# ECE 6416 Assignment 1

1. Two resistors $R_1$ and $R_2$ are connected in parallel. The two resistors are in thermal equilibrium.

   (a) Suppose that only $R_1$ generates thermal noise and $R_2$ is noiseless, show that the average thermal noise power delivered by $R_1$ to $R_2$ in the band $\Delta f$ is given by

   $$P_{12} = \frac{4kTR_1R_2\Delta f}{(R_1 + R_2)^2}$$

   (b) Suppose that only $R_2$ generates thermal noise and $R_1$ is noiseless, show that the average thermal noise power $P_{21}$ delivered by $R_2$ to $R_1$ in the band $\Delta f$ is given by the same expression obtained above.

   (c) Note that $P_{12} = P_{21}$. If the two answers were not the same, could the two resistors be in thermal equilibrium? How would the temperatures of the individual resistors vary with time if $P_{12} > P_{21}$?

2. A lossy inductor having an air core can be modeled as and ideal inductor $L$ in series with a resistor $R$, where $R$ is the winding inductance. Let a capacitor $C$ be connected in parallel with the lossy inductor.

   (a) With $s = j\omega$, where $\omega = 2\pi f$, solve for the complex impedance of the network. Use the general Nyquist equation $v_n^2 = 4kT\,\mathrm{Re}\,(Z)\,\Delta f$ to show that the mean-square thermal noise voltage across the circuit in the frequency band $\Delta f$ is given by

   $$v_n^2 = \frac{4kTR\Delta f}{(1 - \omega^2 LC)^2 + (\omega RC)^2}$$

   (b) Replace the resistor with its Thévenin noise model. Use voltage division to show that the phasor noise voltage $V_n$ across the circuit is given by

   $$V_n = \frac{V_t}{1 - \omega^2 LC + j\omega RC}$$

   where $V_t$ is the thermal noise voltage generated by the resistor. Show that $v_n^2 = \overline{V_n V_n^*}$ gives the same answer as the one obtained above.

3. A resistor $R$ and an ideal capacitor $C$ are connected in parallel. The two are in thermal equilibrium. This means that the thermal noise power generated by the resistor that is absorbed by the capacitor must equal the thermal noise power generated by the capacitor that is absorbed by the resistor. Otherwise, one would be heating up while the other is cooling off and the two would not be in thermal equilibrium.

   (a) Use the Thévenin noise model of the resistor and denote its thermal phasor noise voltage by $V_t$, where $v_t^2 = \overline{V_t V_t^*} = 4kTR\Delta f$ is the mean-square noise voltage in the band $\Delta f$ at the frequency of analysis. Show that the phasor spot noise voltage $V_n$ across the capacitor and phasor spot noise current $I_n$ through the capacitor are given by

   $$V_n = V_t \frac{1}{1 + j\omega RC} \qquad I_n = V_t \frac{j\omega C}{1 + j\omega RC}$$

(b) The power absorbed by the capacitor is given by $P_C = \operatorname{Re}\left(\overline{V_n I_n^*}\right)$. Show that this is zero. (Note, we are using the convention that the magnitude of a noise phasor is the rms value, not the peak value, so that there is no factor of $1/2$ in the expression for $P_C$.)

(c) Because $P_C = 0$, it follows that the capacitor cannot absorb power from the resistor. How does this imply that the capacitor cannot generate noise power?

(d) Repeat the problem for an ideal inductor $L$ in parallel with a resistor $R$. Show that $V_n$ and $I_n$ are given by

$$V_n = \frac{V_t}{R}\frac{j\omega L}{1 + j\omega L/R} \qquad I_n = \frac{V_t}{R}\frac{1}{1 + j\omega L/R}$$

4. An lossy inductor $L$ has a series winding resistance $R$. The inductor can be modeled as an ideal inductor in series with a discrete resistor.

(a) Draw the circuit with a thermal phasor noise voltage $V_t$ in series with the resistor. Calculate the short-circuit noise current $I_n = V_t/Z$, where $Z$ is the complex impedance of the series circuit. Show that the mean-square short-circuit noise current is given by

$$i_n^2 = \overline{I_n I_n^*} = \frac{4kT\Delta f}{R}\frac{1}{1 + (2\pi f L/R)^2}$$

(b) Show that the answer above can be obtained from the expression

$$i_n^2 = 4kT \operatorname{Re}(Y)\,\Delta f$$

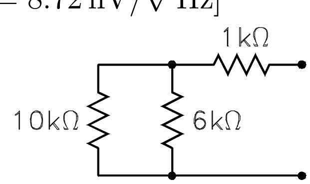
where $Y$ is the complex admittance of the circuit. Note, this result can be thought of as the dual of the formula $v_n^2 = 4kT \operatorname{Re}(Z)\,\Delta f$ that was used in class to calculate the mean-square open-circuit voltage of a parallel $RC$ network.

(c) Integrate the mean-square noise current to show that the total mean-square thermal noise current generated by the inductor, i.e. the noise in the band $0 \le f \le \infty$, is given by

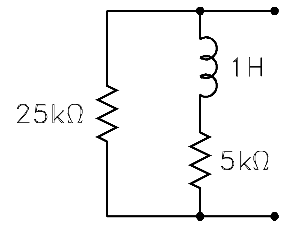
$$i_{total}^2 = \frac{kT}{L}$$

Hint: Let $x = 2\pi f L/R$ and $df = (R/2\pi L)\,dx$. The integral which must be evaluated can be put into the form $\int_0^\infty dx/\left(1 + x^2\right) = \left[\tan^{-1} x\right]_0^\infty = \pi/2$.

5. Calculate the thermal spot noise voltage in $\mathrm{V}/\sqrt{\mathrm{Hz}}$ at the standard temperature across the terminals of the circuit $[v_{rms} = 8.72\,\mathrm{nV}/\sqrt{\mathrm{Hz}}]$



6. Calculate the spot noise voltage at the output of the circuit at the frequency $f = 1.5\,\text{kHz}$. Assume $T = T_0 = 290\,\text{K}$. [$9.83\,\text{nV}/\sqrt{\text{Hz}}$]




7. A $1\,\text{M}\Omega$ resistor has a dc voltage across it of $4\,\text{V}$. At the frequency $f = 100\,\text{Hz}$, the spot noise voltage across the resistor is $v_n/\sqrt{\Delta f} = 400\,\text{nV}/\sqrt{\text{Hz}}$.

(a) Show the flicker noise coefficient is $K_f = 9 \times 10^{-13}$.

(b) Show that the noise index is $NI = 3.17\,\text{dB}$.

(c) The mean-square short-circuit noise current generated by the resistor is given by

$$i_n^2 = \frac{4kT\Delta f}{R} + \frac{K_f I_{DC}^2 \Delta f}{f}$$

Show that the flicker noise corner frequency is $f_{flk} = 900\,\text{Hz}$.

8. A $100\,\text{mH}$ lossy inductor has a measured impedance magnitude of $8\,\text{k}\Omega$ at the frequency $f = 10\,\text{kHz}$. Show that the open-circuit thermal spot noise voltage generated by the inductor at $10\,\text{kHz}$ is $v_t/\sqrt{\Delta f} = 8.9\,\text{nV}/\sqrt{\text{Hz}}$. Note that $|Z|^2 = R^2 + (\omega L)^2$ for the inductor.

9. A resistor $R$ and a capacitor $C$ are connected in parallel to form a two-terminal network. Use the Norton noise model of the resistor to show that the phasor short-circuit noise current $I_{n(sc)}$ and the phasor open-circuit noise voltage $V_{n(oc)}$ are given by

$$I_{n(sc)} = I_t \qquad V_{n(oc)} = \frac{I_t R}{1 + j\omega RC}$$

Show that the mean-square spot noise values are given by

$$i_{n(sc)}^2 = \frac{1}{\Delta f}\overline{I_{n(sc)} I_{n(sc)}^*} = \frac{4kT}{R} \qquad v_{n(oc)}^2 = \frac{1}{\Delta f}\overline{V_{n(oc)} V_{n(oc)}^*} = \frac{4kTR}{1 + (\omega RC)^2}$$

the complex correlation coefficient between $V_{n(oc)}$ and $I_{n(sc)}$ is given by

$$\gamma = \frac{\overline{V_{n(oc)} I_{n(sc)}^*}}{v_{n(oc)} i_{n(sc)}} = \frac{\sqrt{1 + (\omega RC)^2}}{1 + j\omega RC}$$

10. If the diode generates only shot noise and the resistor generates only thermal noise, solve for the rms noise output voltage over the band from $1\,\text{kHz}$ to $3.5\,\text{kHz}$. The diode is modeled as a shot noise current source in parallel with a small-signal resistance given

by $r_d = \eta V_T/I_D$, where $\eta$ is the emission coefficient or idealty factor and $I_D$ is the dc current in the diode. Assume $\eta = 2$ and $V_T = 25\,\mathrm{mV}$. $[v_{rms} = 23.9\,\mathrm{nV}]$