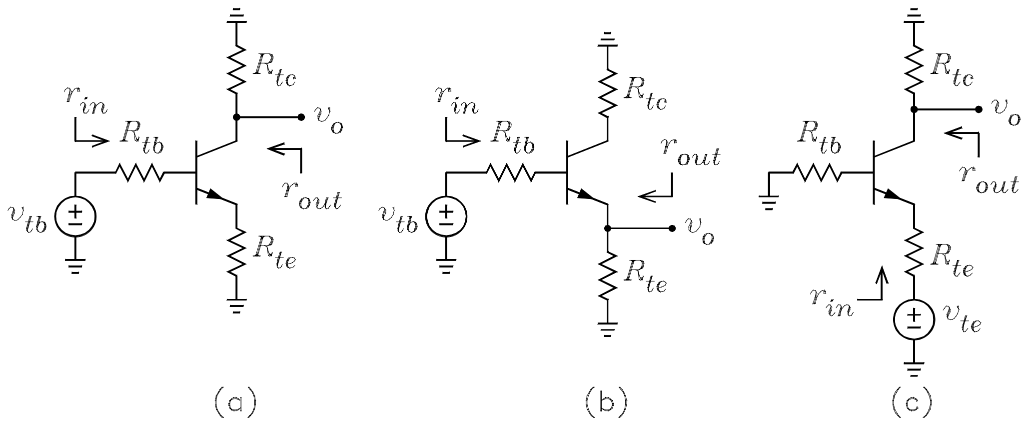
ECE3050 Homework Set 6

The figures show a common-emitter amplifier, a common-collector amplifier, and a common-base amplifier. For each circuit, it is given that $R_{tb} = 1\,\text{k}\Omega$, $R_{te} = 100\,\Omega$, and $R_{tc} = 10\,\text{k}\Omega$. The transistors have the values $I_E = 1.5\,\text{mA}$, $V_T = 25\,\text{mV}$, $\beta = 99$, $r_x = 50\,\Omega$, and $r_0 = \infty$.



1. For each transistor, show that $g_m = 59.4\,\text{mS}$, $r_\pi = 1.667\,\text{k}\Omega$, $\alpha = 0.99$, and $r_e = 16.67\,\Omega$.

2. For the common-emitter amplifier of Figure (a):

   (a) Replace the BJT with the π model. Label the controlled source in the collector i'_c, the base current i'_c/β, and the emitter current i'_c/α. Let the voltage across r_π be written $v_\pi = i'_c/g_m$. Write a loop equation around the base-emitter loop and solve for i'_c. Use the circuit to show that

$$\frac{i'_c}{v_{tb}} = \frac{1}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{1}{g_m} + \dfrac{R_{te}}{\alpha}} = 7.785\,\text{mS}$$

$$\frac{v_o}{v_{tb}} = \frac{-R_{tc}}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{1}{g_m} + \dfrac{R_{te}}{\alpha}} = -77.85$$

   Label the base current i_b and the emitter current $(1 + \beta)\, i_b$. Write the loop equation and show that

$$r_{in} = \frac{v_{tb}}{i_b} = R_{tb} + r_x + r_\pi + (1 + \beta)\, R_{te} = 12.72\,\text{k}\Omega$$

   Show that

$$r_{out} = R_{tc} = 10\,\text{k}\Omega$$

   (b) Replace the BJT with the T model. Label the controlled source in the collector i'_c, the base current i'_c/β, and the emitter current i'_c/α. Write a loop equation around the base-emitter loop and solve for i'_c. Use the circuit to show that

$$\frac{i'_c}{v_{tb}} = \frac{1}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{r_e + R_{te}}{\alpha}} = 7.785\,\text{mS}$$

$$\frac{v_o}{v_{tb}} = \frac{-R_{tc}}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{r_e + R_{te}}{\alpha}} = -77.85$$

Label the base current i_b and the emitter current $(1 + \beta) i_b$. Write the loop equation and show that

$$r_{in} = \frac{v_{tb}}{i_b} = R_{tb} + r_x + (1 + \beta)(r_e + R_{te}) = 12.72\,\mathrm{k}\Omega$$

Show that

$$r_{out} = R_{tc} = 10\,\mathrm{k}\Omega$$

(c) Show that the simplified T model and simplified π model give the same answers for v_o/v_{tb} and r_{out}.

3. For the common-collector amplifier of Figure (b):

(a) Replace the BJT with the π model. Label the controlled source in the collector i'_c, the base current $i'_e/(1 + \beta)$, and the emitter current i'_e. Write a loop equation around the base-emitter loop and solve for i'_e. Use the circuit to show that

$$\frac{i'_e}{v_{tb}} = \frac{1}{\dfrac{R_{tb} + r_x + r_\pi}{1 + \beta} + R_{te}} = 7.864\,\mathrm{mS}$$

$$\frac{v_o}{v_{tb}} = \frac{R_{te}}{\dfrac{R_{tb} + r_x + r_\pi}{1 + \beta} + R_{te}} = 0.786$$

Label the base current i_b and the emitter current $(1 + \beta) i_b$. Write the loop equation and show that

$$r_{in} = \frac{v_{tb}}{i_b} = R_{tb} + r_x + r_\pi + (1 + \beta) R_{te} = 12.72\,\mathrm{k}\Omega$$

The output resistance can be written $r_{out} = R_{te} \| r_{ie}$, where r_{ie} is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit output voltage with $R_{te} = \infty$ to the short-circuit output current with $R_{te} = 0$. Show that r_{ie} is given by

$$r_{ie} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{v_o|_{R_{te}=\infty}}{\left.\dfrac{v_o}{R_{te}}\right|_{R_{te}=0}} = \frac{R_{tb} + r_x + r_\pi}{1 + \beta} = 27.17\,\Omega$$

and that r_{out} is

$$r_{out} = R_{te} \| r_{ie} = 21.36\,\Omega$$

(b) Replace the BJT with the T model. Label the controlled source in the collector i'_c, the base current $i'_e/(1 + \beta)$, and the emitter current i'_e. Write a loop equation around the base-emitter loop and solve for i'_e. Use the circuit to show that

$$\frac{i'_e}{v_{tb}} = \frac{1}{\dfrac{R_{tb} + r_x}{1 + \beta} + r_e + R_{te}} = 7.864\,\mathrm{mS}$$

$$\frac{v_o}{v_{tb}} = \frac{R_{te}}{\dfrac{R_{tb}+r_x}{1+\beta} + r_e + R_{te}} = 0.786$$

Label the base current i_b and the emitter current $(1+\beta)\, i_b$. Write the loop equation and show that

$$r_{in} = \frac{v_{tb}}{i_b} = R_{te} + r_x + (1+\beta)(r_e + R_{te}) = 12.72\,\mathrm{k}\Omega$$

The output resistance can be written $r_{out} = R_{te} \| r_{ie}$, where r_{ie} is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit voltage with $R_{te} = \infty$ to the short-circuit current with $R_{te} = 0$. Show that r_{ie} is given by

$$r_{ie} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{v_o|_{R_{te}=\infty}}{\left.\dfrac{v_o}{R_{te}}\right|_{R_{te}=0}} = \frac{R_{tb}+r_x}{1+\beta} + r_e = 27.17\,\Omega$$

and that r_{out} is

$$r_{out} = R_{te} \| r_{ie} = 21.36\,\Omega$$

(c) Show that the simplified T model gives the same answers for v_o/v_{tb} and r_{out}. Note that the simplified π model is not convenient because the v_o node does not appear in the circuit.

4. For the common-base amplifier of Figure (c):

(a) Replace the BJT with the π model. Label the controlled source in the collector i'_c, the base current i'_b, and the emitter current $i'_e = i'_c/\alpha$. Let the voltage across r_π be written $v_\pi = i'_c/g_m$. Write a loop equation around the base-emitter loop and solve for i'_c to show that

$$\frac{i'_c}{v_{tb}} = \frac{-1}{\dfrac{R_{tb}+r_x+r_\pi}{\beta} + \dfrac{R_{te}}{\alpha}} = 7.785\,\mathrm{mS}$$

$$\frac{v_o}{v_{te}} = \frac{R_{tc}}{\dfrac{R_{tb}+r_x+r_\pi}{\beta} + \dfrac{R_{te}}{\alpha}} = 77.85$$

Label the base current $i'_e/(1+\beta)$ and the emitter current i'_e. Write the loop equation and show that

$$r_{in} = \frac{v_{te}}{-i'_e} = \frac{R_{te}+r_x+r_\pi}{1+\beta} + R_{te} = 127.2\,\Omega$$

Show that

$$r_{out} = R_{tc} = 10\,\mathrm{k}\Omega$$

(b) Replace the BJT with the T model. Label the controlled source in the collector i'_c, the base current i'_c/β, and the emitter current i'_c/α. Write a loop equation around the base-emitter loop and solve for i'_e. Use the circuit to show that

$$\frac{i'_c}{v_{te}} = \frac{-1}{\dfrac{R_{tb}+r_x}{\beta} + \dfrac{r_e+R_{te}}{\alpha}} = 7.785\,\mathrm{mS}$$

$$\frac{v_o}{v_{te}} = \frac{R_{tc}}{\dfrac{R_{tb} + r_x}{\beta} + \dfrac{r_e + R_{te}}{\alpha}} = 77.85$$

Label the base current $i'_e/(1+\beta)$ and the emitter current i'_e. Write the loop equation and show that

$$r_{in} = \frac{v_{te}}{-i'_e} = \frac{R_{te} + r_x}{1+\beta} + r_e + R_{te} = 127.2\,\Omega$$

Show that

$$r_{out} = R_{tc} = 10\,\mathrm{k}\Omega$$

(c) Show that the simplified T model and the simplified π model give the same answers.