The Common-Emitter Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-emitter amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

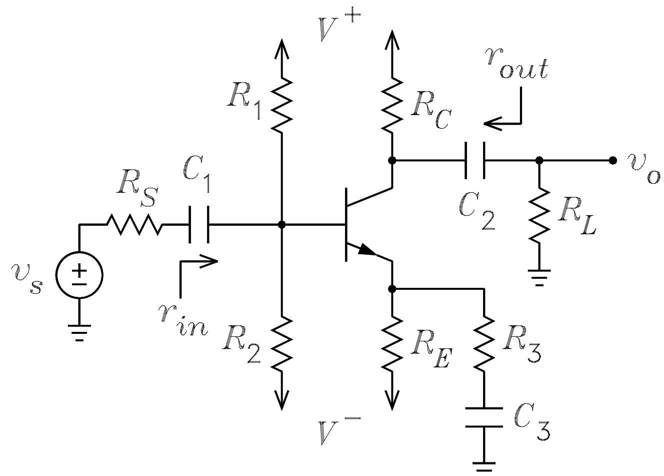

Figure 1: Single-stage common-emitter amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$V_{BB} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} \qquad R_{BB} = R_1 \| R_2 \qquad V_{EE} = V^- \qquad R_{EE} = R_E$$

(b) Make an "educated guess" for V_{BE}. Write the loop equation between the V_{BB} and the V_{EE} nodes.

$$V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE}$$

(c) Solve the loop equation for the currents.

$$I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha}$$

(d) Verify that $V_{CB} > 0$ for the active mode.

$$V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C \left(R_{CC} - R_{BB}/\beta\right)$$

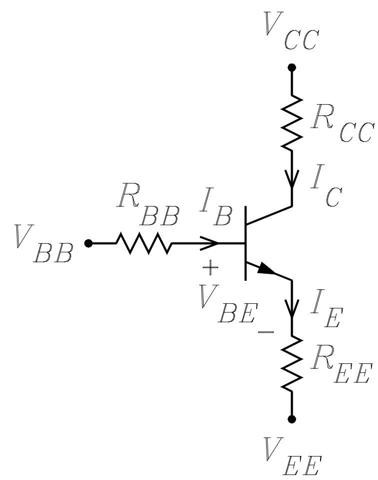

Figure 2: Bias circuit.

Small-Signal Solution

It will be assumed that the base spreading resistance r_x is non zero. This is a resistance in series with the base lead in the small signal models.

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

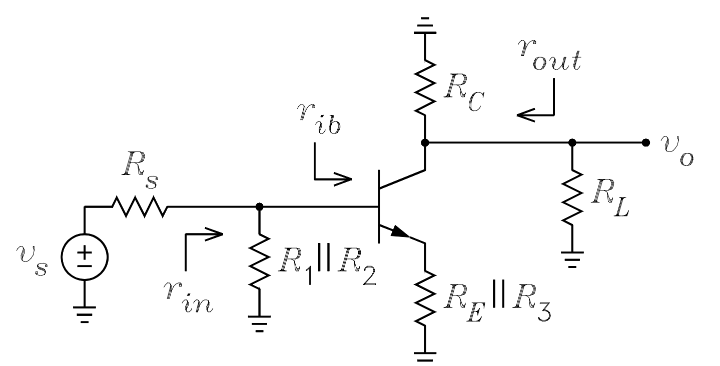

Figure 3: Signal circuit.

(b) Calculate g_m, r_π, r_e, and r_0 from the DC solution.

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E} \qquad r_0 = \frac{V_A + V_{CE}}{I_C}$$

(c) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

$$v_{tb} = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \qquad R_{tb} = R_s \| R_1 \| R_2 \qquad v_{te} = 0 \qquad R_{te} = R_E \| R_3$$

(a) Replace the BJT in Fig. 4 with the Thévenin base circuit and the Norton collector circuit as shown in Fig. 5.

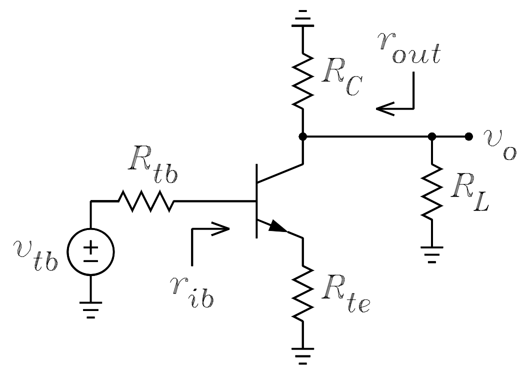

Figure 4: Signal circuit with Thévenin base circuit.

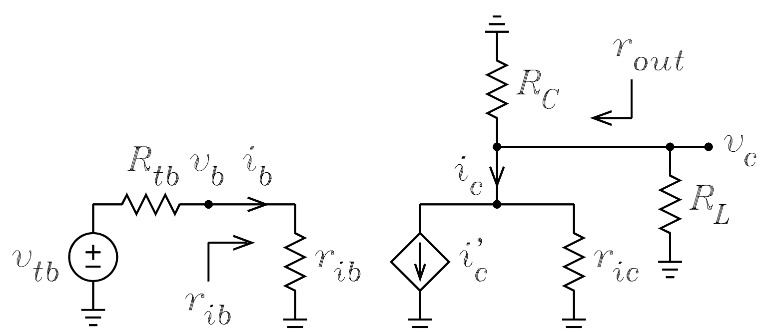

Figure 5: Base and collector equivalent circuits.

(b) Solve for G_m.

$$G_m = \frac{i_{c(sc)}}{v_{tb}} \overset{\text{or}}{=} \frac{i'_c}{v_{tb}} = \frac{1}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{1}{g_m} + \dfrac{R_{te}}{\alpha}}$$

(c) Solve for the voltage gain $A_v = v_o/v_s$. A flow graph is shown in Figure 6.

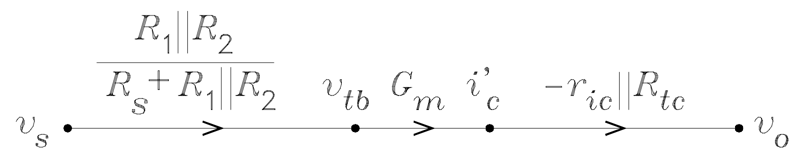

Figure 6: Flow graph for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i'_c}{v_{tb}} \times \frac{v_o}{i'_c} = \frac{R_1\|R_2}{R_s + R_1\|R_2} \times G_m \times -\left(r_{ic}\|R_{tc}\right)$$

(d) Solve for r_{out}.

$$r_{out} = r_{ic}\|R_C \qquad r_{ic} = r_0\left(1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}}\right) + \left(R_{tb} + r_x + r_\pi\right)\|R_{te}$$

(e) Solve for r_{in}.

$$r_{in} = R_1\|R_2\|r_{ib} \qquad r_{ib} = r_x + r_\pi + (1+\beta)\left(R_E\|R_3\right)$$

Second Small-Signal Solution for A_v

The voltage gain can be written

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i_b}{v_{tb}} \times \frac{i'_c}{i_b} \times \frac{v_o}{i'_c} = \frac{R_1\|R_2}{R_s + R_1\|R_2} \times \frac{1}{R_{tb} + r_{ib}} \times \beta \times -\left(r_{ic}\|R_{tc}\right)$$

Third Small-Signal Solution for A_v

The voltage gain can be written

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i_e}{v_{tb}} \times \frac{i'_c}{i_e} \times \frac{v_o}{i'_c} = \frac{R_1\|R_2}{R_s + R_1\|R_2} \times \frac{1}{r_{ie} + R_{te}} \times \alpha \times -\left(r_{ic}\|R_{tc}\right)$$

Example 1 *For the CE amplifier of Fig. 1, it is given that* $R_s = 5\,\text{k}\Omega$, $R_1 = 120\,\text{k}\Omega$, $R_2 = 100\,\text{k}\Omega$, $R_C = 4.3\,\text{k}\Omega$, $R_E = 5.6\,\text{k}\Omega$, $R_3 = 100\,\Omega$, $R_L = 20\,\text{k}\Omega$, $V^+ = 15\,\text{V}$, $V^- = -15\,\text{V}$, $V_{BE} = 0.65\,\text{V}$, $r_x = 20\,\Omega$, $\beta = 99$, $\alpha = 0.99$, $V_A = 100\,\text{V}$ *and* $V_T = 0.025\,\text{V}$. *Solve for the gain* $A_v = v_o/v_s$*, the input resistance* r_{in}*, and the output resistance* r_{out}*. The capacitors can be assumed to be ac short circuits at the operating frequency.*

Solution. For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the base are

$$V_{BB} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} = -1.364\,\text{V} \qquad R_{BB} = R_1\|R_2 = 54.55\,\text{k}\Omega$$

The Thévenin voltage and resistance seen looking out of the emitter are $V_{EE} = V^-$ and $R_{EE} = R_E$. The bias equation for I_E is

$$I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/(1+\beta) + R_{EE}} = 2.113\,\text{mA} \qquad I_C = \alpha I_E = 2.092\,\text{mA} \qquad I_B = \frac{I_E}{1+\beta} = 21.13\,\mu\text{A}$$

To test for the active mode, we calculate the collector-base voltage

$$V_{CB} = V_C - V_B = \left(V^+ - \alpha I_E R_C\right) - \left(V_{BB} - \frac{I_E}{1+\beta} R_{BB}\right) = 8.521\,\text{V}$$

Because this is positive, the BJT is biased in its active mode.

For the small-signal ac analysis, we need r_0 and r_e. To calculate r_0, we first calculate the collector-emitter voltage

$$V_{CE} = V_{CB} + V_{BE} = 9.171\,\text{V}$$

It follows that r_0, g_m, r_π, and r_e have the values

$$r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 52.18\,\text{k}\Omega \qquad g_m = \frac{I_C}{V_T} = \frac{2.092}{25} = \frac{1}{11.95}\,\text{S}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{\beta V_T}{I_C} = \frac{99 \times 25}{2.113} = 1.183\,\text{k}\Omega \qquad r_e = \frac{V_T}{I_E} = 11.83\,\Omega$$

For the small-signal analysis, V^+ and V^- are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the base are given by

$$v_{tb} = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} = 0.916 v_s \qquad R_{tb} = R_s \| R_1 \| R_2 = 4.58\,\text{k}\Omega$$

The Thévenin resistances seen looking out of the emitter and the collector are

$$R_{te} = R_E \| R_3 = 98.25\,\Omega \qquad R_{tc} = R_C \| R_L = 3.539\,\text{k}\Omega$$

Note that the base spreading resistance r_x is not zero for this problem. Next, we calculate G_m, r_{ic}, and r_{ib}.

$$G_m = \frac{1}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{1}{g_m} + \dfrac{R_{te}}{\alpha}} = \frac{1}{\dfrac{4580 + 20}{99} + 11.83 + \dfrac{98.25}{0.99}} = \frac{1}{157.7}\,\text{S}$$

$$r_{ic} = r_0 \left(1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}}\right) + \left(R_{tb} + r_x + r_\pi\right) \| R_{te} = 138.6\,\text{k}\Omega$$

$$r_{ib} = r_x + r_\pi + (1+\beta) R_{te} = 11.03\,\text{k}\Omega$$

The voltage is

$$A_v = \frac{v_o}{v_s} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \times G_m \times -\left(r_{ic} \| R_{tc}\right) = 0.916 \times \frac{1}{157.7} \times -\left(138.6\text{k} \| 3.539\text{k}\right) = -20.05$$

The input and output resistances are

$$r_{in} = R_1 \| R_2 \| r_{ib} = 9.173\,\text{k}\Omega \qquad r_{out} = r_{ic} \| R_C = 4.17\,\text{k}\Omega$$