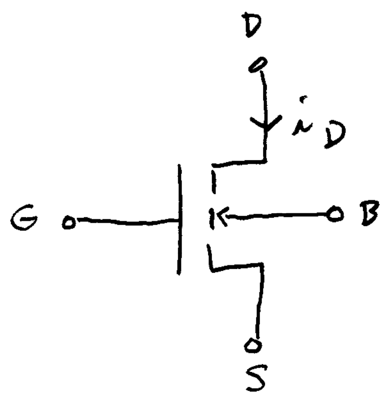
Modeling the Body Effect

The pi model must be modified if the body is not connected to the source.



$$i_D = K(v_{GS} - V_{TH})^2$$

$$V_{TH} = V_{TO} + \gamma\left[\sqrt{\phi - v_{BS}} - \sqrt{\phi}\right]$$

If v_{BS} varies, V_{TH} varies and i_D varies. We define the body transconductance as follows

$$g_{mb} = \frac{dI_D}{dV_{BS}}$$

This can be shown to be

$$g_{mb} = \frac{\gamma}{2\sqrt{\phi - v_{BS}}} \, 2\sqrt{K I_D}$$

We define the transconductance ratio as η

$$\eta = \frac{g_{mb}}{g_m}$$

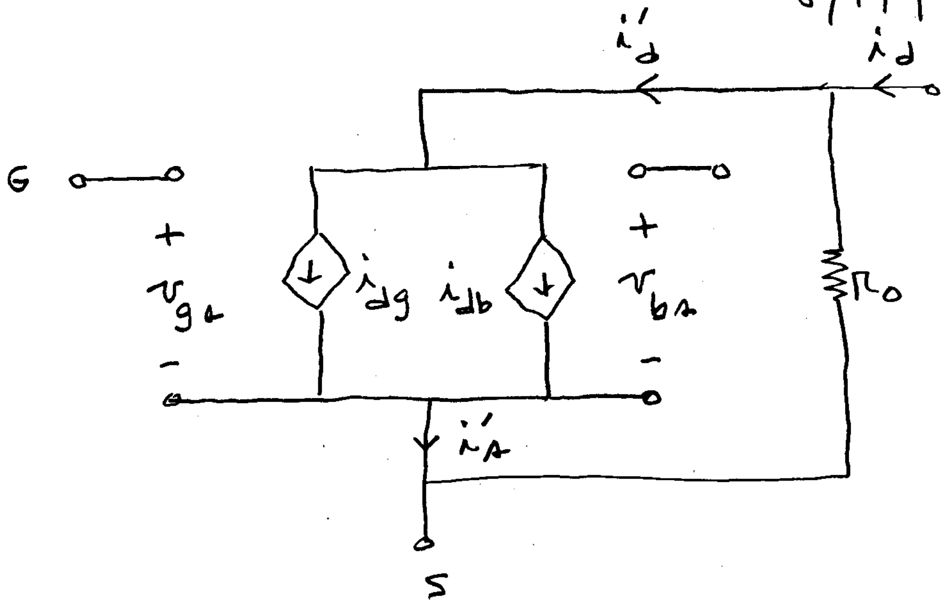
where $g_m = 2\sqrt{K I_D}$. It follows that η is given by

$$\eta = \frac{\gamma}{2\sqrt{\phi - v_{BS}}}$$

The small-signal drain current is given by

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + \frac{v_{ds}}{r_o}$$

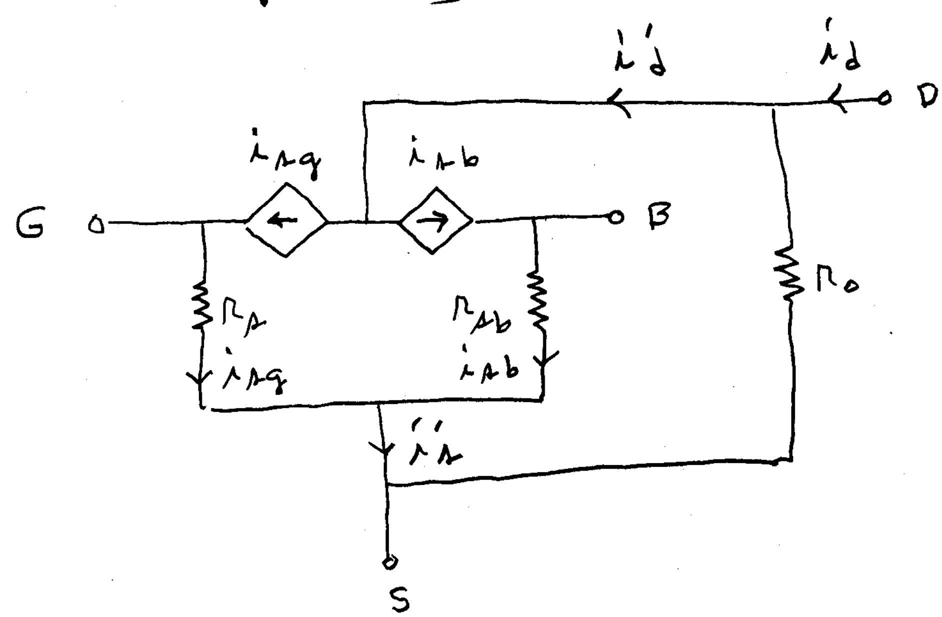
The pi model can be redrawn as follows:



$$i_{dg} = g_m v_{gs}$$

$$i_{db} = g_{mb} v_{bs} = \eta g_m v_{bs}$$

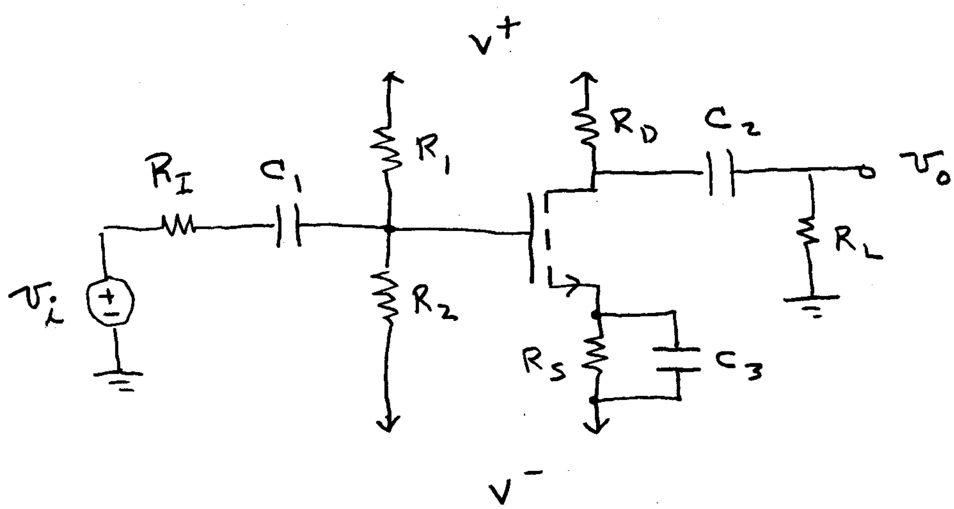
The corresponding T model is



where

$$r_s = \frac{1}{g_m} \qquad r_{sb} = \frac{1}{g_{mb}} = \frac{1}{\eta g_m} = \frac{r_s}{\eta}$$

The Common-Source Amplifier



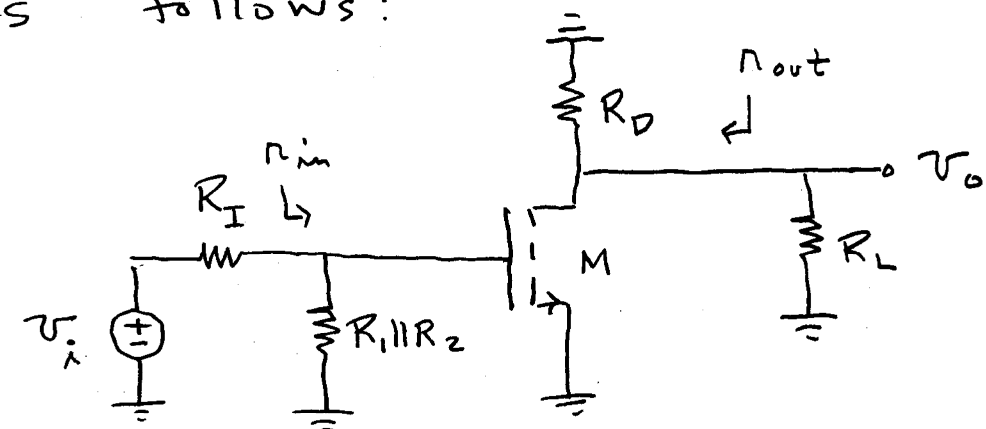
The Q point values are calculated with C_1, C_2, and C_3 open circuited. We assume this solution is known.

For a sinusoidal signal, the complex impedance of a capacitor is given by

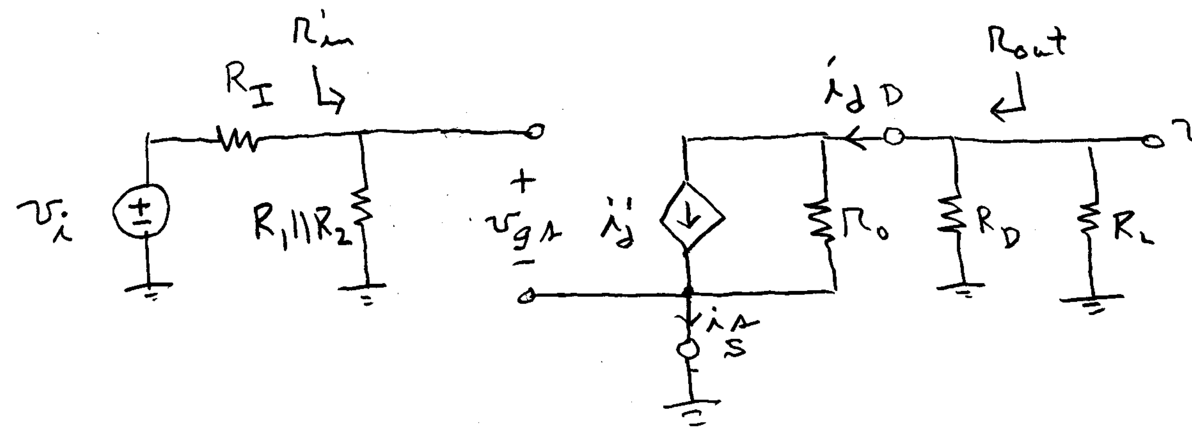
$$Z_c = \frac{1}{j\omega C}$$

If ωC is sufficiently large, Z_c can be made small enough to be considered an ac short circuit. We assume each C can be considered to be an ac short.

The ac equivalent circuit is as follows:



Replace M with its pi model



$$i_d' = g_m v_{gs} \qquad g_m = 2\sqrt{K I_D}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D}$$

We can write

$$v_{gs} = v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2}$$

$$v_o = - i_d' \; r_o \| R_D \| R_L$$

$$= - g_m v_{gs} \; r_o \| R_D \| R_L$$

$$= - g_m v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2} r_o \| R_D \| R_L$$

The voltage gain is given by

$$A_v = \frac{v_o}{v_i}$$

$$= \frac{R_1 \| R_2}{R_I + R_1 \| R_2} g_m \left(- r_o \| R_D \| R_L \right)$$

Note that this is of the form

$$A_v = \frac{v_{gs}}{v_i} \times \frac{i_d'}{v_{gs}} \times \frac{v_o}{i_d'}$$

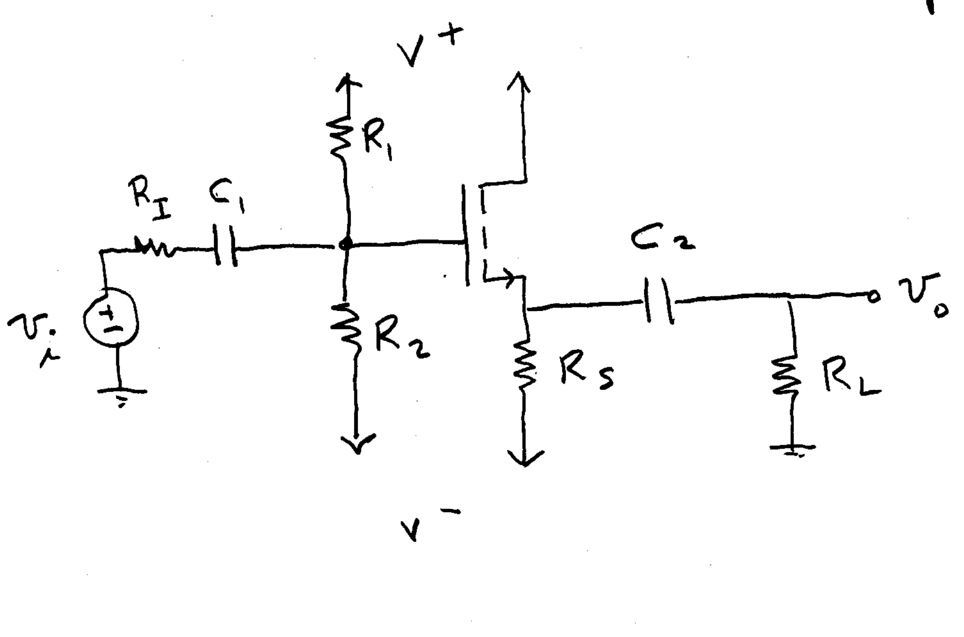
The input and output resistances are given by

$$r_{in} = R_1 \| R_2$$

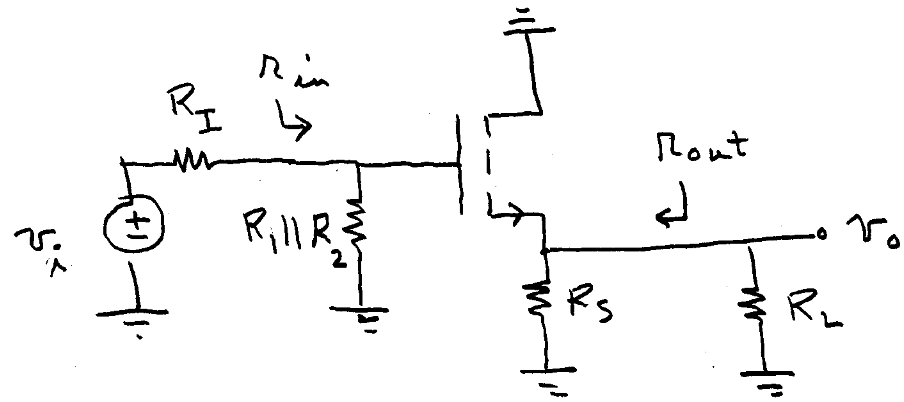
$$r_{out} = r_o \| R_D$$

Note that A_v is negative. This means that the output voltage is inverted when compared to the input voltage.

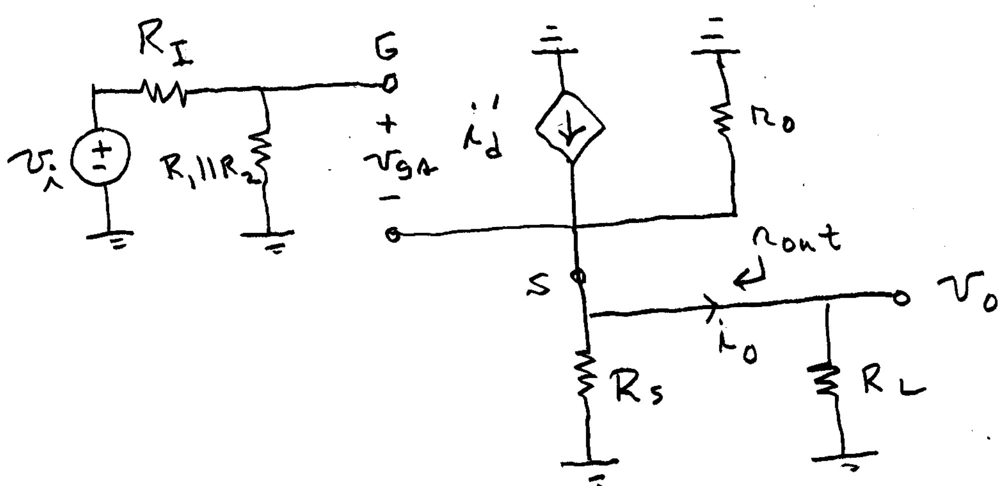
The Common Drain Amplifier



We assume that we know the Q point solution, g_m, and r_o. Assume the capacitors are ac short circuits. The ac circuit is



Replace the MOSFET with its pi model



$$v_{gs} = v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2} - v_o$$

$$v_o = i_d' \; r_o \| R_S \| R_L$$

$$= g_m \left(v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2} - v_o \right) r_o \| R_S \| R_L$$

$$\Rightarrow \quad v_o \left[1 + g_m (r_o \| R_S \| R_L) \right]$$

$$= g_m v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2} \; r_o \| R_S \| R_L$$

Thus the voltage gain is given by

$$A_v = \frac{v_o}{v_i}$$

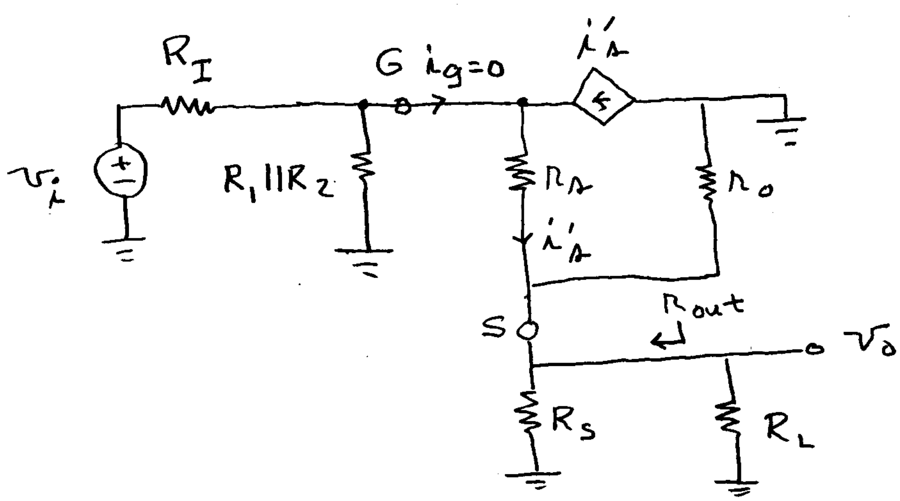
$$= \frac{R_1 \| R_2}{R_I + R_1 \| R_2} \; \frac{g_m (r_o \| R_S \| R_L)}{1 + g_m (r_o \| R_S \| R_L)}$$

The gain is positive or non-inverting

The input resistance is given by

$$r_{in} = R_1 \| R_2$$

To solve for the output resistance, it is much simpler to use the T model. The circuit is

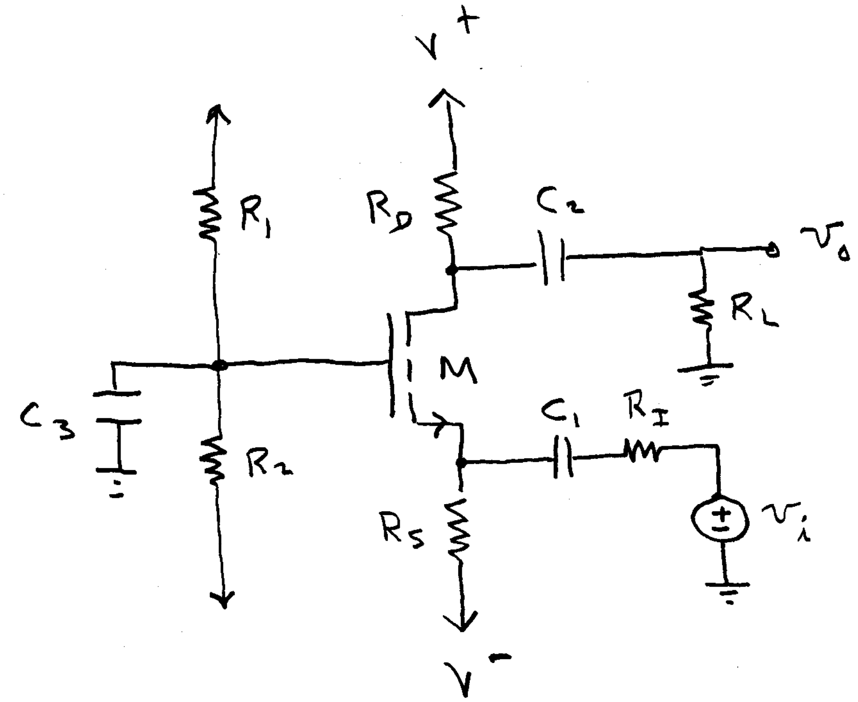


Set $v_g = 0$ to solve for r_{out}. Because $i_g = 0$, the voltage at the gate node is zero. Thus r_s connects from S to ground and we have

$$r_{out} = r_s \| r_0 \| R_S$$

where $r_s = 1/g_m$

The Common Gate Amplifier

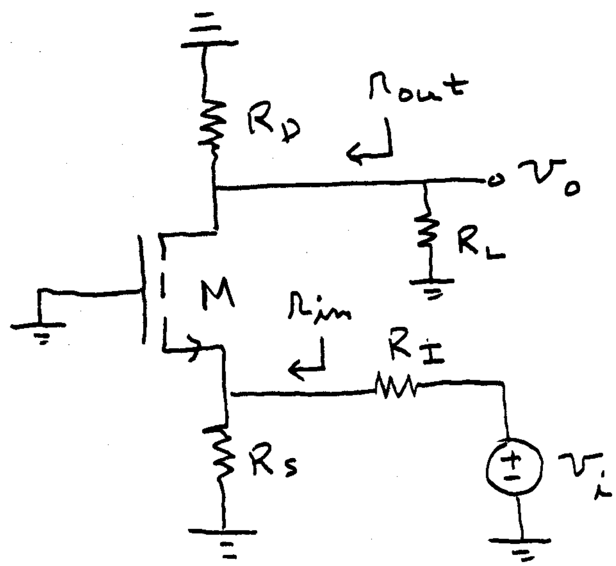


Assume we know the Q point solution from which we can calculate

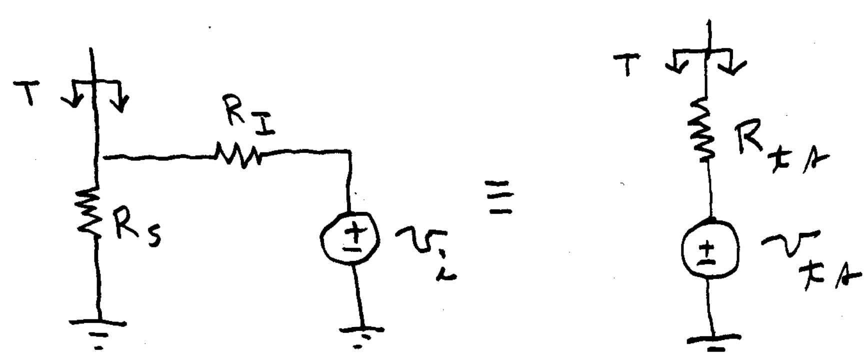
$$g_m = 2\sqrt{K I_D}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D}$$

The ac circuit is obtained by zeroing the dc supplies and replacing the capacitors with ac short circuits.

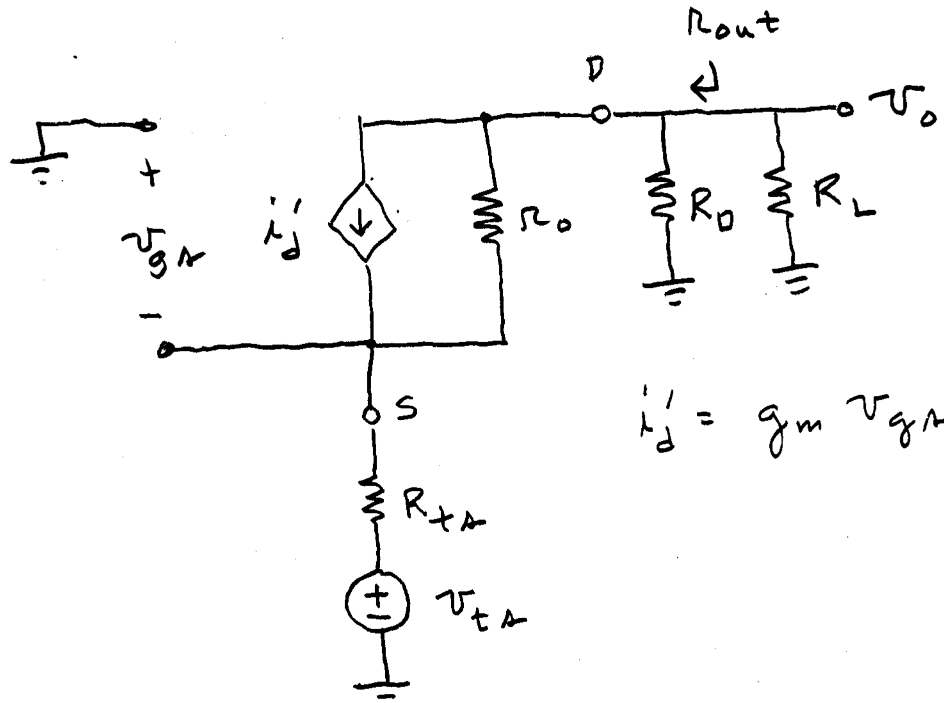


The solution for v_o and r_{out} is simplified by replacing the circuit looking out of the source by a Thévenin equivalent



$$v_{ts} = v_i \frac{R_S}{R_I + R_S} \qquad R_{ts} = R_S \parallel R_I$$

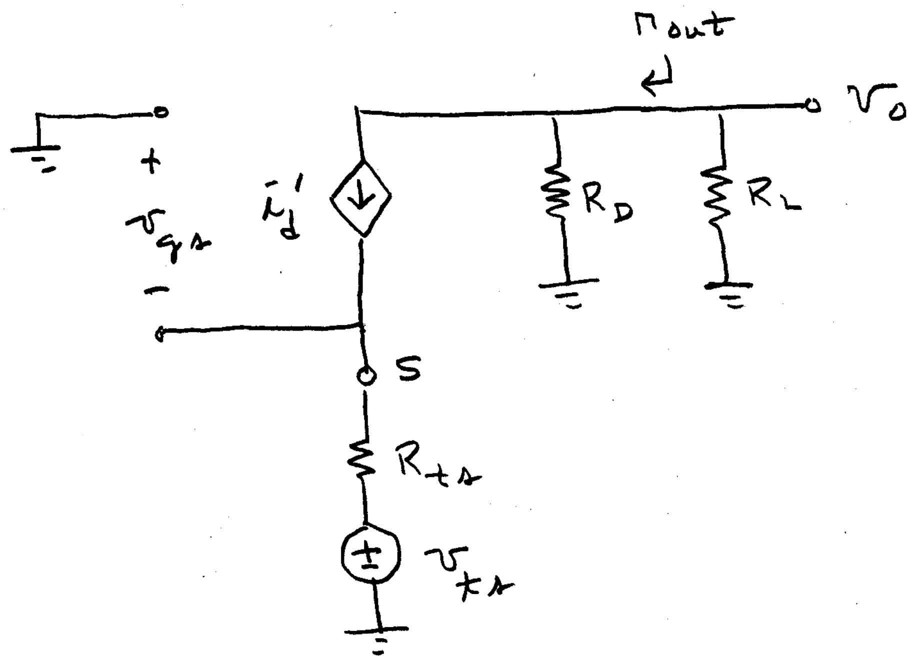
Next we replace M with its pi model



$i_d' = g_m v_{gs}$

Note that neither end of r_o connects to ground. This greatly complicates the analysis. We will obtain an approximate solution by assuming the current through r_o can be neglected. That is, we let $r_o = \infty$.

The circuit becomes



$$v_o = -i_d' \, R_D \| R_L$$

$$i_d' = g_m v_{gs}$$

$$v_{gs} = v_g - v_s$$

$$= 0 - (v_{ts} + i_d' R_{ts})$$

$$\Rightarrow i_d' = -g_m (v_{ts} + i_d' R_{ts})$$

$$\Rightarrow i_d' [1 + g_m R_{ts}] = -g_m v_{ts}$$

$$\Rightarrow \quad i_d' = \frac{-g_m}{1+g_m R_{ts}} v_{ts}$$

$$\Rightarrow \quad v_o = \frac{+g_m R_D \| R_L}{1+g_m R_{ts}} v_{ts}$$

$$= \frac{g_m R_D \| R_L}{1+g_m R_S \| R_I} v_i \frac{R_S}{R_I+R_S}$$

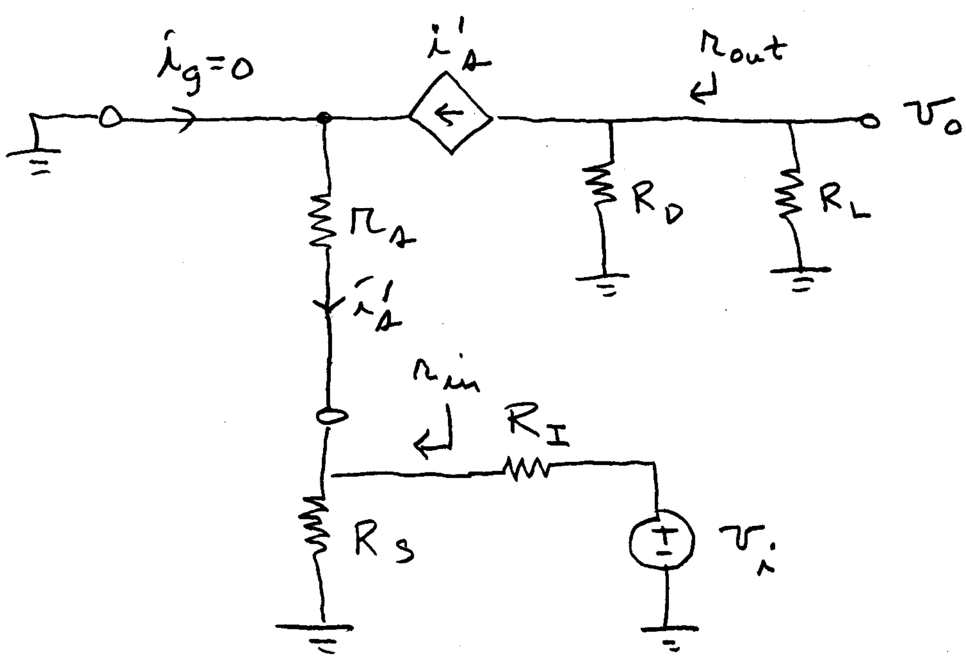
$$\Rightarrow \quad A_v = \frac{v_o}{v_i}$$

$$= \frac{R_S}{R_I+R_S} \frac{g_m R_D \| R_L}{1+g_m R_S \| R_I}$$

Note that the gain is non-inverting. The output resistance is

$$R_{out} = R_D$$

To obtain the input resistance, we replace M with its T model.



With $v_i = 0$, we have

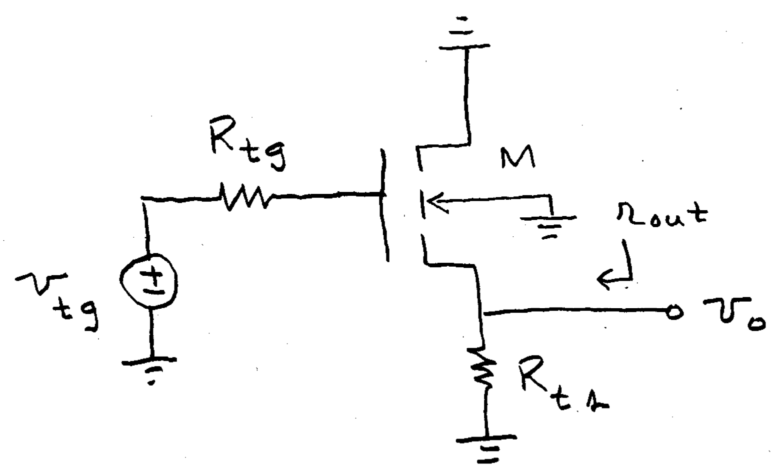
$$r_{in} = r_d \parallel R_S$$

where $r_d = \frac{1}{g_m}$

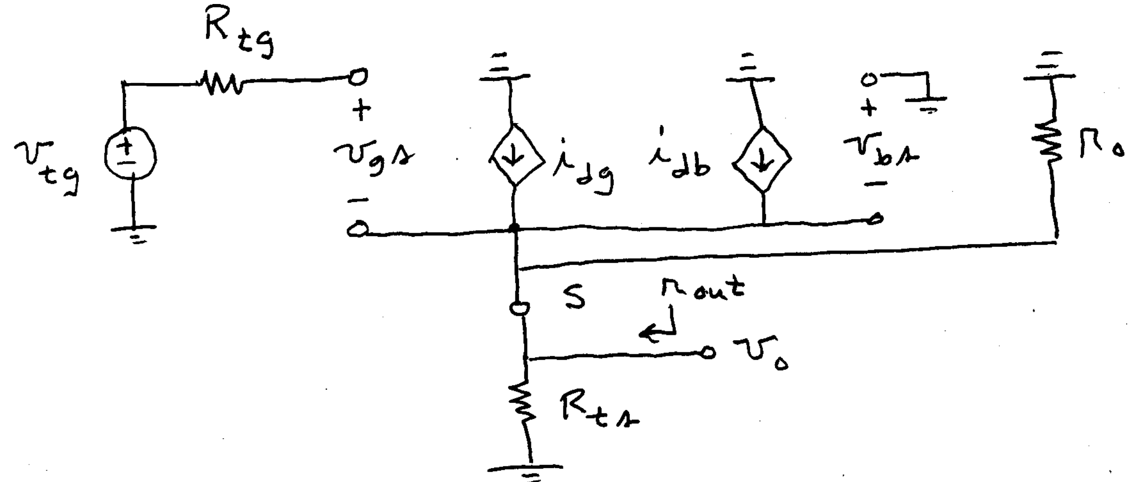
Note that the common-gate analysis is approximate because we have omitted r_o. This is often done in hand calculations when one end of r_o does not connect to ac ground.

The Body Effect

So far we have neglected the body effect. To illustrate how it affects the solution, we will consider the common drain amplifier. Consider the ac circuit



Replace M with its pi model



$$v_o = (i_{dg} + i_{db})(r_o \| R_{ts})$$

$$i_{dg} = g_m v_{gs} = g_m (v_{tg} - v_o)$$

$$i_{db} = g_{mb} v_{bs} = g_{mb} (0 - v_o)$$

$$\Rightarrow v_o = \left[g_m (v_{tg} - v_o) - g_{mb} v_o \right] r_o \| R_{ts}$$

$$\Rightarrow v_o \left[1 + (g_m + g_{mb}) r_o \| R_{ts} \right] = g_m (r_o \| R_{ts}) v_{tg}$$

$$\Rightarrow A_v = \frac{v_o}{v_{tg}}$$

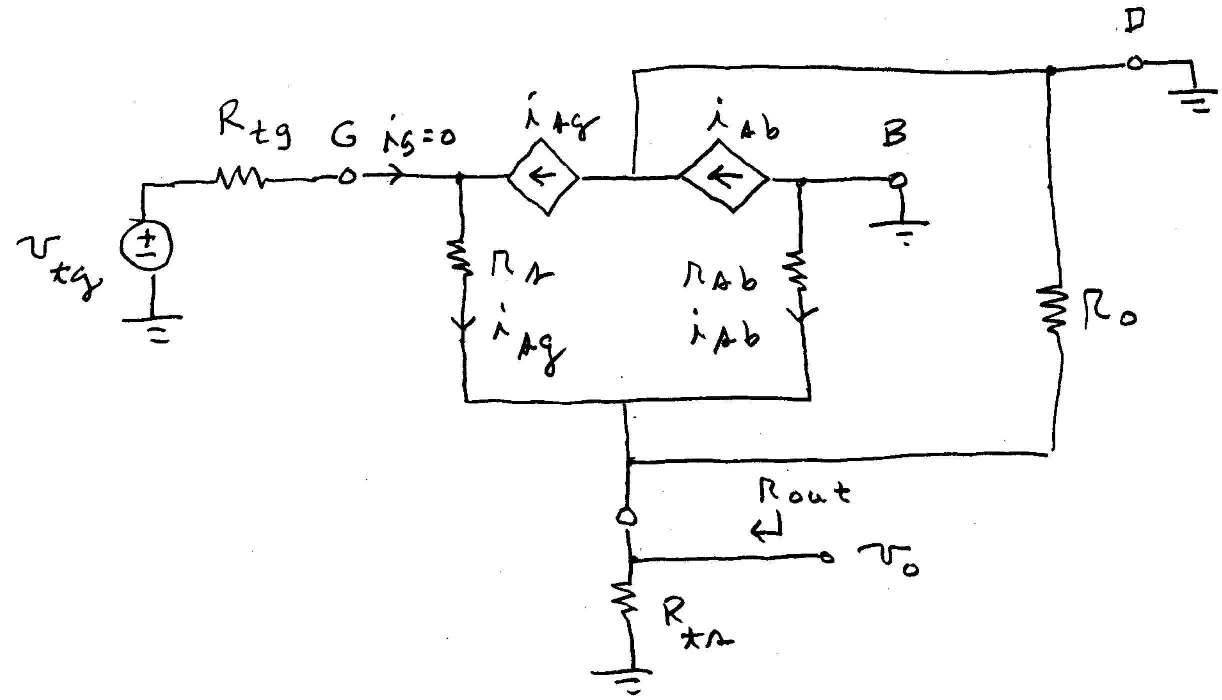
$$= \frac{g_m (r_o \| R_{ts})}{1 + (g_m + g_{mb})(r_o \| R_{ts})}$$

But $g_{mb} = \eta g_m$

$$\Rightarrow A_v = \frac{g_m (r_o \| R_{ts})}{1 + g_m (1 + \eta)(r_o \| R_{ts})}$$

Note that the body effect has reduced the voltage gain.

To obtain the output resistance, we use the T model.



Set $v_{tg} = 0$ to solve for r_{out}. Because $i_g = 0$, the top of r_s connects to ac ground.

$$\Rightarrow \quad r_{out} = r_s \,\|\, r_{sb} \,\|\, R_{ts}$$

where $r_s = \dfrac{1}{g_m}$

$$r_{sb} = \frac{1}{g_{mb}} = \frac{1}{\eta g_m} = \frac{r_s}{\eta}$$

Note that

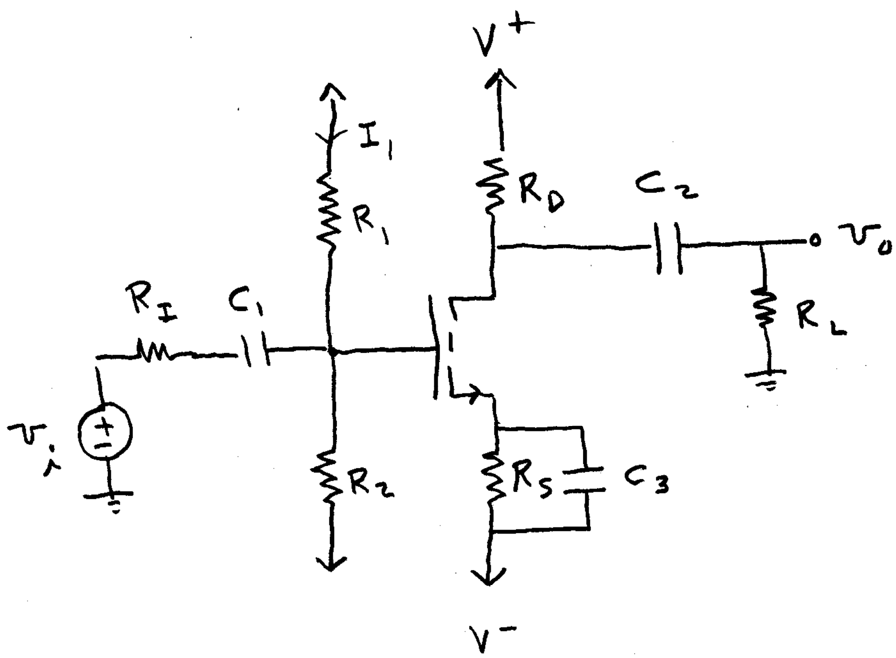
$$r_s \| r_{sb} = \frac{r_s * \frac{r_s}{\eta}}{r_s + \frac{r_s}{\eta}} = \frac{\frac{r_s}{\eta}}{1 + \frac{1}{\eta}} = \frac{r_s}{1+\eta}$$

Thus another expression for r_{out} is

$$r_{out} = \frac{r_s}{1+\eta} \| R_{ts}$$

Numerical Example

Common Source Amplifier



We are given $V^+ = +18V$, $V^- = -18V$, $K = 0.001\ A/V^2$, $V_{TH} = 1.5\ V$, $r_0 = 50\ k\Omega$, $R_I = 1\ k\Omega$, $R_L = 1\ k\Omega$

We desire $I_D = 2$ mA and $V_{DS} = 10$ V.

To calculate a value for R_D and R_S, we need V_D and V_S. Let

$$V_D = -4 \text{ V.}$$

$$\Rightarrow \quad V_S = V_D - V_{DS} = -14 \text{ V.}$$

$$R_D = \frac{V^+ - V_D}{I_D} = 11 \text{ k}\Omega$$

$$R_S = \frac{V_S - V^-}{I_D} = 2 \text{ k}\Omega$$

To calculate R_1 and R_2, we need V_G and I_1.

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TH} = 2.914 \text{ V.}$$

$$V_G = V_{GS} + V_S = -11.086 \text{ V}$$

Let $I_1 = 23.05\ \mu A$

$$\Rightarrow \quad R_1 = \frac{V^+ - V_G}{I_1} = 1.262 \text{ M}\Omega$$

$$R_2 = \frac{V_G - V^-}{I_1} = 300\ k\Omega$$

$$g_m = 2\sqrt{K I_D} = \frac{1}{353.553}$$

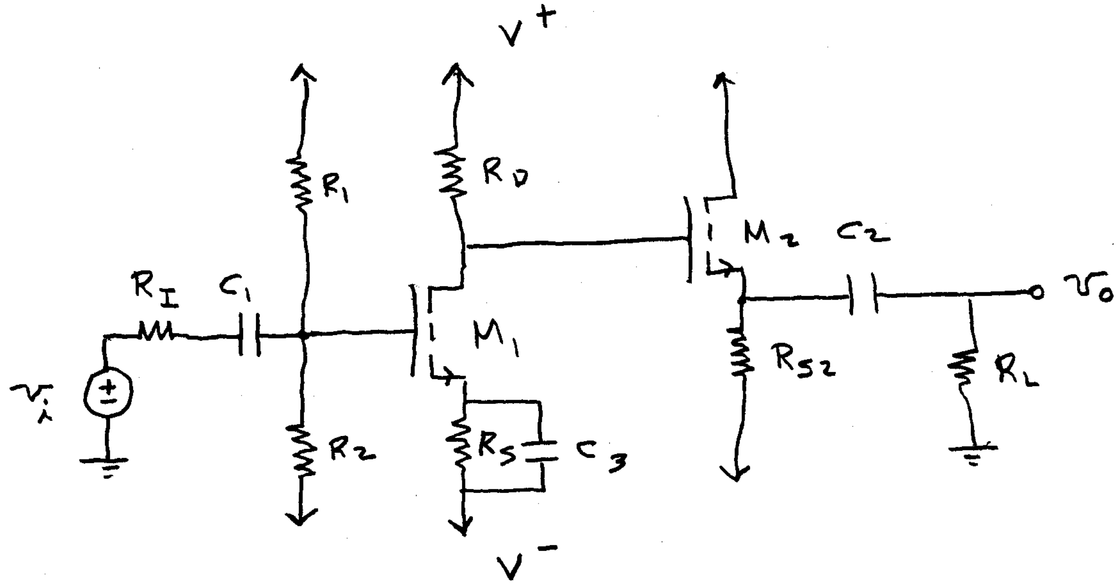
The voltage gain is given by

$$A_v = \frac{v_o}{v_i}$$

$$= \frac{R_1 \| R_2}{R_I + R_1 \| R_2}\ g_m \left[-\left(r_o \| R_D \| R_L \right) \right]$$

$$= -2.536$$

This is a fairly low gain. Let us investigate the effect of adding a common drain stage between the amplifier and the load resistor R_L. The new circuit is



Let us choose R_{S2} to bias M_2 at $I_{D2} = I_{D1} = 2\ mA$

$\Rightarrow \quad V_{GS2} = V_{GS1} = 2.914\ V.$

$$V_{S2} = V_{G2} - V_{GS2}$$
$$= (V^+ - I_{D1} R_{D1}) - V_{GS2}$$
$$= -6.914\ V.$$

$$R_2 = \frac{V_{S2} - V^-}{I_{D2}} = 5.543\ k\Omega$$

Because $I_{D2} = I_{D1}$, it follows that

$$g_{m2} = g_{m1} = \frac{1}{353.553}$$

We also assume that $r_{o2} = r_{o1} = 50\,k\Omega$

The new gain of the circuit is

$$A_v = \frac{R_1 \| R_2}{R_I + R_1 \| R_2} * g_{m1} * (-r_{o1} \| R_D)$$

$$* \quad \frac{g_{m2}(r_{o2} \| R_{S2} \| R_L)}{1 + g_{m2}(r_{o2} \| R_{S2} \| R_L)}$$

$$= -17.83$$

The gain has increased by a factor

$$\frac{17.83}{2.536} = 7.032$$

In decibels, the change in gain is

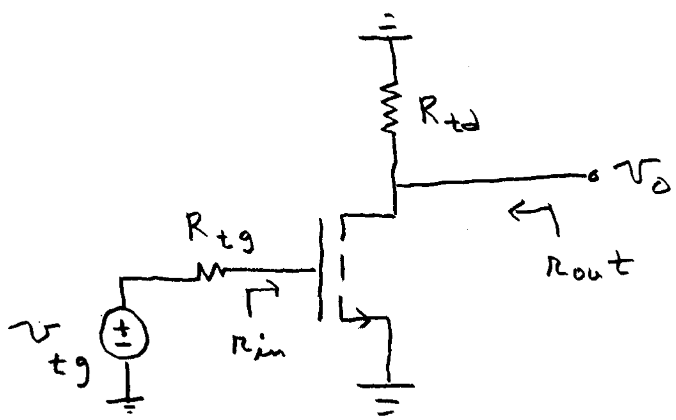
$$20 \log(7.032) = 16.942 \text{ dB}$$

Thus the addition of a stage which has a gain less than unity has increased the overall gain of the amplifier. In effect, M_2 has removed the load R_L from the drain output of M_1. This causes the gain of M_1 to increase. Be cause M_2 has a low output resistance, it can drive R_L without its gain changing much. Thus an overall increase in gain is achieved.

M_2 is commonly called a buffer output stage. It isolates R_L from M_1 so that a higher gain can be obtained from M_1.

Summary Results

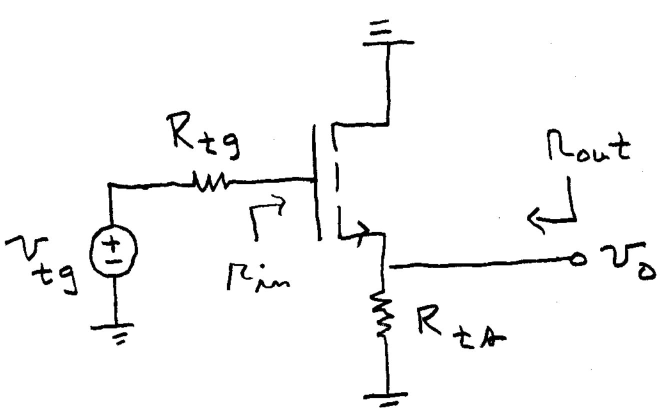
CS Amplifier



$$A_v = \frac{v_o}{v_{tg}} = -g_m(r_o \| R_{td})$$

$r_{in} = \infty$ $\qquad$ $r_{out} = r_o \| R_{td}$

CD Amplifier



$$A_v = \frac{v_o}{v_{tg}} = \frac{g_m (r_o \| R_{ts})}{1 + g_m (r_o \| R_{ts})}$$

$r_{in} = \infty$ $\quad$ $r_{out} = r_s \| r_o \| R_{ts}$

$r_s = 1/g_m$

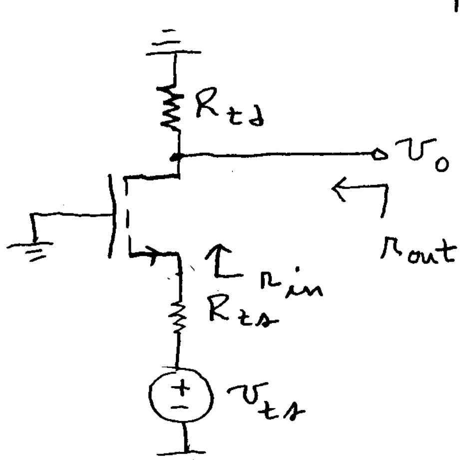
If the body lead connects to ac ground rather than to the source

$$A_v = \frac{v_o}{v_{tg}} = \frac{g_m (r_o \| R_{ts})}{1 + (1+\eta) g_m (r_o \| R_{ts})}$$

$r_{in} = \infty$ $\quad$ $r_{out} = \dfrac{r_s}{1+\eta} \| r_o \| R_{ts}$

$\eta = \dfrac{g_{mb}}{g_m}$ $\quad$ $r_s = \dfrac{1}{g_m}$

Common-Gate Amplifier



Because neither end of r_o connects to ac ground, we go with an approximate solution by assuming $r_o = \infty$.

$$A_v = \frac{v_o}{v_{ts}} = \frac{g_m R_{td}}{1 + g_m R_{ts}}$$

$$r_{in} = r_s = \frac{1}{g_m} \qquad r_{out} = R_{td}$$

Let us compare the CS and CG gains. We assume $r_o = \infty$ for each.

$$CS - \quad A_v = -g_m R_{td}$$

$$CG - \quad A_v = \frac{+g_m R_{td}}{1 + g_m R_{ts}}$$

Aside from the difference in signs, the CG amplifier gain is lower by a factor

$$\frac{1}{1 + g_m R_{ts}}$$

If $R_{ts} = 0$, the gains are the same. As R_{ts} increases, the CG gain decreases. This if a high gain is desired, the CG stage should not be used with a source having a high R_{ts}.