Effective impedance model for analysis of reflection at the interfaces of photonic crystals

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We present an alternative definition of impedance to describe the reflection at the interfaces of photonic crystals. We show that this effective impedance can be defined only by the properties of the photonic crystal modes and is independent of the properties of the incident region. This approximate model successfully explains the main features in the reflection spectrum and of various interface terminations of photonic crystals. In particular, we show an impedance matching condition at which reflectionless transmission of power to a low-group-velocity photonic crystal mode is possible, a property that is attractive for various dispersion-based applications of photonic crystals. © 2007 Optical Society of America

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With recent development in the theory and fabrication of photonic crystals (PCs) as synthetic optical material, new applications have been proposed that rely on PCs as optical materials with controllable dispersive properties (e.g., self-collimation, diffraction compensation, and superprism-based demultiplexing). For all these PC applications, it is important to analyze (and possibly suppress) the reflection at the interfaces of the PC regions (e.g., the interface between bulk and a PC or that between two different PCs).

The brute-force approach of calculating the reflection at the interfaces of the PC regions usually involves either direct electromagnetic simulation of the structure in the time domain[1] or using a modal approach.[2] Both these approaches require massive calculations and give little insight into the physics of the coupling process from the incident region to the PC mode. Here, we propose an approximate effective impedance model that can be used to estimate the reflection at the interface of a PC structure. We will show that this model has two main attractive features: first, it is independent of the properties of the incident region, and second, it can be readily applied to analyze different terminations of the PC interface (i.e., location of the interface relative to the PC unit cell). A different impedance model, inspired by the rigorous definition of impedance, has been already reported by Biswas *et al.*[3] We use an alternative definition in this Letter based on the continuity of power and field that provides more insight into the physics of reflection and enables us to optimize the structure at the interface to reduce reflection.

To find the reflection at the interface between two distinctive materials, the common practice is to apply the continuity of electromagnetic field (transverse components of electric and magnetic fields) along the interface. Alternatively, one can use a combination of these fields as new parameters to apply the electromagnetic boundary conditions. We limit our discussion to two-dimensional (2D) PC structures with transverse electric (TE) field excitation (i.e., electric field perpendicular to the plane of periodicity). In this particular case, we choose continuity of the tangential electric field and continuity of the normal Poynting vector at the interface as the boundary conditions. The choice of the continuity of the Poynting vector at the interface, encouraged by the physical concepts of the continuity of power, proves to be more suitable for PCs since the Poynting vector has the aggregated effect of the periodicity of the structure included in it. Using the analogy with homogeneous bulk media, the effective impedance for propagating modes of a PC can be defined as

$$\eta = |\langle E_{\text{int}}\rangle^2|/(2S_n), \tag{1}$$

where $\langle E_{\text{int}}\rangle$ is the spatial average amplitude of the electric field along the interface and S_n is the time-averaged component of the Poynting vector normal to the interface. For the equivalent impedance model to be valid, the incident and transmitted regions need to be single mode (or they need to have a dominant mode). Furthermore, the accuracy of the model degrades when the variations of the field along the interface in the two regions (i.e., incidence and transmission regions) differ significantly. These considerations limit the range of structures that our effective impedance model covers, but the model remains valid in the vicinity of the first photonic bandgap of the PCs, on which most of the activity in the PC dispersion engineering has been focused.

To verify the effective impedance model, we consider a 2D structure in which light is incident from a dielectric region to an interface of a square lattice PC. The PC structure consists of a periodic array of air holes in a dielectric with normalized radii of $r/a = 0.35$ (a being the lattice constant), and its band structure in the direction normal to the interface is shown in Fig. 1(a). Since we analyze the structure using a 2D model, the effect of the finite PC thickness is taken into account by selecting the effective permittivity of $\epsilon_r = 7.4$ for Si as the dielectric material. Here, normal incidence is assumed, and the PC termination at the interface is assumed to go through the middle of the holes [i.e., $y_0 = a/2$, as shown in the inset of Fig. 1(b)]. The transmission results are shown in Fig. 1(b) for direct simulation (by modifying a cas-

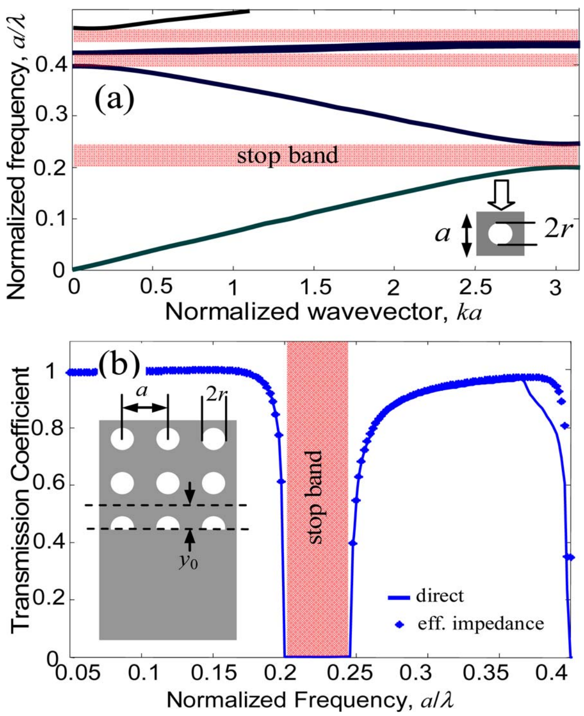

Fig. 1. (Color online) (a) Band structure of a square lattice PC of air holes ($r=0.35a$) in a dielectric with $\epsilon_r=7.4$ for the direction specified by the arrow in the inset. (b) Transmission coefficient (i.e., transmitted power divided by the incident power, $T=1-R$) of light from the incident region ($\epsilon_r=7.4$) to the PC in (a) for direct grating analysis and the effective impedance model. The interface is located halfway through the holes (i.e., $y_0=a/2$).

caded grating analysis[4] to incorporate termination to a periodic structure) and for those calculated using our proposed approximate effective impedance model [$R=|(\eta_i-\eta_{pc})/(\eta_i+\eta_{pc})|^2$, where η_i and η_{pc} are the effective impedance of the incident region and that of the PC interface, respectively]. To calculate the effective impedance, we use the plane wave expansion method to find the eigenmodes of the structure, and then the effective impedance for each PC mode is found directly from Eq. (1) for any given location of interface. Figure 1(b) shows that the effective impedance model provides a very good estimate of the transmission coefficient up to the point that the reflected grating orders appear (at $a/\lambda=0.37$). In particular, the transmission characteristics in the vicinity of the stop band are accurately represented by the effective impedance model.

One main advantage of the effective impedance model is that it is independent of the permittivity of the incident region and is defined only by the properties (permittivity, geometry, lattice type, etc.) of the PC region. To verify this, the same PC structure in Fig. 1 is used when light is incident from air. Figure 2 shows the transmission coefficient calculated with air as the incident region with different terminations of the PC with good agreement between the effective impedance model and direct numerical simulation visible.

Figure 2 shows the existence of high transmission zones in the vicinity of the stop band [around $a/\lambda=0.19$ in Fig. 2(a) and around $a/\lambda=0.26$ in Fig. 2(b)]. Such behavior is of special interest in the dispersion-based applications of PCs, where coupling of light to PC modes with low group velocity is desired. Figure 3 shows the electric field profile of the relevant mode of the PC studied in this Letter at frequencies close to the stop band in Fig. 2. To calculate the PC effective impedance, we use Eq. (1) and note that the Poynting vector is related to the group velocity normal to the interface (v_{gn}) and the average energy density stored in the medium (W_E) as

$$S_n = W_E v_{gn}. \quad (2)$$

In Eq. (2), the average stored energy over a unit cell, W_E, is given by

$$W_E = \frac{1}{2}\int\int_{u.c.} \mathrm{d}\mathbf{r}\,\epsilon(\mathbf{r})|E(\mathbf{r})|^2, \quad (3)$$

where $\epsilon(\mathbf{r})$ represents the permittivity and the 2D integration is performed over a PC unit cell [i.e., the structure in Fig. 3(a)]. Combining Eqs. (1) and (2) results in

$$\eta = |\langle E_{\mathrm{int}}\rangle^2|/(2W_E v_{gn}). \quad (4)$$

In Fig. 3(a) the electric field is more concentrated in the dielectric around the edges of the unit cell where the interface is located. Continuity of the electric field along the interface states that the field along this line is the same inside the PC and in the

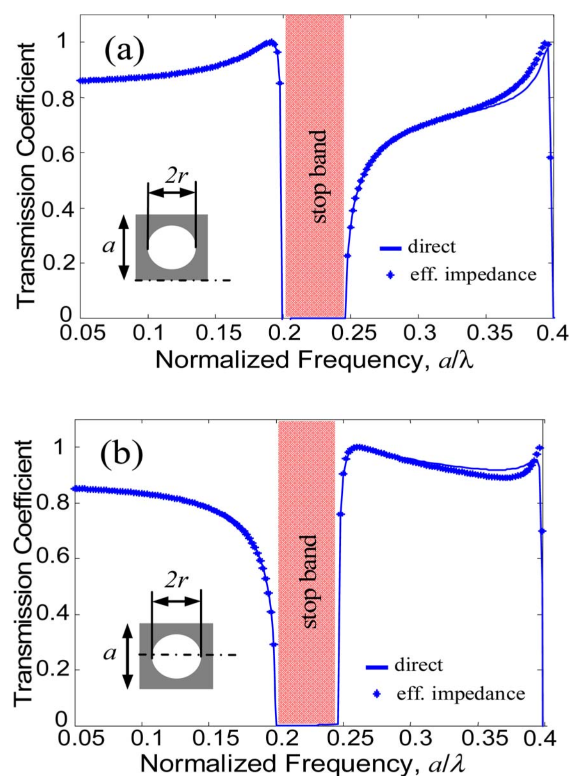

Fig. 2. (Color online) The transmission coefficient of light from air ($\epsilon_{air}=1.0$) to the square lattice PC in Fig. 1 is calculated using direct grating analysis and the effective impedance model. The interface (dashed–dotted line) is at (a) $y_0=0$ and (b) $y_0=a/2$, with y_0 defined in Fig. 1.

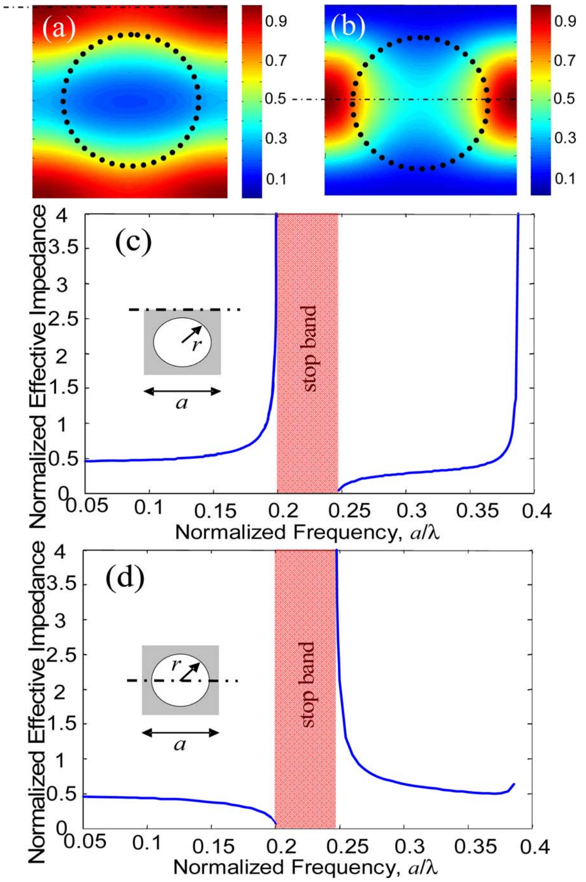

Fig. 3. (Color online) Electric field profiles of the PC modes at high transmission points are shown for (a) $a/\lambda = 0.19$ and (b) $a/\lambda = 0.26$. The dashed–dotted lines show the relative location of the interface for low reflection when incident light is coming from air in each case. The effective impedance of the PC under study (normalized to that of air) is shown for the interface with (c) $y_0 = 0$ and (d) $y_0 = a/2$.

air. Therefore, using Eq. (3), one can see that $(|\langle E\rangle_{\text{int}}^2|/W_E)_{air} > (|\langle E\rangle_{\text{int}}^2|/W_E)_{PC}$ due to large W_E in the PC region caused by higher permittivity, $\epsilon(\mathrm{r})$. Combining this result with Eq. (4), it is clear that impedance matching at the PC–air interface requires a small group velocity for the PC mode compared with air. This can be realized by using the PC in the frequencies close to the edge of the stop band. Similarly, in the frequencies above the first stop band, the field is more concentrated in the dielectric in the middle of the PC unit cell as shown in Fig. 3(b). Therefore, for these frequencies and for an interface going through the middle of the unit cell [as shown in Fig. 3(b)], again $(|\langle E\rangle_{\text{int}}^2|/W_E)_{air} > (|\langle E\rangle_{\text{int}}^2|/W_E)_{PC}$, and by using frequencies close to the edge of the stop band and thus lowering the group velocity in the PC, effective PC impedance of unity for impedance matching to the incident region (i.e., air in this case) can be achieved. Another important behavior in the vicinity of the stop band can be seen when the field profile of the PC mode goes to zero along the interface [e.g., $y_0 = a/2$ interface and $y_0 = 0$ interface in Figs. 3(a) and 3(b), respectively]. In this case, $(|\langle E\rangle_{\text{int}}^2|/W_E)$ has a second-order zero at the band edge, while v_{gn} has a first-order zero; therefore, as we approach the bandgap, the effective impedance of the structure goes to zero.

Our results clearly show that the PC effective impedance can be considerably varied by the PC termination at the interface. The possibility to approach both zero and infinite effective impedances (and thus any impedance value in between) based on the PC termination at the interface provides the opportunity to impedance match the PC to any incident region by proper choice of the interface. In many dispersion-based applications of PCs, a low-group-velocity mode of the PC is of interest, and the impedance matching discussed above provides a practical way to achieve reflection-free coupling of light to these modes. The bandwidth of the high transmission window (>95%) in this case is around 10% of the operation wavelength, which is wide enough for almost all practical purposes. Note that this particular impedance matching happens in the vicinity of the bandgap, where other matching schemes face complications (e.g., Ref. 5).

In conclusion, the effective impedance model can be used to estimate the reflection at the interfaces of PCs with acceptably low error for most practical cases. The fact that an effective impedance can be defined for a PC interface regardless of the properties of the incident region makes it possible to assess the reflection for a given PC mode without going into lengthy simulations. In addition, the use of effective impedance also enables us to describe a unique impedance matching condition for reflection-free coupling to PCs, which is of significant value in designing heterostructure dispersion-based PC devices.

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