

# Adiabatic matching stage for coupling of light to extended Bloch modes of photonic crystals

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In this letter, we present a matching stage for reflection reduction based on the principle of gradual change to efficiently couple light to propagating modes of photonic crystals (PCs). Basic physical considerations in designing these matching stages are investigated and a systematic yet simple design procedure is suggested. We show that matching stages obtained using this method are wideband in frequency, have a wide acceptance angle, and are robust against fabrication imperfections. Therefore, they are the preferred choice in general-purpose matching stages to be used along with dispersion-based PC devices. © 2005 American Institute of Physics.

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Photonic crystals (PCs) are synthetic optical materials in which the ability to modify the subwavelength features of PC unit cells makes it possible to control dispersion properties. Examples of functionalities based on dispersion properties of PCs are superprism-based demultiplexing,<sup>1</sup> diffraction-free propagation,<sup>2</sup> and diffraction compensation.<sup>3</sup> One crucial issue in implementing dispersion-based PC devices is to overcome the reflection loss in coupling the light into or out of the PC structures. Different techniques that have been proposed to reduce this reflection can be divided into two groups: Methods based on interference,<sup>4,5</sup> and those based on gradual change.<sup>6,7</sup>

Interference-based methods are those in which multiple reflections throughout the matching stage are used to achieve a destructive interference for the reflection wave (e.g., in antireflection multilayer coatings). In the matching stages based on gradual change (also known as adiabatic transition), however, the initial mode passes through slightly modified intermediate steps that transform it into the final mode, keeping the reflection low at all intermediate stages. A main concern in proposed interference-based techniques to date is the small range of frequencies or incident angles over which low reflection is achieved. In this letter, we present a design for a reflection reduction stage based on adiabatic transition that is wideband (in frequency), tolerant to fabrication imperfections, compatible with planar fabrication technologies, and has a wide acceptance angle.

In adiabatic transitions in electromagnetic structures, the characteristic properties of the medium vary smoothly in a way that there is negligible coupling of the initial mode to other modes (reflecting or of different properties) of the structure. In theory, for such a smooth variation, in the limit for an infinitely long transition region, complete transfer of energy is possible. In practice, however, the length of the transition stage is limited, and an approximation to the ultimate adiabatic case is achieved. Furthermore, relatively strong dispersion effects in the behavior of the modes of the PC make it necessary to consider the smooth variation of mode properties in addition to the smooth variation of geometrical features. In the following discussions, we will restrict ourselves to two-dimensional (2D) PCs of air holes in a

dielectric material with a fixed permittivity to comply with the practical limitations in fabricating planar PC devices. Thus, the design parameters are the radius of holes and the aspect ratio (ratio of the lattice vector in the two perpendicular directions).

Assuming that the variations are slow and following the basic approximation in Wentzel–Kramers–Brillouin method,<sup>8</sup> the wave in the transition region can be considered to be the local mode of the corresponding PC at each location. Thus, in order to design a smooth transition, one needs to know how large the mismatch is in any of the intermediate interfaces. To get this information, we first consider the reflection at the interface of two slightly different PCs shown in the inset of Fig. 1. Assuming that the difference between the field profiles of the modes at the interface is negligible, the field in these two regions can be written as

$$\begin{cases} f_1(\mathbf{r}) = u_1(\mathbf{r})\exp(-j\mathbf{k}_i \cdot \mathbf{r}) + \rho u_1(\mathbf{r})\exp(-j\mathbf{k}_r \cdot \mathbf{r}) \\ f_2(\mathbf{r}) = \tau u_2(\mathbf{r})\exp(-j\mathbf{k}_t \cdot \mathbf{r}), \end{cases} \quad (1)$$

where the field is  $f_n(\mathbf{r}) = \sqrt{\epsilon_n(\mathbf{r})}E_{n,z}(\mathbf{r})$  ( $n=1,2$  shows the region that the field corresponds to) for transverse electric (TE) polarization (electric field perpendicular to the plane of periodicity), and  $f_n(\mathbf{r}) = \sqrt{\mu_0}H_{n,z}(\mathbf{r})$  for transverse magnetic polarization (magnetic field perpendicular to the plane of peri-

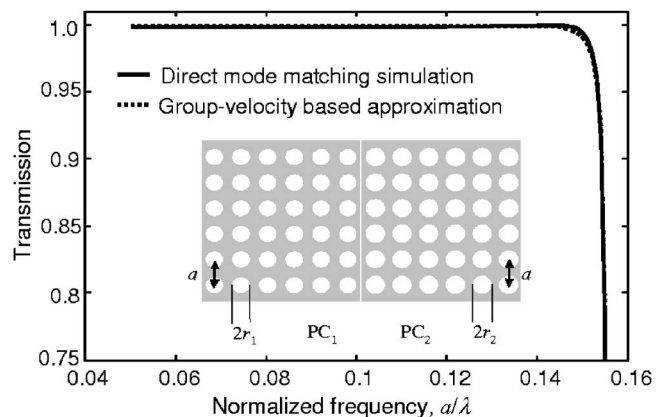


FIG. 1. Transmission coefficients (TE polarization) for the light coupling between two 2D square lattice PCs with slightly different parameters ( $r_1/a=0.30$  and  $r_2/a=0.35$ , as shown in the inset) using the direct mode matching simulation and group-velocity-based approximation.

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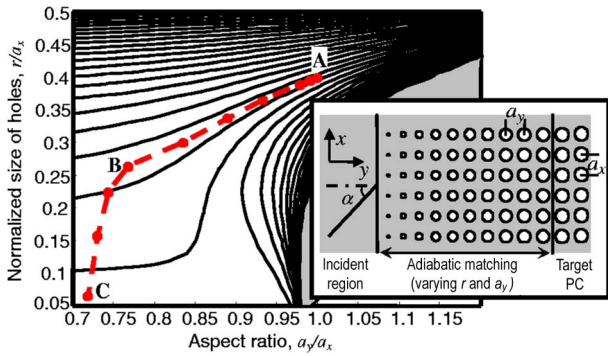


FIG. 2. (Color online) Contours of constant group velocities (at  $a/\lambda=0.15$  with TE polarization) for different design parameters of 2D rectangular lattice PCs. The lattice constant parallel to the interface,  $a_x$ , is kept intact and normal incidence (along the  $y$  direction) is considered. The dashed line shows the suggested path for an adiabatic transition and the circles highlight the designed parameters for the successive layers. The gray shaded region represents PC structures for which the operation frequency lies inside the band gap.

odicity). The coefficients  $\rho$  and  $\tau$  represent the reflection and transmission relative field amplitudes, and  $u_1(\mathbf{r})$  and  $u_2(\mathbf{r})$  are the Bloch envelope functions corresponding to the PC modes. In addition,  $\mathbf{k}_i$ ,  $\mathbf{k}_r$ , and  $\mathbf{k}_t$  are the incident, reflected, and transmitted wave vectors, respectively.

Assuming  $S_i$ ,  $S_r$ , and  $S_t$  to be the component of Poynting vector normal to the interface corresponding to incident, reflected, and transmitted contributions, respectively, the boundary conditions at the interface can be written as

$$\begin{cases} f_1(\mathbf{r})|_{\text{interface}} = f_2(\mathbf{r})|_{\text{interface}} \\ S_i|_{\text{interface}} = S_r|_{\text{interface}} + S_t|_{\text{interface}} \end{cases} \quad (2)$$

The Poynting vector can be related to the group velocity using  $\mathbf{S} = E\mathbf{v}_g$ , in which  $E$  is the average energy density of the mode, given by

$$E = 2 \int_{\text{uc}} |f(\mathbf{r})|^2 d\mathbf{r} / \int_{\text{uc}} d\mathbf{r} = 2 \int_{\text{uc}} |u(\mathbf{r})|^2 d\mathbf{r} / \int_{\text{uc}} d\mathbf{r}, \quad (3)$$

where the integration is performed over a unit cell (uc) of the PC. Starting from this relation for incident, reflected, and transmitted waves, and using the assumption that the field profiles in the two regions have a negligible difference, one can directly deduce that  $\rho$  and  $\tau$  are real valued and that  $\int_{\text{uc}} |u_1(\mathbf{r})|^2 d\mathbf{r} \approx \int_{\text{uc}} |u_2(\mathbf{r})|^2 d\mathbf{r}$ . Therefore,  $E_{1,i} = E_{1,r} / \rho^2 = E_{2,t} / \tau^2$ , and substituting this into Eq. (2) results in

$$\begin{cases} 1 + \rho = \tau \\ \tau^2 v_{g2,n} + \rho^2 v_{g1,n} = v_{g1,n} \end{cases} \quad (4)$$

where  $v_{g1,n}$  and  $v_{g2,n}$  are the components of group velocities normal to the interface for the PC modes in Regions 1 and 2, respectively. Using Eq. (4), one can obtain the reflection coefficient as

$$\rho = (v_{g1,n} - v_{g2,n}) / (v_{g1,n} + v_{g2,n}). \quad (5)$$

This relation relates the reflection at the interface between slightly different PCs to the mismatch in the group velocity of their modes. To verify the validity of Eq. (5), we analyze the reflection for TE polarized light between two 2D square lattice PCs of air holes in Si ( $\epsilon_r=11.4$ ) with  $r_1/a=0.30$  and  $r_2/a=0.35$ , as shown in Fig. 1. The results for direct calcu-

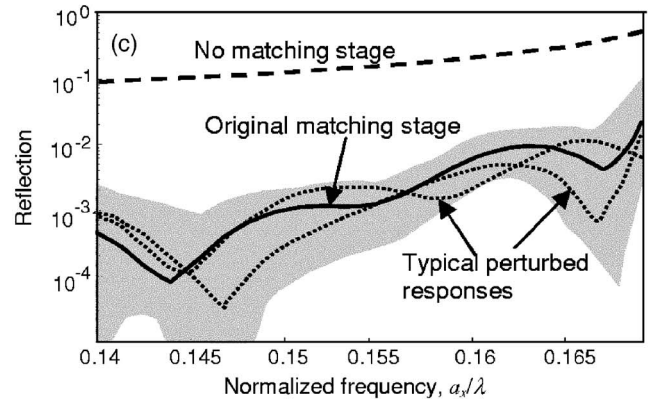
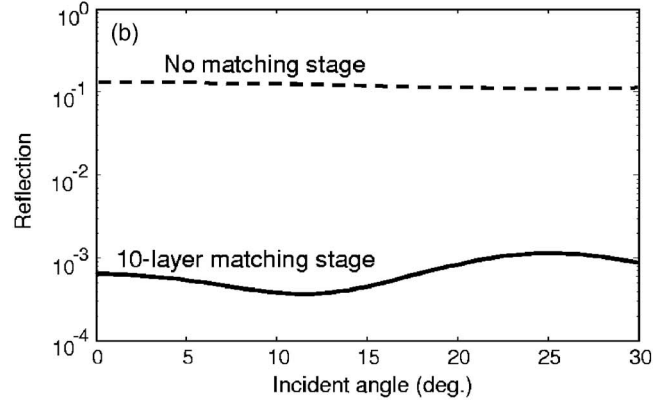
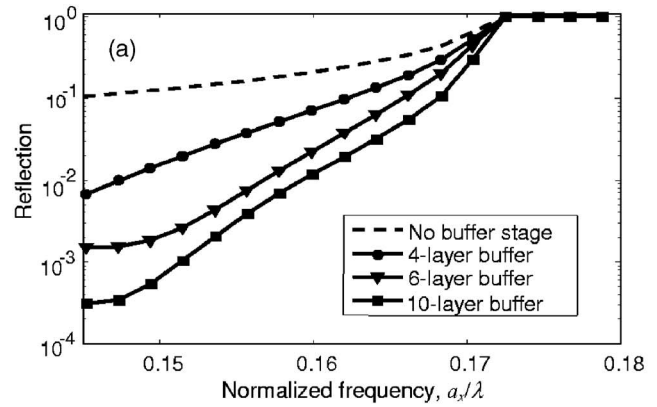


FIG. 3. (a) Reflection from a square lattice PC structure (circular air holes in Si,  $r/a=0.40$ ) is shown with no matching stage and with adiabatic matching stages. (b) For the same structure, the angular response of the ten-layer matching at  $a_x/\lambda=0.15$  is compared with that of the PC with no matching stage. (c) The shaded region shows reflections for 50 different random buffer stages (with 12 PC layers) with up to 5% perturbation in the sizes of holes compared to the original design.

lations (using plane-wave expansion and mode matching method)<sup>3</sup> are compared with those obtained by the group-velocity approximation in Eq. (5), as shown in Fig. 1. Transmission coefficients (i.e., transmitted power divided by the incident power) calculated using the approximate relation in this case agrees well (within a relative error less than 1%) with those of the exact calculations.

In PCs, the reflection between slightly different regions can be attributed to two sources: The difference between their group velocities and the difference between their field profiles. Thus, the design of a smooth transition region (i.e., designing the pattern of variation for the radius of holes and aspect ratio) should be performed in two aspects: (1) Smooth variation in field profile, and (2) smooth variation in group velocity. Here, the focus is on coupling from an incident

homogeneous region to a PC structure. For this case, the strategy that we propose involves: (1) initially changing the geometry of the PC structure smoothly to keep the group velocity intact until we reach a PC structure with low dispersion effects, and (2) in the second step, changing the structure to match the group velocity of the incident region. Using this scheme, the field profile matching is done at first, and then group-velocity matching is performed in the regime that dispersion effects are no longer strong. To demonstrate this design scheme, coupling at normal incidence from a homogeneous Si region to a square lattice PC of air holes in Si with  $r/a=0.40$  under TE polarization is considered. For this lattice type, constant group-velocity contours with respect to different values of hole size and aspect ratio are plotted in Fig. 2. Normalized frequency ( $\omega_n=a_x/\lambda$ ,  $a_x$  being the lattice constant parallel to the interface) of 0.15 is assumed for the calculation of the results shown in Fig. 2. The path proposed for the adiabatic transition is shown by a dashed line in this figure. Point A in Fig. 2 corresponds to the PC structure that we plan to efficiently couple light to (target PC). The path between A and B falls on a constant group-velocity contour, and each point—marked by a circle—corresponds to one of the intermediate PC layers. The path from B to C is chosen to have equal reflection at all intermediate stages.

The approximate length of the matching stage,  $L$ , for long transition regions,  $k_0L \gg c/v_{g2,n}$ , can be obtained based on general approximations of adiabatic transition,<sup>8</sup> as

$$L \approx (1/2k_0)(c/v_{g2,n})\ln(1/R_g), \quad (6)$$

in which  $R_g$  is the desired (or acceptable maximum) power reflection,  $c$  is the velocity of light in a vacuum, and  $k_0$  is the free-space wave number. Equation (6) for a PC structure is different from the formula for an adiabatic transition in bulk materials (given in Ref. 8) by a factor of  $2\pi$ , which was calculated by interpolating the simulation results to include the effects of the discretization of the properties of the PC structures (i.e., layered structure instead of a continuous adiabatic change), and the difference between initial and final field profiles in the PC structure (which is not present for bulk structures in Ref. 8).

Figure 3(a) shows the reflection performance of the matching stages designed for different lengths of the transition for the PC structure described in Fig. 2. A combination of plane-wave expansion and multilayer-grating analysis is used to calculate the reflection in these structures. The exponential reduction in reflection by adding the buffer layer can be qualitatively seen from Fig. 3(a) at each normalized frequency as suggested by Eq. (6). Figure 3(a) shows that a

buffer stage with a small size (less than ten lattice constants) results in a considerable reflection reduction over a wide frequency range. Figure 3(b) compares the angular reflection response of the PC structure having a ten-layer adiabatic matching stage with that of the structure with no matching stage. The incident region is a homogeneous Si medium. It can be observed from Fig. 3(b) that, in a wide range of angles, two orders of magnitude reflection reduction is achieved by adding the buffer stage with only ten layers. To study the robustness of the adiabatic matching stage under fabrication imperfection, Fig. 3(c) shows the reflection performance of a 12-layer adiabatic matching stage and 50 different randomly perturbed versions of it obtained by randomly varying the radii of holes in all layers by up to 5% using a random variable with uniform distribution. Figure 3(c) shows that at the center frequency of  $\omega_n=a/\lambda=0.15$ , the reflection remains at the same order of magnitude as the original design. Furthermore, the modified structures maintain the wideband performance expected from an adiabatic design.

In conclusion, we showed here that the principle of gradual change can be used to design matching stages for coupling light into and out of PC structures. The two main considerations for designing these structures are the smooth variation of the field profile and the group velocity of the PC modes throughout the matching region. We also demonstrated a simple design scheme to realize these matching stages in the form of multiple PC layers with chirped size of holes and periodicity. Resulting adiabatic matching stages have a wide frequency band, have a large acceptance angle, and are robust against fabrication imperfections. These properties make adiabatic matching stages perfect candidates for all dispersion-based applications of PCs.

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<sup>1</sup>H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, *J. Lightwave Technol.* **17**, 2032 (1999).

<sup>2</sup>D. N. Chigrin, S. Enoch, C. M. Sotomayor Torres, and G. Tayeb, *Opt. Express* **11**, 1203 (2003).

<sup>3</sup>B. Momeni and A. Adibi, *J. Lightwave Technol.* **23**, 1522 (2005).

<sup>4</sup>T. Baba and D. Ohsaki, *Jpn. J. Appl. Phys., Part 1* **40**, 5920 (2001).

<sup>5</sup>J. Witzens, M. Hochberg, T. Baehr-Jones, and A. Scherer, *Phys. Rev. E* **69**, 046609 (2004).

<sup>6</sup>P. Rabiei and A. F. J. Levi, *Proceedings of CLEO'01*, Baltimore, MD (2001), p. 590.

<sup>7</sup>S. Kuchinsky, V. Golyatin, and A. Kutikov, *Proc. SPIE* **5000**, 59 (2003).

<sup>8</sup>M. V. Berry and K. E. Mount, *Rep. Prog. Phys.* **35**, 315 (1972).