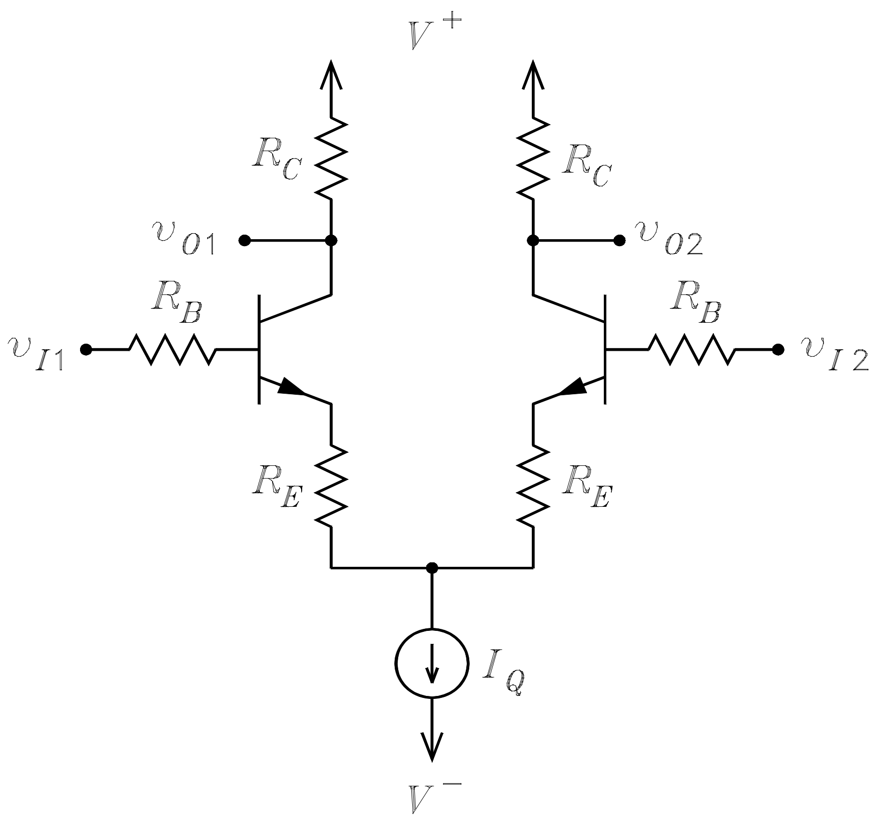
BJT Differential Amplifier Example

$R_P(x,y) := \frac{x \cdot y}{x+y}$ Function for calculating parallel resistors.

$R_C := 20000$ $R_B := 1000$ $R_E := 100$ $I_Q := 0.001$

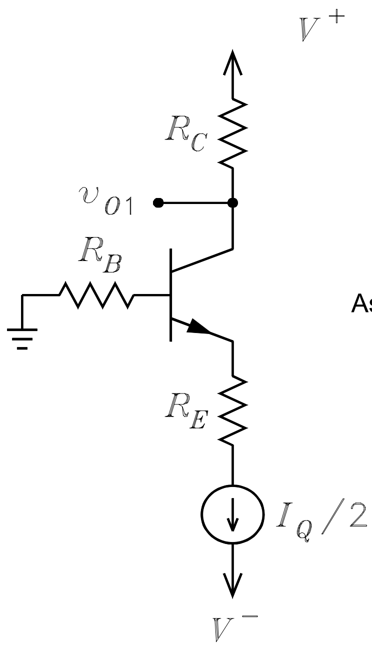
$V_p := 20$ $V_m := -20$ $V_{BE} := 0.65$ $V_T := 0.025$ $\beta := 199$ $\alpha := \frac{\beta}{1+\beta}$

$r_x := 20$ $r_0 := 50000$



There are two ac solutions, one for the second input zeroed and one for the first input zeroed. By superposition, the total solution would the be sum of these two. To keep Mathcad happy, all source voltages are taken to be equal to 1 V so that the output voltage is equal to the voltage gain. In general, the output voltage is equal to the voltage gain multiplied by the source voltage.

DC Bias Solution



Assume the dc value of the sources is zero.

$I_{E1} := \frac{I_Q}{2}$ $\quad I_{E1} = 5 \cdot 10^{-4}$ $\quad I_{E2} := I_{E1}$

$V_{C1} := V_p - \alpha \cdot I_{E1} \cdot R_C$ $\quad V_{C1} = 10.05$

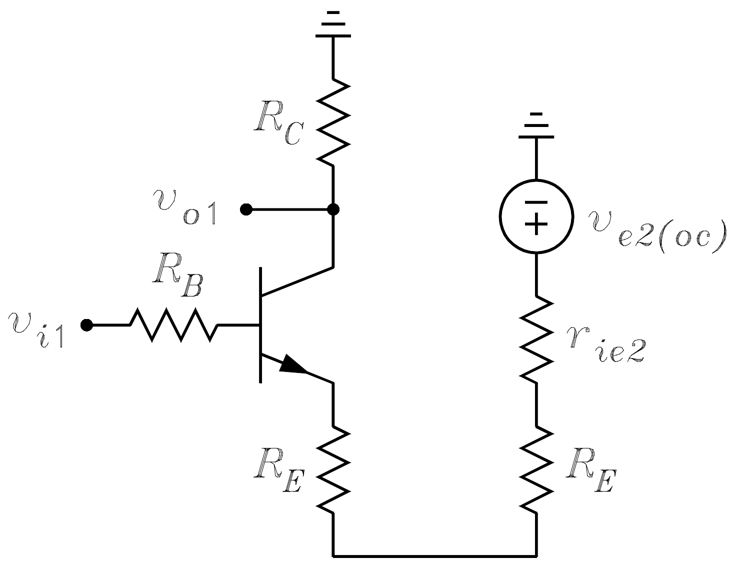
$V_{B1} := \frac{-I_{E1}}{1+\beta} \cdot R_B$ $\quad V_{B1} = -2.5 \cdot 10^{-3}$

$V_{CB1} := V_{C1} - V_{B1}$ $\quad V_{CB1} = 10.0525$ Thus active mode. Same for Q2.

$r_{e1} := \frac{V_T}{I_{E1}}$ $\quad r_{e1} = 50$ $\quad r_{e2} := r_{e1}$

$r'_{e1} := \frac{R_B + r_x}{1+\beta} + r_{e1}$ $\quad r'_{e1} = 55.1$ $\quad r'_{e2} := r'_{e1}$

AC Solutions



Circuit for the first output.

$v_{i1} := 1$ $\quad v_{i2} := 1$

With the input equal to 1, the voltage gain is equal to the output voltage.

$$v_{e2oc} := v_{i2} \cdot \frac{r_0 + \frac{R_C}{1+\beta}}{r'_{e2} + r_0 + \frac{R_C}{1+\beta}}$$

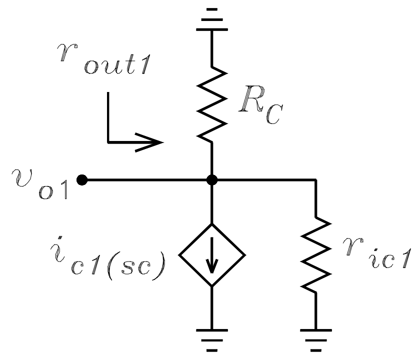
$v_{e2oc} = 0.9989$

$$r_{ie2} := r'_{e2} \cdot \frac{r_0 + R_C}{r'_{e2} + r_0 + \frac{R_C}{1+\beta}}$$

$r_{ie2} = 76.9015$

$v_{tb1} := v_{i1}$ $\quad R_{tb1} := R_B$

$v_{te1} := v_{e2oc}$ $\quad R_{te1} := 2 \cdot R_E + r_{ie2}$ $\quad R_{te1} = 276.9015$



$$G_{mb1} := \frac{\alpha}{r'_{e1} + R_{te1}} \cdot \frac{r_0 - \frac{R_{te1}}{\beta}}{r_0 + R_P(r'_{e1}, R_{te1})}$$ $G_{mb1} = 2.9941 \bullet 10^{-3}$

$$G_{me1} := \frac{\alpha}{r'_{e1} + R_{te1}} \cdot \frac{r_0 + \frac{r'_{e1}}{\alpha}}{r_0 + R_P(r'_{e1}, R_{te1})}$$ $G_{me1} = 2.9975 \bullet 10^{-3}$

$$r_{ic1} := \frac{r_0 + R_P(r'_{e1}, R_{te1})}{1 - \frac{\alpha \cdot R_{te1}}{r'_{e1} + R_{te1}}}$$ $r_{ic1} = 2.9416 \bullet 10^{5}$

Voltage gain from first input to first output:

$i_{c1sc} := G_{mb1} \cdot v_{tb1}$ $i_{c1sc} = 2.9941 \bullet 10^{-3}$

$v_{o1} := -i_{c1sc} \cdot R_P(r_{ic1}, R_C)$ $A_{v1} := v_{o1}$

$A_{v1} = -56.0705$

This is the voltage gain from the first input to the first output. The gain from the second input to the second output is the same.

Voltage gain from the second input to the first output.

$i_{c1sc} := -G_{me1} \cdot v_{te1}$ $\qquad$ $i_{c1sc} = -2.9942 \bullet 10^{-3}$

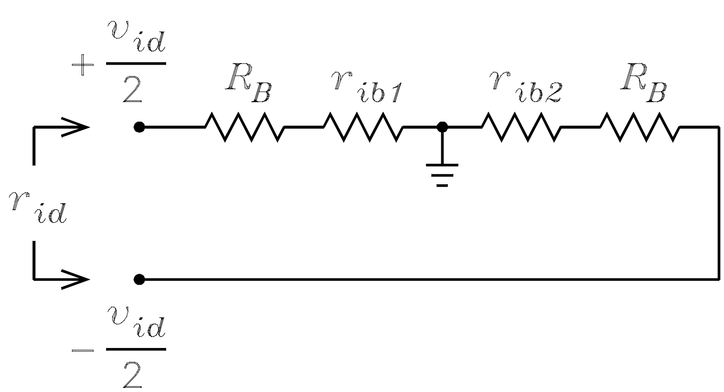
$v_{o1} := -i_{c1sc} \cdot R_P(r_{ic1}, R_C)$ $\qquad$ $A_{v2} := v_{o1}$

$A_{v2} = 56.0725$ $\qquad$ This is the voltage gain from the second input to the first output. The gain from the first input to the second output is the same.

$v_{o1} := -56.0705 \cdot v_{i1} + 56.0725 \cdot v_{i2}$ $\qquad$ This is the sum ac output from Q1.

$v_{o2} := -56.0705 \cdot v_{i2} + 56.0725 \cdot v_{i1}$ $\qquad$ This is the sum ac output from Q2.

Differential input resistance.



$$r_{ib1} := r_x + (1+\beta) \cdot \left(r_{e1} + R_P\left(R_{te1}, r_0 + R_C\right) \right) - \frac{\beta \cdot R_{te1} \cdot R_C}{R_{te1} + r_0 + R_C}$$

$r_{ib1} = 4.95 \bullet 10^{4}$ $\qquad$ $r_{ib2} := r_{ib1}$

$r_{id} := 2 \cdot R_B + r_{ib1} + r_{ib2}$ $\qquad$ $r_{id} = 1.01 \bullet 10^{5}$

Common-Mode Rejection Ratio

$A_{v1} = -56.0705$ $A_{v2} = 56.0725$

Let us take the output from the collector of the first transistor. Because neither β nor r_0 is infinity, the two voltage gains are not equal. This causes the CMRR to be non infinite. We calculate it below.

$v_{id} := 1$ $v_{i1} := \frac{v_{id}}{2}$ $v_{i2} := \frac{-v_{id}}{2}$

$v_{o1} := A_{v1} \cdot v_{i1} + A_{v2} \cdot v_{i2}$ $A_d := v_{o1}$

$A_d = -56.0715$ This is the differential voltage gain.

$v_{icm} := 1$ $v_{i1} := v_{icm}$ $v_{i2} := v_{icm}$

$v_{o1} := A_{v1} \cdot v_{i1} + A_{v2} \cdot v_{i2}$ $A_{cm} := v_{o1}$

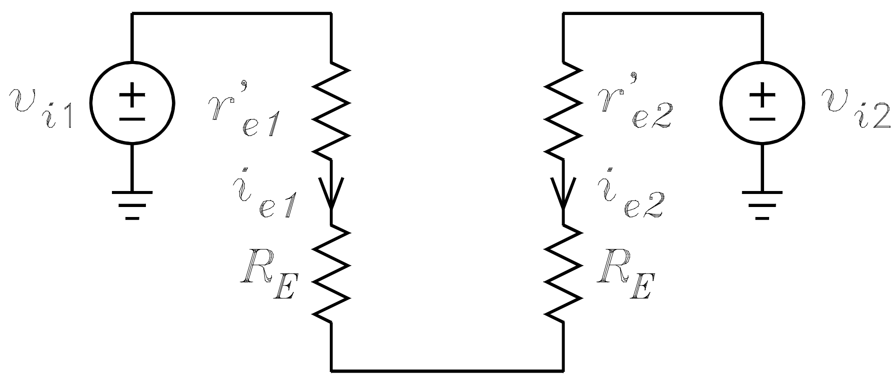
$A_{cm} = 1.9938 \cdot 10^{-3}$ This is the common mode voltage gain.

$CMRR := \left| \frac{A_d}{A_{cm}} \right|$ $CMRR = 2.8123 \cdot 10^{4}$

$CMRR_{dB} := 20 \cdot \log(CMRR)$ $CMRR_{dB} = 88.9811$

If R_Q (the ac resistance of the current source) is not infinity, the CMRR would be lower.

Solution with the r_0 approximations. We neglect r_0 except in calculating r_{ic}. Thus we can use the emitter equivalent circuit to solve for i_{e1} and i_{e2}, then multiply by α to solve for the collector currents. Because the common mode gain is zero if we neglect r_0, we will assume a differential input signal.



$v_{id} := 1$ $\quad v_{i1} := \frac{v_{id}}{2}$ $\quad v_{i2} := \frac{-v_{id}}{2}$ Differential input signal of 1 V.

$i_{e1} := \frac{v_{i1} - v_{i2}}{r'_{e1} + 2 \cdot R_E + r_{ie2}}$ $\quad i_{e1} = 3.012 \cdot 10^{-3}$ $\quad i_{e2} := -i_{e1}$

$v_{o1} := -\alpha \cdot i_{e1} \cdot R_P(R_C, r_{ic1})$ $\quad v_{o1} = -56.1236$ This is the differential voltage gain to the first output.

$v_{o2} := -v_{o1}$ $\quad v_{o2} = 56.1236$ This is the differential voltage gain to the second output.

$r_{ib1} := r_x + (1 + \beta) \cdot (r_{e1} + R_{te1})$ $\quad r_{ib1} = 6.54 \cdot 10^4$ $\quad r_{ib2} := r_{ib1}$

$r_{id} := 2 \cdot R_B + r_{ib1} + r_{ib2}$ $\quad r_{id} = 1.328 \cdot 10^5$ This is the differential input resistance.

There is more error using the r_0 approximations than I had expected for this problem. Usually the answers are much closer. The major cause of the error here is the effect of r_0 on r_{ie}. If r_0 is infinity, then r_{ie} is equal to r'_e. There is a fairly big difference between these two resistances in this problem.