ECE3050 Mason's Flow Graph Formula

Amplifier analysis using superposition is often facilitated by the use of signal flow graphs. A signal flow graph, or flow graph for short, is a graphical representation of a set of linear equations which can be used to write by inspection the solution to the set of equations. For example, consider the set of equations

$$x_2 = Ax_1 + Bx_2 + Cx_5 \tag{1}$$

$$x_3 = Dx_1 + Ex_2 \tag{2}$$

$$x_4 = Fx_3 + Gx_5 \tag{3}$$

$$x_5 = Hx_4 \tag{4}$$

$$x_6 = Ix_3 \tag{5}$$

where x_1 through x_6 are variables and A through I are constants. These equations can be represented graphically as shown in Fig. 1. The graph has a node for each variable with branches connecting the nodes labeled with the constants A through I. The node labeled x_1 is called a source node because it has only outgoing branches. The node labeled x_6 is called a sink node because it has only incoming branches. The path from x_1 to x_2 to x_3 to x_6 is called a forward path because it originates at a source node and terminates at a non-source node and along which no node is encountered twice. The path gain for this forward path is AEI. The path from x_2 to x_3 to x_4 to x_5 and back to x_2 is called a feedback path because it originates and terminates on the same node and along which no node is encountered more than once. The loop gain for this feedback path is $EFHC$.

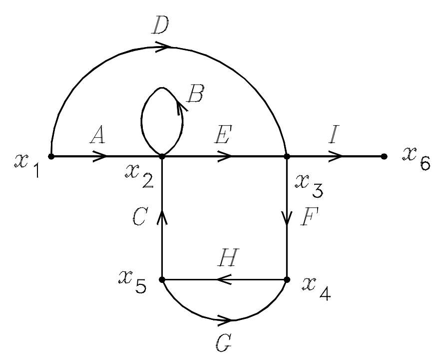

Figure 1: Example signal flow graph.

Mason's formula can be used to calculate the transmission gain from a source node to any non-source node in a flow graph. The formula is

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k \tag{6}$$

where P_k is the gain of the kth forward path, Δ is the graph determinant, and Δ_k is the determinant with the kth forward path erased. The determinant is given by

$$\begin{aligned}\Delta &= 1 - (\text{sum of all loop gains}) \\ &\quad + \left(\begin{array}{c}\text{sum of the gain products of all possible} \\ \text{combinations of two non-touching loops}\end{array}\right) \\ &\quad - \left(\begin{array}{c}\text{sum of the gain products of all possible} \\ \text{combinations of three non-touching loops}\end{array}\right) \\ &\quad + \left(\begin{array}{c}\text{sum of the gain products of all possible} \\ \text{combinations of four non-touching loops}\end{array}\right) \\ &\quad - \cdots \end{aligned} \tag{7}$$

For the flow graph in Fig. 1, the objective is to solve for the gain from node x_1 to node x_6. There are two forward paths from x_1 to x_6 and three loops. Two of the loops do not touch each other. Thus the product of these two loop gains appears in the expression for Δ. The path gains and the determinant are given by

$$P_1 = AEI \tag{8}$$

$$P_2 = DI \tag{9}$$

$$\Delta = 1 - (B + CEFH + GH) + B \times GH \tag{10}$$

Path P_1 touches two loops while path P_2 touches one loop. The determinants with each path erased are given by

$$\Delta_1 = 1 - GH \tag{11}$$

$$\Delta_2 = 1 - (B + GH) + B \times GH \tag{12}$$

Thus the overall gain from x_1 to x_6 is given by

$$\frac{x_6}{x_1} = \frac{AEI \times (1 - GH) + DI \times [1 - (B + GH) + B \times GH]}{1 - (B + CEFH + GH) + B \times GH} \tag{13}$$