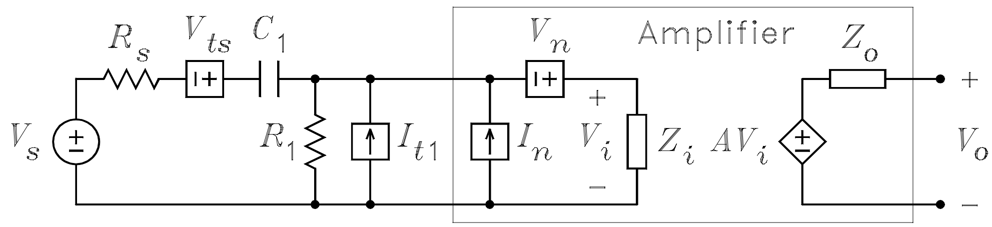
# ECE 6416 Assignment 2

1. The figure shows the noise model of an amplifier. The frequency is $f = 1\,\text{kHz}$. It is given that $R_s = 1\,\text{k}\Omega$, $R_1 = 2\,\text{k}\Omega$, $C_1 = 0.1\,\mu\text{F}$, $v_n/\sqrt{\Delta f} = 4\,\text{nV}/\sqrt{\text{Hz}}$, $i_n/\sqrt{\Delta f} = 8\,\text{pA}/\sqrt{\text{Hz}}$, $\gamma = 0$, $Z_i = 2800\angle -45\,°$, $A = 20\angle 45\,°$, $Z_o = 1200\angle 30\,°$. The open-circuit input voltage with $Z_i = \infty$ can be written $V_{i(oc)} = KV_s + V'_{ni} = K\,(V_s + V'_{ni}/K)$, where $K = V_{i(oc)}/V_s$ when all noise sources are neglected and $V'_{ni}$ is the open-circuit input voltage due to all noise sources. The latter is given by $V'_{ni} = V_{teq} + V_n + I_n Z_{eq}$, where $V_{teq}$ is the thermal noise generated by the real part of the impedance $Z_{eq} = R_1 \| (R_s + 1/j\omega C_1)$. It follows that the equivalent noise voltage in series with $V_s$ is $V_{ni} = V'_{ni}/K$.



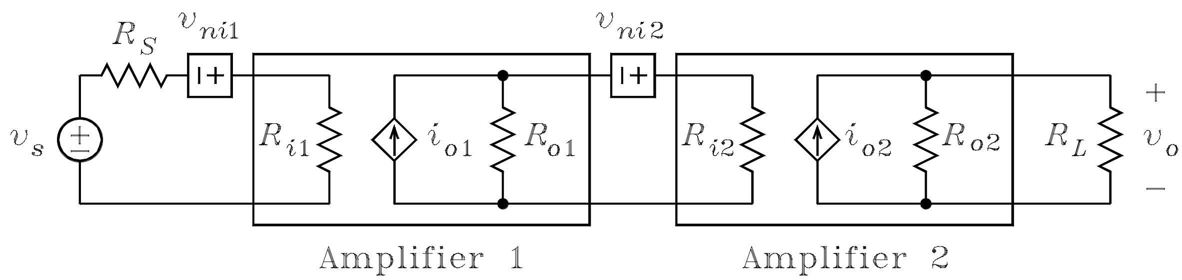

   (a) Show that $|K| = 0.5889$ and $v'_{ni}/\sqrt{\Delta f} = 10\,\text{nV}$.

   (b) Show that $v_{ni}/\sqrt{\Delta f} = 17.79\,\text{nV}$.

   (c) If $R_1$ is replaced by an open circuit and $C_1$ is replaced by a short circuit, show that $v_{ni}/\sqrt{\Delta f} = 9.798\,\text{nV}$. This calculation illustrates how the addition of series and parallel elements increase the noise.

   (d) Show that the addition of $C_1$ and $R_1$ increases the noise by 5.18 dB.

2. The figure shows the model of a two-stage amplifier. It is given that $R_S = 100\,\Omega$, $R_{i1} = R_{i2} = 1.2\,\text{k}\Omega$, $R_{o1} = R_{o2} = 5\,\text{k}\Omega$, $R_L = 2\,\text{k}\Omega$, $I_{o1} = g_{m1}V_{i1}$, $I_{o2} = g_{m2}V_{i2}$, $g_{m1} = g_{m2} = 13^{-1}\,\text{S}$, $v_{n1}/\sqrt{\Delta f} = v_{n2}/\sqrt{\Delta f} = 2\,\text{nV}/\sqrt{\text{Hz}}$, and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 100\,\text{pA}/\sqrt{\text{Hz}}$. The noise sources can be assumed to be uncorrelated. $V_{ni1}$ models the noise generated both by $R_S$ and the first amplifier. It is given by $V_{ni1} = V_{ts} + V_{n1} + I_{n1}R_S$. $V_{ni2}$ models the noise generated by the second stage and is given by $V_{ni2} = V_{n2} + I_{n2}R_{o1}$. The noise generated by $R_L$ can be neglected. All other resistors are noiseless because their noise is contained in $V_{ni1}$ and $V_{ni2}$.



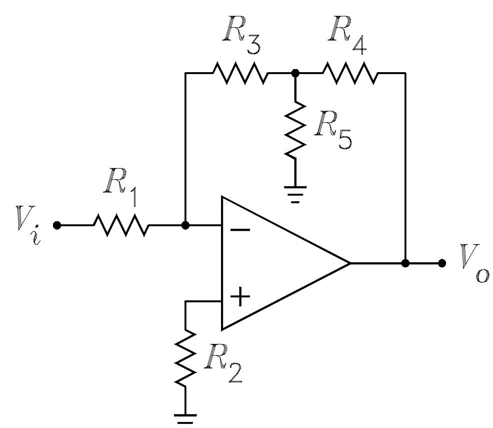

   (a) Show that the voltage gain can be written as a product of terms as follows:

$$A = \frac{g_{m1}R_{i1}}{R_S + R_{i1}} \times R_{o1} \times \frac{g_{m2}R_{i2}}{R_{o1} + R_{i2}} \times (R_{o2} \| R_L) = 7551 \;\; (77.6\,\text{dB})$$

(b) Show that components of $v_{ni}/\sqrt{\Delta f}$ due to $v_{ts}$, $v_{n1}$, $i_{n1}$, $v_{n2}$, and $i_{n2}$, respectively, are $1.265\,\text{nV}/\sqrt{\Delta f}$, $2\,\text{nV}/\sqrt{\Delta f}$, $10\,\text{nV}/\sqrt{\Delta f}$, $5.633\,\text{pV}/\sqrt{\Delta f}$, and $1.408\,\text{nV}/\sqrt{\Delta f}$.

(c) Show that $v_{ni}/\sqrt{\Delta f} = 10.37\,\text{nV}$.

3. The figure shows an op amp circuit. Except for noise, the op amp is ideal. It is given that $R_1 = 1\,\text{k}\Omega$, $R_2 = 500\,\Omega$, $R_3 = 901\,\text{k}\Omega$, $R_4 = 9.9\,\text{k}\Omega$, and $R_5 = 100\,\Omega$. The op amp noise is modeled by a noise voltage source $V_n$ in series with the non-inverting input and two noise current sources $I_{n1}$ and $I_{n2}$ from each input to ground. It is given that $v_n/\sqrt{\Delta f} = 10\,\text{nV}/\sqrt{\text{Hz}}$ and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 1\,\text{pA}/\sqrt{\text{Hz}}$. All correlation effects can be neglected.




(a) Show that the voltage at the non-inverting input is given by

$$V_+ = V_n + (I_{n1} + I_{t2})\,R_2$$

(b) Show that the voltage at the inverting input is given by

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_4 + R_3\|R_5}\frac{R_5}{R_3 + R_5} + I_{n2} + I_{teq}\right)[R_1\|\,(R_3 + R_4\|R_5)]$$

where $I_{teq}$ is the noise current generated by the resistance $R_{eq} = R_1\|\,(R_3 + R_4\|R_5)$.

(c) Set $V_+ = V_-$. With all noise terms zeroed, show that the voltage gain has the value $A = V_o/V_i = -90.1 \times 10^3$.

(d) With $V_i = 0$ and the thermal noise sources for the resistors activated, use the equation from parts 3a and 3b to show that the total spot noise output voltage due to the thermal noise of all resistors has the value $v_{no}/\sqrt{\Delta f} = 442\,\mu\text{V}/\sqrt{\text{Hz}}$.

(e) Use the equation from parts 3a and 3b to show that the spot noise output voltage due to the op-amp noise sources alone has the value $v_{no}/\sqrt{\Delta f} = 908\,\mu\text{V}/\sqrt{\text{Hz}}$.

(f) Use rms addition of the thermal noise and op-amp noise to show that the total spot noise output voltage is $v_{no}/\sqrt{\Delta f} = 1.01\,\text{mV}/\sqrt{\text{Hz}}$.

(g) Divide $v_{no}/\sqrt{\Delta f}$ by $A$ to show that the equivalent spot noise voltage in series with $v_i$ is $v_{ni}/\sqrt{\Delta f} = 11.2\,\text{nV}/\sqrt{\text{Hz}}$.

4. An amplifier has a voltage gain $V_o/V_i = 10$ (20 dB) and an input resistance $R_i = 10\,\mathrm{k}\Omega$. It is driven from a voltage source which has an open-circuit rms output voltage $V_s = 1\,\mathrm{V}$ and an output resistance $R_s = 1\,\mathrm{k}\Omega$. The rms noise at the amplifier output in the band from 100 Hz to 100 kHz is $v_{no} = 28.4\,\mu\mathrm{V}$. When a $9\,\mathrm{k}\Omega$ resistor is added in series with $R_s$, the rms output noise increases to $v'_{no} = 67.5\,\mu\mathrm{V}$. If the $V_n$ and $I_n$ of the amplifier are not correlated, show that $v_n = 2.56\,\mu\mathrm{V}$ and $i_n = 1.26\,\mathrm{nA}$, where each are calculated over the band from 100 Hz to 100 kHz.

5. $N$ identical amplifiers having $v_n/\sqrt{\Delta f} = 1.09\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and $i_n/\sqrt{\Delta f} = 11.4\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ are driven from a source having the output resistance $R_s = 10\,\Omega$. Show that it would take 10 amplifiers in parallel to minimize the noise. For $N = 10$, show that $v_{ni}/\sqrt{\Delta f} = 639\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$.

6. An input transformer is to be used to connect the source of problem 5 to the amplifier. If the winding resistances of the transformer can be neglected, show that the noise is a minimum if the transformer has the turns ratio $n = 3.1$. Show that $v_{ni}/\sqrt{\Delta f} = 639\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$.