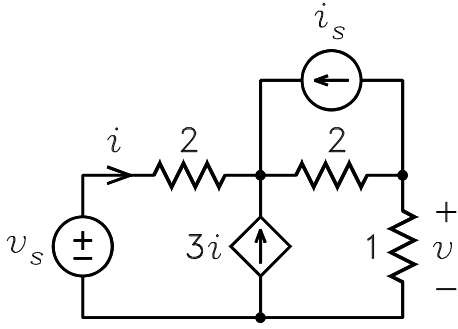Fig. 8. Circuit for Example 7.

H. Example 8

This example illustrates the use of superposition in solving for the dc bias currents in a BJT. The object is to solve for the collector current I_C in the circuit of Fig. 9. Although no explicit dependent sources are shown, the three BJT currents are related by $I_C = \beta I_B = \alpha I_E$, where β is the current gain and $a = \beta/(1+\beta)$. If any one of the currents is zero, the other two must also be zero. However, the currents can be treated as independent variables in using superposition.
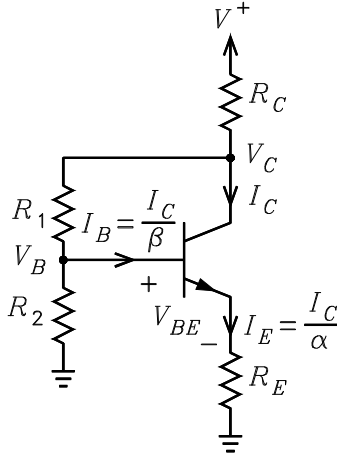

Fig. 9. Circuit for example 8.

By superposition of V^+, $I_B = I_C/\beta$, and I_C, the voltage V_B is given by

$$V_B = V^+\frac{R_2}{R_C+R_1+R_2} - \frac{I_C}{\beta}\left[(R_C+R_1)\|R_2\right]$$
$$-I_C\frac{R_CR_2}{R_C+R_1+R_2}$$

A node-voltage solution for V_B requires the solution of two simultaneous equations to obtain the same answer which superposition yields by inspection. This equation and the equation

$$V_B = V_{BE} + \frac{I_C}{\alpha}R_E$$

can be solved for I_C to obtain

$$I_C = \frac{V^+\frac{R_2}{R_C+R_1+R_2} - V_{BE}}{\frac{(R_C+R_1)\|R_2}{\beta} + \frac{R_CR_2}{R_C+R_1+R_2} + \frac{R_E}{\alpha}}$$

In most contemporary electronics texts, the value $V_{BE} = 0.7\,\text{V}$ is assumed in BJT bias calculations.

I. Example 9

This example illustrates the use of superposition to solve for the small-signal base input resistance of a BJT. Fig. 10 shows the small-signal BJT hybrid-pi model with a current source i_b connected from the base to ground, a resistor R_E from emitter to ground, and a resistor R_C from collector to ground. In the model, $r_\pi = V_T/I_B$ and $r_0 = (V_A + V_{CE})/I_C$, where V_T is the thermal voltage, I_B is the dc base current, V_A is the Early voltage, V_{CE} is the dc collector-emitter voltage, and I_C is the dc collector current. The base input resistance is given by $r_{ib} = v_b/i_b$.
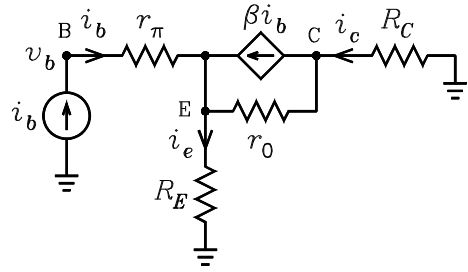

Fig. 10. Circuit for example 9.

By superposition of i_b and βi_b, the base voltage v_b is given by

$$v_b = i_b\left[r_\pi + R_E\|(r_0+R_C)\right] + \beta i_b\frac{r_0}{R_E+r_0+R_C}R_E$$

This can be solved for the base input resistance $r_{ib} = v_b/i_b$ to obtain

$$r_{ib} = r_\pi + R_E\|(r_0+R_C) + \frac{\beta r_0 R_E}{R_E+r_0+R_C}$$

which simplifies to

$$r_{ib} = r_\pi + R_E\frac{(1+\beta)r_0+R_C}{R_E+r_0+R_C}$$

A node-voltage solution for r_{ib} requires the solution of three simultaneous equations to obtain the same answer which follows almost trivially by superposition.

J. Example 10

This example illustrates the use of superposition to solve for the small-signal collector input resistance of a BJT. Fig. 11 shows the small-signal BJT hybrid-pi model with a current source i_c connected from collector to ground, a resistor R_B from base to ground, and a resistor R_E from emitter to ground. In the model, $r_\pi = V_T/I_B$ and $r_0 = (V_A + V_{CE})/I_C$, where V_T is the thermal voltage, I_B is the dc base current, V_A is the Early voltage, V_{CE} is the dc collector-emitter voltage, and I_C is the dc collector current. The collector input resistance is given by $r_{ic} = v_c/i_c$.
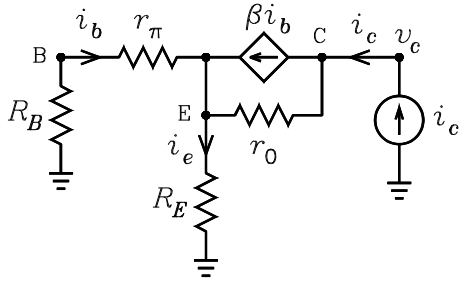
Fig. 11.   Circuit for Example 10.

Fig. 12.   Circuit for Example 11.

By superposition of i_c and βi_b, the collector voltage is given by

$$v_c = i_c \left[r_0 + (R_B + r_\pi) \, \| \, R_E \right] - \beta i_b r_0$$

Current division can be used to express i_b in this equation in terms of i_c as follows:

$$i_b = -i_c \frac{R_E}{R_B + r_\pi + R_E}$$

Substitution of this equation for i_b into the the equation for v_c yields

$$v_c = i_c \left[r_0 + (R_B + r_\pi) \, \| \, R_E + \frac{\beta R_E}{R_B + r_\pi + R_E} r_0 \right]$$

It follows that the collector resistance is given by

$$r_{ic} = \frac{v_c}{i_c} = r_0 \left(1 + \frac{\beta R_E}{R_B + r_\pi + R_E} \right) + (R_B + r_\pi) \, \| \, R_E$$

The drain resistance r_{id} of a FET can be obtained from this expression by using the relation $r_\pi = \beta / g_m$ and taking the limit as $\beta \to \infty$ to obtain

$$r_{id} = r_0 \left(1 + g_m R_S \right) + R_S$$

In this equation, the resistance R_S in series with the FET source replaces the resistance R_E in series with the BJT emitter.

K. Example 11

This example illustrates the use of superposition to solve for the small-signal emitter input resistance of a BJT. Fig. 12 shows the small-signal BJT hybrid-pi model with a voltage source v_e connected from emitter to ground, a resistor R_B from base to ground, and a resistor R_C from collector to ground. In the model, $r_\pi = V_T / I_B$ and $r_0 = (V_A + V_{CE}) / I_C$, where V_T is the thermal voltage, I_B is the dc base current, V_A is the Early voltage, V_{CE} is the dc collector-emitter voltage, and I_C is the dc collector current. The emitter input resistance is given by $r_{ie} = -v_e / i_e$.

By superposition of v_e and βi_b, the emitter current is given by

$$i_e = -\frac{v_e}{(R_B + r_\pi) \, \| \, (r_0 + R_C)} + \beta i_b \frac{r_0}{r_0 + R_C}$$

The base current is a function of v_e only and is given by

$$i_b = -\frac{v_e}{R_B + r_\pi}$$

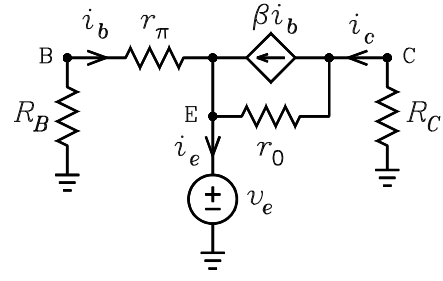Substitution of this equation for i_b into the the equation for i_e yields

$$
\begin{aligned}
i_e &= -\frac{v_e}{(R_B + r_\pi) \, \| \, (r_0 + R_C)} - \beta \frac{v_e}{R_B + r_\pi} \frac{r_0}{r_0 + R_C} \\
&= -v_e \frac{1 + \beta r_0 / (R_B + r_\pi + r_0 + R_C)}{(R_B + r_\pi) \, \| \, (r_0 + R_C)}
\end{aligned}
$$

It follows that the emitter resistance is given by

$$r_{ie} = \frac{v_e}{-i_e} = \frac{(R_B + r_\pi) \, \| \, (r_0 + R_C)}{1 + \beta r_0 / (R_B + r_\pi + r_0 + R_C)}$$

If $r_0 \to \infty$, the equation for the emitter resistance becomes $r_{ie} = (R_B + r_\pi) / (1 + \beta)$.

The source resistance r_{is} of a FET can be obtained from this expression by using the relation $r_\pi = \beta / g_m$ and taking the limit as $\beta \to \infty$ to obtain

$$r_{is} = \frac{r_0 + R_D}{1 + g_m r_0}$$

In this equation, the resistance R_D in series with the FET drain replaces the resistance R_C in series with the BJT collector. If $r_0 \to \infty$, the equation for the source resistance becomes $r_{is} = 1/g_m$.

L. Example 12

This example illustrates the use of superposition with an op-amp circuit. The circuit is shown in Fig. 13. The object is to solve for v_O. With $v_2 = 0$, it follows that $v_A = v_1$, $v_B = 0$, and $v_C = [1 + R_4 / (R_3 \| R_5)] v_1$. By superposition of v_A and v_C, v_O can be written

$$v_O = -\frac{R_2}{R_5} v_A - \frac{R_2}{R_1} v_C = -\left[\frac{R_2}{R_5} + \frac{R_2}{R_1} \left(1 + \frac{R_4}{R_3 \| R_5} \right) \right] v_1$$

With $v_1 = 0$, it follows that $v_A = 0$, $v_B = v_2$, and $v_C = -(R_4 / R_5) v_2$. By superposition of v_2 and v_C, v_O can be written

$$
\begin{aligned}
v_O &= \left(1 + \frac{R_2}{R_1 \| R_5} \right) v_2 - \frac{R_2}{R_1} v_C \\
&= \left(1 + \frac{R_2}{R_1 \| R_5} + \frac{R_2}{R_1} \frac{R_4}{R_5} \right) v_2
\end{aligned}
$$

Thus the total expression for v_O is

$$v_O = -\left[\frac{R_2}{R_5} + \frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3\|R_5}\right)\right]v_1$$
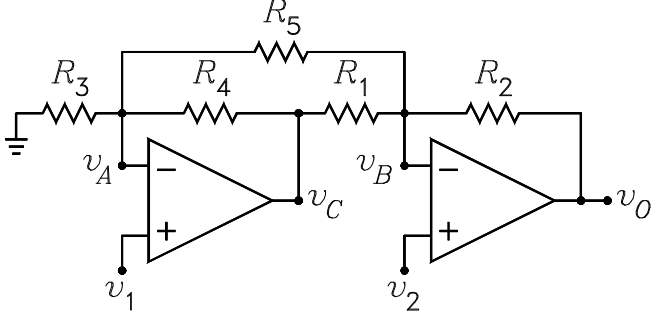$$+ \left(1 + \frac{R_2}{R_1\|R_5} + \frac{R_2}{R_1}\frac{R_4}{R_5}\right)v_2$$

Fig. 13. Circuit for Example 12.

M. Example 13

Figure 14 shows a circuit that might be encountered in the noise analysis of amplifiers. The amplifier is modeled by a z-parameter model. The square sources represent noise sources. V_{ts} and I_{tA}, respectively, model the thermal noise generated by Z_s and Z_A. V_n and I_n model the noise generated by the amplifier. The amplifier load is an open circuit so that $I_2 = 0$. The open-circuit output voltage is given by

$$V_{o(oc)} = z_{12}I_1 + I_A Z_A$$

By superposition, the currents I_1 and I_A are given by

$$I_1 = \frac{V_s + V_{ts} + V_n}{Z_S + Z_A + z_{11}} + I_n\frac{Z_S + Z_A}{Z_S + Z_A + z_{11}}$$
$$- I_{tA}\frac{Z_A}{Z_S + Z_A + z_{11}}$$

$$I_A = \frac{V_s + V_{ts} + V_n}{Z_S + Z_A + z_{11}} - I_n\frac{z_{11}}{Z_S + Z_A + z_{11}}$$
$$+ I_{tA}\frac{Z_S + z_{11}}{Z_S + Z_A + z_{11}}$$

Note that when $I_n = 0$, the sources V_s, V_{ts}, and V_n are in series and can be considered to be one source equal to the sum of the three. When these are substituted into the equation for $V_{o(oc)}$ and the equation is simplified, we obtain

$$V_{o(oc)} = \frac{z_{21} + Z_A}{Z_S + Z_A + z_{11}}\left[V_s + V_{ts} + V_n\right.$$
$$+ I_n\frac{(Z_S + Z_A)z_{21} - Z_A z_{11}}{z_{21} + Z_A}$$
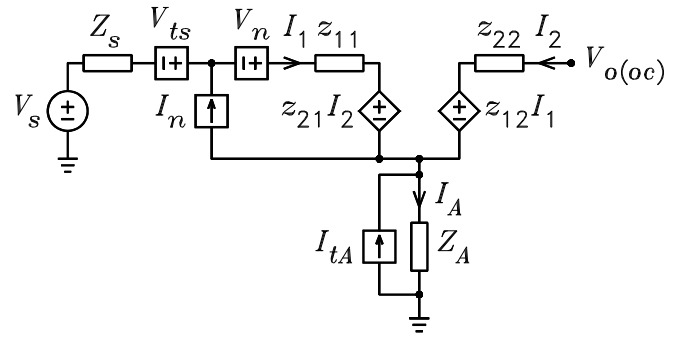$$\left. - I_{tA}\frac{Z_A z_{21} - (Z_S + z_{11})Z_A}{z_{21} + Z_A}\right]$$

Fig. 14. Circuit for Example 13.

N. Example 14

It is commonly believed that superposition can only be used with circuits that have more than one source. This example illustrates how it can be use with a circuit having one. Consider the first-order all-pass filter shown in Fig. 15(a). An equivalent circuit is shown in Fig. 15(b) in which superposition can be used to write by inspection

$$V_o = \left(1 + \frac{R_1}{R_1}\right)\frac{RCs}{1 + RCs}V_i - \frac{R_1}{R_1}V_i = \frac{RCs - 1}{RCs + 1}V_i$$
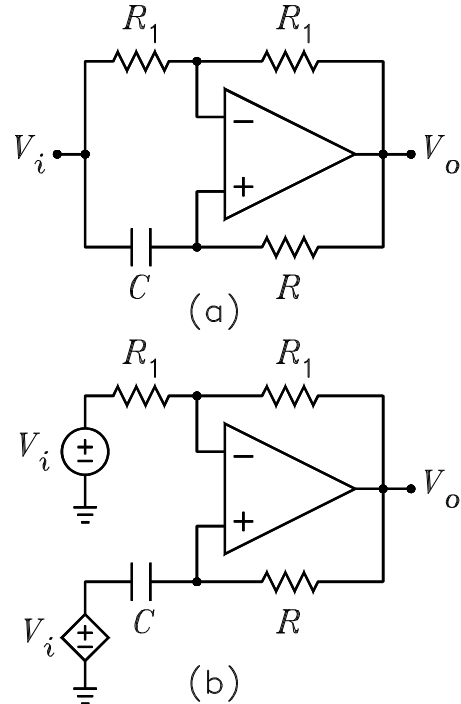

Fig. 15. Circuit for Example 14.

V. CONCLUSION

Superposition of dependent sources is valid in writing circuit equations if it is applied correctly. Often, it can be used to obtain solutions almost trivially by inspection. When all sources but one are deactivated, the circuit must not contain a node at

which the voltage is indeterminate or a branch in which the current is indeterminate. In such cases, superposition cannot be used even if all sources are independent.

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IEEE TRANSACTIONS ON EDUCATION

Prof. W. Marshall Leach
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Atlanta, GA   30332-0250

Re: Trans. on Ed. ms. # 94-103

Dear Prof. Leach,

After evaluation of your manuscript, 'On the Application of Superposition to Circuits
Containing Controlled Sources,' I feel that I must reject it for publication in the IEEE
Transactions on Education. Enclosed are reviewer comments to help explain our decision.

We appreciate your interest in the Transactions and hope that you will continue to consider it a
desirable place to publish your work.

Very truly yours,

Frank S. Barnes
Editor

FSB/ck

June 30, 1994

Dr. Frank Barnes   Re: #94-103
Dept. of Elec. & Comp. Engineering
University of Colorado
Boulder, CO 80309-0425

Dear Frank:

   I have read the paper by Marshall Leach and it is not correct. He states that controlled sources can be considered the same as independent sources and considered one at a time to analyze a circuit using "superposition." He picks two simple examples where his method gives the right answer.

   Since a resistor can be considered as a voltage controlled current source with the current source equal to the voltage across the resistor multiplied by 1/R, or as a current controlled voltage source with the parameter R, then all circuit elements should be ameanable to superposition.

   As a counter example consider the circuit below. It has a perfectly deterministic set of voltages and currents. It cannot be analyzed by Marshall Lynch's method.