# ECE 6416 Assignment 3

1. For Problem 3 of Assignment 2, show that $F = 19.78$ and $NF = 12.96\,\text{dB}$ with $R_1$ and $C_1$ in the circuit. Show that $F = 6$ and $NF = 7.782\,\text{dB}$ if $R_1$ is replaced by an open circuit and $C_1$ is replaced by a short circuit.

2. For Problem 4 of Assignment 2:

   (a) Use the $v_{ni}$ found in part (c) of the problem to show that $F = 67.24$ and $NF = 18.28\,\text{dB}$.

   (b) Show that Eq. (6.71) gives the same results. The intermediate answers are $F_1 = 66$, $F_2 = 3125$, $G_{a1} = 2521$.

3. The noise figure of an op amp is $NF = 5\,\text{dB}$ with a source resistance of $R_s = 10\,\text{k}\Omega$.

   (a) Show that $v_{ni}/\sqrt{\Delta f} = 22.49\,\text{nV}/\sqrt{\text{Hz}}$.

   (b) Show that the noise temperature is $T_n = 627\,\text{K}$.

   (c) Show that a resistor of value $21.6\,\text{k}\Omega$ at the op amp input would generate the same noise as the op amp.

4. Given $G_n$, $F_{min}$, and $Z_{opt} = R_{opt} + jX_{opt}$ for an amplifier, use Eqs. (6.8), (6.13), and (6.14) to show that

$$i_n^2 = 4kTG_n\Delta f \qquad \gamma_i = \frac{-\operatorname{sgn}(X_{opt})}{\sqrt{1 + (R_{opt}/X_{opt})^2}}$$

$$v_n^2 = \left(\frac{X_{opt}}{\gamma_i}\right)^2 i_n^2 \qquad \gamma_r = \frac{2kT_0\Delta f}{\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}}(F_{min} - 1) - \sqrt{1 - \gamma_i^2}$$

   where $\operatorname{sgn}(X_{opt}) = X_{opt}/|X_{opt}|$.

5. An amplifier has an input resistance of $150\,\Omega$. Its noise parameters are $v_n/\sqrt{\Delta f} = 2\,\text{nV}/\sqrt{\text{Hz}}$, $i_n/\sqrt{\Delta f} = 10\,\text{pA}/\sqrt{\text{Hz}}$, and $\gamma = 0$. It is driven from a source having an output resistance of $75\,\Omega$.

   (a) Show that $v_{ni}/\sqrt{\Delta f} = 2.401\,\text{nV}/\sqrt{\text{Hz}}$.

   (b) Show that $F = 4.802$ and $NF = 6.814\,\text{dB}$.

   (c) A resistor $R_1$ is added in series with the source to make the source impedance seen by the amplifier equal to $Z_{opt}$. Show that $R_1 = 125\,\Omega$. If the resistor is considered to be part of the source, not the amplifier, show that $F = 3.5$ and $NF = 5.441\,\text{dB}$.

   (d) The result for $F$ above illustrates the noise factor fallacy. For a proper noise analysis, $R_1$ must be considered to be part of the amplifier, not the source. Show that the correct values are $F = 9.333$ and $NF = 9.7\,\text{dB}$.

   (e) Show that $R_1$ reduces the $SNR$ by $2.886\,\text{dB}$.

6. The source in the amplifier of problem 5 is changed to one having an output resistance $R_s = 1\,\mathrm{k}\Omega$.

   (a) Show that $v_{ni}/\sqrt{\Delta f} = 10.95\,\mathrm{nV}$.

   (b) Show that $F = 7.5$ and $NF = 8.751\,\mathrm{dB}$.

   (c) A resistor $R_2$ is added in parallel with the source to make the source impedance seen by the amplifier equal to $Z_{opt}$. Show that $R_2 = 250\,\Omega$. If the resistor is considered to be part of the source, not the amplifier, show that $F = 3.5$ and $NF = 5.441\,\mathrm{dB}$.

   (d) The result for $F$ above illustrates the noise factor fallacy. For a proper noise analysis, the parallel resistor must be considered to be part of the amplifier, not the source. Show that the correct values are $F = 17.5$ and $NF = 12.43\,\mathrm{dB}$.

   (e) Show that $R_2$ reduces the $SNR$ by 3.68 dB.

7. An amplifier has a voltage gain of 200 and an input resistance of $5\,\mathrm{k}\Omega$. With a resistor of value $5\,\mathrm{k}\Omega$ connected in parallel with its input, the output noise measures $447\,\mu\mathrm{V}$ over a noise bandwidth of 100 kHz. The $5\,\mathrm{k}\Omega$ resistor is removed and a white noise source is connected through an attenuator to the input of the amplifier. The attenuator consists of a series $30\,\mathrm{k}\Omega$ resistor and a shunt $6\,\mathrm{k}\Omega$ resistor. The output resistance of the attenuator is $5\,\mathrm{k}\Omega$. The source voltage over a noise bandwidth of 100 kHz has the value $v_n = 53.7\,\mu\mathrm{V}$. With the source activated, show that the noise output voltage from the amplifier increases to 1 mV. Use this information to show that $F = 2.5$ and $NF = 3.98\,\mathrm{dB}$.

8. An amplifier is connected to a source with an output resistance $R_s$ through a lossy transmission line having a characteristic impedance $Z_c = R_s$. If the loss in the cable is $k$ dB, show that the noise figure is increased by $k$ dB. Hint: Let $K = 10^{-k/20}$. The open-circuit voltage at the amplifier input is $V_{i(oc)} = KV_s + V_{ts} + V_n + I_n R_s = K\left(V_s + V_{ni}\right)$. It follows that $V_{ni} = \left(V_{ts} + V_n + I_n R_s\right)/K$ and $F = v_{ni}^2/v_{ts}^2$. Compare the value of $F$ with $k = 0$ to the value for $k > 0$.