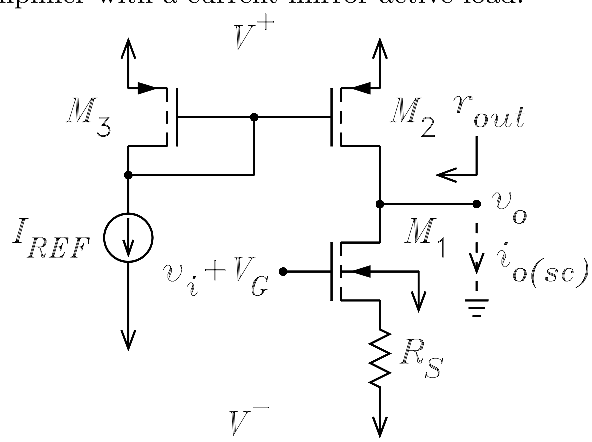
**ECE3050 – Assignment 14**

1. The figure shows a CS amplifier with a current-mirror active load.




   (a) If the $r_0$ approximations are used, show that the small-signal short-circuit output current is given by

$$i_{o(sc)} = -\frac{v_i}{1+\chi_1}\frac{1}{r'_{s1}+R_S} = \frac{g_{m1}}{1+g_{m1}(1+\chi_1)R_S}v_i$$

   (b) Show that the small-signal output resistance is given by

$$r_{out} = r_{id1}\|r_{02} = \left[r_{01}\left(1+\frac{R_S}{r'_{s1}}\right)+R_S\right]\|r_{02}$$

   (c) Show that the small-signal open-circuit voltage is given by

$$v_{o(oc)} = i_{o(sc)}r_{out} = -\frac{g_{m1}}{1+g_{m1}(1+\chi_1)R_S}\left[r_{01}\left(1+\frac{R_S}{r'_{s1}}\right)+R_S\right]\|r_{02}\times v_i$$

2. For the CS amplifier of Problem 1, each MOSFET has the parameters $K_0 = 0.002\,\mathrm{A/V^2}$, $V_{TO} = 1.4\,\mathrm{V}$, $\lambda = 0.02\,\mathrm{V^{-1}}$, $\gamma = 1.5\,\mathrm{V^{1/2}}$, and $\varphi = 0.6\,\mathrm{V}$. It is given that $V^+ = 10\,\mathrm{V}$, $V^- = -10\,\mathrm{V}$, $I_{\mathrm{ref}} = 1\,\mathrm{mA}$, $R_S = 200\,\Omega$.

   (a) Use the equation $I_D = K_0(1+\lambda V_{SD})(V_{SG}-V_{TO})^2$ to show that $V_{SG3} = 2.09\,\mathrm{V}$. Note that $V_{SD3} = V_{SG3}$.

   (b) Use the equation $I_D = K_0(1+\lambda V_{SD})(V_{SG}-V_{TO})^2$ to show that $I_{D2} = 1.15\,\mathrm{mA}$. Note that $V_{SG2} = V_{SG3}$.

   (c) Show that $V_{DS1} = 9.77\,\mathrm{V}$ and $V_{BS1} = -0.23\,\mathrm{V}$. Note that the current through $R_S$ is equal to $I_{D2}$.

   (d) Use the equation $\chi = 0.5\gamma/\sqrt{\varphi - V_{BS}}$ to show that $\chi_1 = 0.823$.

   (e) Use the equation $K = K_0(1+\lambda V_{DS})$ to show that $K_1 = 4.78\times10^{-3}\,\mathrm{A/V^2}$.

   (f) Use the equations $g_m = 2\sqrt{KI_D}$ and $r_0 = (V_{DS}+1/\lambda)/I_D$ to show that $g_{m1} = 3.32\times10^{-3}\,\mathrm{S}$ and $r_{01} = 51.9\,\mathrm{k\Omega}$.

   (g) Use the equation $r_0 = (V_{SD}+1/\lambda)/I_D$ to show that $r_{02} = 52.1\,\mathrm{k\Omega}$.

(h) Show that the small-signal short-circuit output current is

$$i_{o(sc)} = -\frac{v_i}{1+\chi_1}\frac{1}{r'_{s1}+R_S} = 1.50 \times 10^{-3} v_i$$

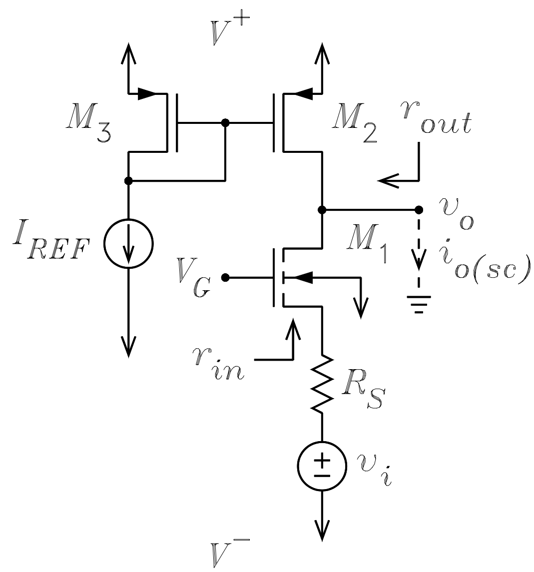
Assume $r_{01} = \infty$ and use the simplified T model.

(i) Show that the small-signal output resistance and open-circuit output voltage are $r_{out} = r_{id1} \| r_{02} = 35.8\,\mathrm{k}\Omega$ and $v_{o(oc)} = i_{o(sc)} \times r_{out} = -53.8 v_i$.

(j) If a load resistor $R_L = 10\,\mathrm{k}\Omega$ is connected from output to ground, show that the output voltage changes by the factor $R_L/(r_{out}+R_L) = 0.218$ (or by $-13.2\,\mathrm{dB}$) and the new voltage gain is $v_o/v_i = -11.7$.

(k) If $R_S = 0$, show that $v_{o(oc)}$ increases to the value $v_{o(oc)} = -86.28 v_i$. Show that the gain increases by 4.10 dB.

3. The figure shows a CG amplifier with a current-mirror active load. The voltage $V_G$ is a dc bias voltage. Each MOSFET has the parameters $g_m = 2.5 \times 10^{-3}\,\mathrm{S}$, $r_0 = 40\,\mathrm{k}\Omega$, and $\chi = 0.5$. It is given that $R_S = 200\,\Omega$. It can be assumed that the dc value of the output voltage is zero.



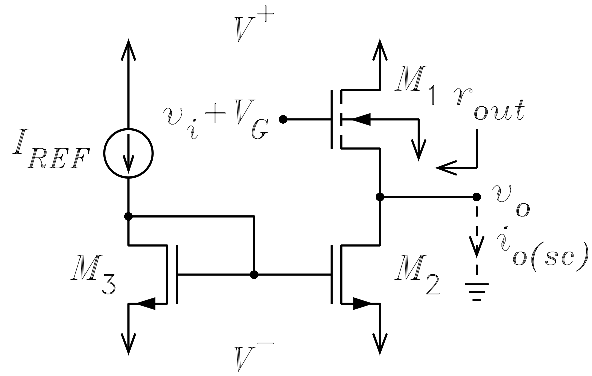

(a) Solve for the Norton short-circuit output current. Assume $r_{01} = \infty$ and use the simplified T model. Answer: $i_{o(sc)} = v_i/(r'_{s1}+R_S) = 2.14 \times 10^{-3} v_i$.

(b) Solve for the Thévenin equivalent circuit seen looking into the $v_o$ node, i.e. solve for $r_{out}$ and $v_{o(oc)}$. Answers: $r_{out} = r_{id1} \| r_{02} = 25.5\,\mathrm{k}\Omega$ and $v_{o(oc)} = i_{o(sc)} \times r_{out} = 54.6 v_i$.

(c) By what factor does $v_o$ change if a load resistor $R_L = 10\,\mathrm{k}\Omega$ is connected from output to ground? What is the new voltage gain? Answer: $R_L/(r_{out}+R_L) = 0.282$ or by $-11\,\mathrm{dB}$ and $v_o/v_i = 15.4$.

(d) Show that the input resistance is $r_{in} = r'_{s1} = 267\,\Omega$.

4. The figure shows a CD amplifier with a current-mirror active load. The voltage $V_G$ is a dc bias voltage. Each MOSFET has the parameters $g_m = 2.5 \times 10^{-3}\,\mathrm{S}$, $r_0 = 40\,\mathrm{k}\Omega$, and $\chi = 0.5$.

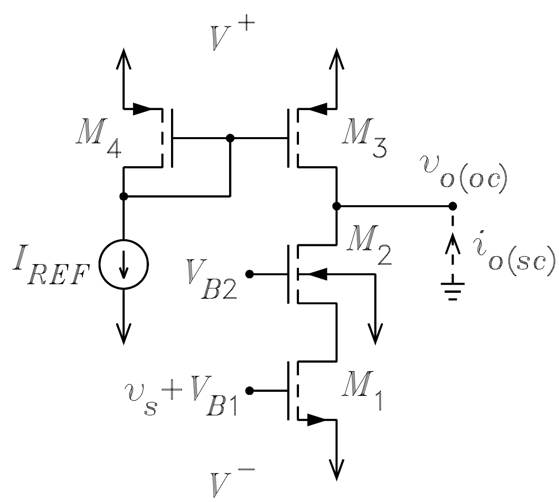



(a) Solve for the Norton short-circuit output current. Use the simplified T model. Answer: $i_{o(sc)} = v_i/r_{s1} = g_{m1}v_i = 2.5 \times 10^{-3}v_i$. Note that the body effect cancels when $v_o = 0$.

(b) Solve for the Thévenin equivalent circuit seen looking into the $v_o$ node, i.e. solve for $r_{out}$ and $v_{o(oc)}$. Answers: $r_{out} = r'_{s1} \| r_{01} \| r_{02} = 263\,\Omega$ and $v_{o(oc)} = i_{o(sc)} \times r_{out} = 0.658v_i$.

(c) By what factor does $v_o$ change if a load resistor $R_L = 10\,\text{k}\Omega$ is connected from output to ground? What is the new voltage gain? Answers: $R_L/(r_{out} + R_L) = 0.974$ or by $-0.226\,\text{dB}$ and $v_o/v_i = 0.641$.

5. The figure shows a cascode amplifier. $M_1$ is operated as a CS amplifier with a small-signal voltage $v_s$ and a dc bias voltage $V_{B1}$ applied to its gate. $M_2$ is operated as a CG amplifier with a dc bias voltage $V_{B2}$ applied to its gate. $M_3$ and $M_4$ form a current mirror with an input dc current $I_{REF}$. For each MOSFET, it is given that $g_m = 0.005\,\text{S}$, $g_{mb} = 0.0025\,\text{S}$, and $r_0 = 50\,\text{k}\Omega$.




(a) To simplify the solution for $i_{o(sc)}$, assume $r_{01} = r_{02} = \infty$. Show that $i_{o(sc)}/v_s = i'_{d2} = i'_{d1} = 0.005\,\text{S}$.

(b) With $r_{01} = r_{02} = 50\,\text{k}\Omega$, show that the output resistance is

$$r_{out} = r_{03} \| \left[ r_{02} \left( 1 + r_{01}/r'_{s2} \right) + r_{02} \right] = 49.87\,\text{k}\Omega$$

(c) Show that $v_{o(oc)}/v_s = -\left[ i_{o(sc)}/v_s \right] \times r_{out} \| r_{03} = -249.3$.