Power-Controlled Capacity for Interfering MIMO Links

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Abstract— **An iterative algorithm is presented to jointly optimize the capacities for a number of interfering array-to-array or MIMO flat-fading links. All signals are co-channel. At each iteration for a given link, water-filling is used to calculate the optimum transmitter signal vector correlation matrices and receiver transformations under a constraint of total transmitted power for that link. Thus, by controlling the relative transmit powers at each of the interfering links the relative capacities can be controlled. A Monte-Carlo simulation was performed for an example three-link geometry. Several statistics for the three capacities are given for various power control conditions.**

I. Introduction

This paper considers joint optimization of power and array weights for a set of interfering co-channel links, where each transmitter and each receiver is assumed to have an array antenna. Flat-fading is assumed. Our present goal is to understand the flexibility in such a network in order to concisely quantify link control parameters that might someday be used by resource allocation algorithms.

Array-to-array links, also known as multiple-input multiple-output (MIMO) wireless links, are well known to provide extremely high spectral efficiency in rich multipath environments [1]. Many studies consider a single point-to-point link, without co-channel interference. The capacity of MIMO systems when the transmitter operates without channel knowledge is given in [2]. The case of an informed transmitter, where channel information is fed back from the receiver to the transmitter, is explored in [3]. The problem of determining the capacity of a MIMO link with interference is considered in [4],[5]. Some studies have considered joint optimization of sets of co-channel links that interfere with each other. These studies assume that only the base-station has an array antenna; [6] considers the uplink and-[7] considers the downlink. In a few of these studies, iterative methods are used to determine optimal power control and weight adaptation [6],[7]. Control over the spatial domain is limited to only one end of the link in these link sets.

In this paper, we allow spatial control at both ends of the link. In this way, a link that has, for example, the highest quality of service (QoS) requirement can devote all the spatial degrees of freedom in both its transmitter and its receiver to achieve the maximum capacity. An interfering link with the lowest QoS must dedicate spatial degrees of freedom at the transmitter simply to avoid making interference on the high QoS link and must also dedicate spatial degrees of freedom at its receiver to suppress the interference being generated by the high-QoS link. Whatever is left after these avoidance and suppression requirements are satisfied can be used for transmission of information.

The capacity of each link and its optimum transmitter signal vector correlation matrix are functions of the channel and the external interference at the receiver. Since the external interference is affected by the transmitter correlation matrices of the interfering transmitters, the optimum transmission strategies and the capacities of all links are mutually dependent.

This paper gives an iterative method for determining the optimum transmitter correlation matrices and receiver transformations for each link in a set of interfering co-channel links. The control parameter for each link is the total transmitted power, normalized by the receiver noise. The solution is optimum in the sense that the link capacities are maximized for given values of the control parameters.

The paper is organized as follows: In Section II, we formulate the MIMO channel capacity given by the water-filling solution assuming additive Gaussian noise and fixed interference. In Section III, the capacities of multiple co-channel links are considered. An iterative method to determine the capacities is introduced. In Section IV, some network considerations are discussed. An example network with three links is considered in Section V, and some results are presented. Section VI concludes the paper.

II. Capacities of MIMO Links

In a flat-fading MIMO system, the output vector of the receiver antenna array can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ denotes the received signal, $\mathbf{x}$ denotes the transmitted signal, $\mathbf{n}$ is a vector of noise samples, and $\mathbf{H}$ is the complex channel matrix from the transmitter node to the receiver. Each element $\mathbf{H}(a,b)$ of the channel matrix gives the complex channel gain from antenna element a of the transmitter to antenna element b of the receiver.

The channel capacity is obtained by maximizing the mutual information between $\mathbf{x}$ and $\mathbf{y}$, as

$$C = \max_{\mathbf{P}} \log_2 |\mathbf{HPH}' + \mathbf{I}| \quad (2)$$

where $\mathbf{P} = \mathbf{E}\{\mathbf{xx}'\}$, is the power allocation matrix, normalized by the received noise power on a single receive antenna element, earlier in the paper referred to as the transmitter signal vector correlation matrix. If the transmitter does not have an estimate of the channel matrix $\mathbf{H}$, an equal power allocation to the channel modes, namely, setting $\mathbf{P} = (P_{Tot}/K)\mathbf{I}$, gives the best result [2],[8]. Here, P_{Tot} is the total transmitted power, normalized by the additive noise power; and K is the number of orthogonal spatial channels formed, or the number of channel modes. However, if the transmitter knows the channel matrix, the classical water-filling method maximizes the expression in (2) [5]. Let the singular-value decomposition of $\mathbf{H}$ be denoted as $\mathbf{H} = \mathbf{USV}'$and the eigenvalue decomposition of $\mathbf{P}$ as $\mathbf{P} = \mathbf{D\Sigma D}'$. Furthermore, let $\sqrt{\lambda_k}$ and $\alpha_k, k = 1,...K$ be the diagonal elements of $\mathbf{S}$ and $\mathbf{\Sigma}$, respectively. In accordance with the convention, the singular values of $\mathbf{H}$, $\sqrt{\lambda_k}$, are ordered such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$.With the choice of $\mathbf{D} = \mathbf{V}$, the expression for the capacity becomes [5]

$$C = \max_{\substack{\alpha_k \\ k=1,...K}} \sum_{k=1}^{K} \log_2(1 + \lambda_k \alpha_k) \quad (3)$$

With a total transmitted power of P_{Tot}, the classical water-filling solution gives

$$\alpha_k = \left[\mu - \frac{1}{\lambda_k}\right]^+, \quad (4)$$

where $[\cdot]^+$ indicates that only non-negative values are acceptable, and μ is chosen so that $\sum_{k=1}^{K} \alpha_k = P_{Tot}$ [5].

The maximum capacity is utilized when the transformation at the receiver equals $\mathbf{U}$, and the power allocation matrix at the transmitter array is given by $\mathbf{P}$ [5].

It is known that for network models with non-stationary channels, such a *smart* utilization of the channel may result in worse performance than what would be obtained by an equal power allocation to the channel modes [4]. Therefore, we assume a stationary or a quasi-stationary channel, where the channel estimation error does not have a significant effect on the results.

In the presence of a fixed interference with a covariance matrix of $\mathbf{R}$, the expression for the channel capacity becomes

$$C = \max_{\mathbf{P}} \log_2 \frac{|\mathbf{I} + \mathbf{R} + \mathbf{HPH}'|}{|\mathbf{I} + \mathbf{R}|}. \quad (5)$$

Applying a spatial whitening transform [4] to the channel yields

$$\widetilde{\mathbf{H}} = [\mathbf{I} + \mathbf{R}]^{-1/2}\,\mathbf{H}, \quad (6)$$

which reduces (5) to the simple form in (2), with a substitution of $\mathbf{H} \rightarrow \widetilde{\mathbf{H}}$. The water-filling method explained in the previous section, again, gives the solution to the problem.

III. Capacities of Interfering Links

In a network with multiple interfering links, the interference correlation matrix seen by each link receiver array varies with the transmitter correlation matrices of the interfering nodes. The whitened channel matrix, $\widetilde{\mathbf{H}}$, for a given link is a function of the interference, $\mathbf{R}$. The transmission strategy, in turn, is dependent on the whitened channel matrix. As a result, a change in the power allocation matrix of one link induces a change in the optimum power allocation matrix of the other co-channel links. Therefore, the optimum transmission strategies and the capacities of interfering co-channel links are mutually dependent, and cannot be calculated directly.

An iterative method is used to determine the optimum transmitter correlation matrices and receiver transformations in such a network. At each iteration, every transmitter-receiver pair optimizes its link capacity for the measured interference at the receiver, and its respective given total transmitted power. In the next iteration, the interference seen at each receiver is different, and the calculated transmission correlation matrices are closer to optimum. Our simulations show that for a given channel realization the capacities converge to a certain optimum set of values. Note that with this iterative algorithm, the optimum transmission strategies for a given link are found based only on the channel gains for that link and the interference correlation matrix observed at the receiver array for that link. The knowledge of the channel gains between other transmitter-receiver pairs is not necessary.

For about 90% of the channel trials of the method for a network with two or three interfering links, and independent Rayleigh fading channel coefficients, convergence was achieved within about 12 iterations. The largest number of iterations we encountered in 100 trials was about 30.

IV. Network Considerations

Two main advantages of using MIMO systems are increased capacity and spatial control at both ends of a link. A MIMO link is known to yield a higher capacity than a link with a single antenna at each end. The channel capacity of a MIMO link increases linearly with SNR (in dB) at high SNR, with a slope that is proportional to the number of significant singular values of the channel matrix [3]. The capacity and its slope of a MIMO link generally decrease as more degrees of freedom in the transmitter and receiver arrays are used for interference suppression.

In spite of the capacity reduction in a particular link because of interference from other co-channel links, the overall utilization of the channel is enhanced, as we will show in a simulation example. The total capacity of the links is more than what would be achieved by time-multiplexing the use of the channel. In addition to the capacity gain, allowing multiple links to share a single channel also reduces the delay jitter of received packets, and eases scheduling of the use of channel. Since the nodes are allowed to transmit at all times, the delays experienced by packets are expected to be uniform. Instead of scheduling the use of time slots, a way of allocating the capacities in accordance to certain network flow configurations is needed.

Under light data traffic, however, it may not be advantageous to have multiple interfering links active. When time-division multiplexed, the nodes use power for transmission only during their scheduled time slots. However, when co-channel links operate simultaneously, all nodes transmit continuously, and each link has a lower capacity than it would have in the absence of interference. Although there is a gain in the total capacity utilized, more power is transmitted in the network.

In order to facilitate optimal network performance under different QoS requirements at each individual link, it might be necessary to allocate different capacity levels to the links that must accommodate certain desired network flows. From a power-aware point-of-view, this can be achieved simply by adjusting the total powers at each of the transmitting nodes. A link requiring less channel capacity can reduce its transmitter power, and cause less interference. On the other hand, a link requiring more capacity can either increase its transmitter power, or request the other transmitters to reduce their powers.

The resulting allocation of the capacities is optimum, since each link in the network gets the maximum capacity for the power used. It is also remarkable that the allocation process can be done without a central controller in the network.

V. Three-Link Example and Results

Shown in Figure 1 is a simple network with six nodes, where the nodes are equally spaced on the corners of a hexagon. The nodes are paired to form three simultaneously active links, shown by the black arrows. Each node has an array of 4 antenna elements, spaced sufficiently far apart to have independent fading. The gray arrows show the sources of interference. Since each node in the sample network has 4 antenna elements, each channel is represented by a 4×4 matrix. In our simulations, channel is assumed to be stationary and Rayleigh faded. The parameter for the Rayleigh distribution is set as a function of the distance between the nodes.

To observe the statistics of the channels and the resulting capacities, 4000 channel realizations are generated for each link, and corresponding capacities are calculated for different sets of total transmitted power levels.

An illustration of the dependence of the link capacities on total transmitted powers is given in Figure 2. The figure shows the means of the capacities of the three links found by the iterative method explained above for different total powers transmitted. For each set of channel realizations, the capacities are found iteratively, and the converged values of the capacity are averaged over 4000 trials. The two horizontal axes represent the total powers transmitted from nodes 3 and 5, respectively, which vary from 0 to 20 dB. The power of node 1 is held constant at 20dB. The vertical axis is the capacity in bits/s/Hz. For each set of powers the capacities found are different. The surfaces are not symmetric with respect to the line of equal powers for nodes 3 and 5. This is because the transmitter in link 1-2 makes stronger interference on the receiver of link 5-6 than it does on the receiver of link 3-4 because the distances are not the same. However, there are some points of symmetry: For example, the mean capacity of link 3-4 at the coordinates (power(3),power(5))=(20,0) equals the mean capacity of link 1-2 at the coordinates (power(3), power(5))=(0,20), because of the symmetry of the hexagon.

Figure 3 shows the 90% confidence intervals for the three link capacities for same set of power configurations as in Figure 2. The surfaces are shown from two opposite viewpoints. The surfaces showing the upper and lower interval

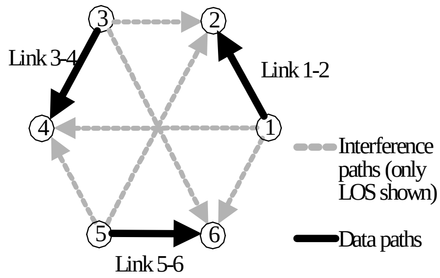

Fig. 1. A simple network with six nodes. Nodes 1, 3, and 5 are transmitting data to 2, 4, and 6, respectively.

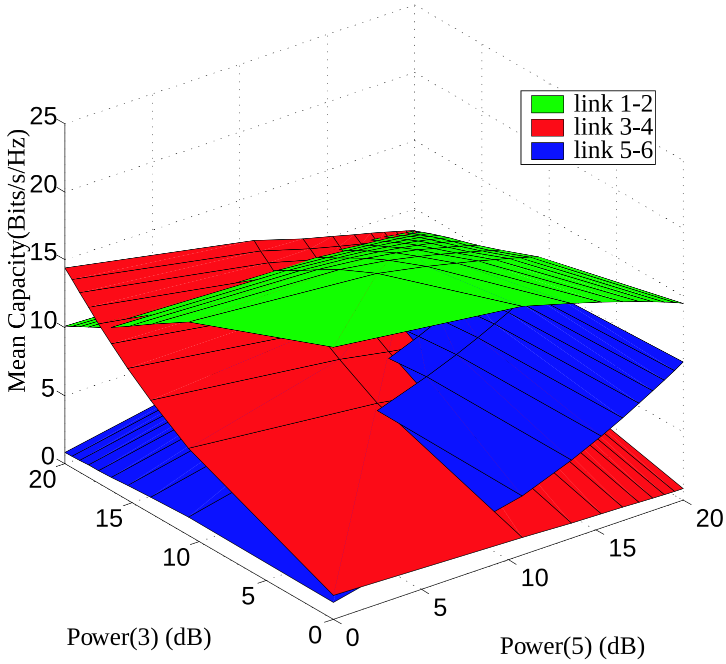

Fig. 2. The expected capacities of three interfering links as a function of the noise-normalized total transmitted powers.

boundaries for the same link have the same color. The interval widths are small, ranging from 2 to 5 bits/s/Hz. The interval width is larger when there is more interference. Therefore, the interval width at the front corner of Figure 3(b) is more than at other power coordinates. The surfaces corresponding to different links intersect at this corner, where all three transmitters have the same power.

Figure 4 shows the cumulative distribution functions (CDFs) of the capacities obtained from the simulations when the links use equal power. The curve in the middle corresponds to the capacity of one link when there is no interference from the transmitters of the other links. This CDF is the same for all three links, because all links have the same Tx-Rx distance. The curve on the left is the CDF of one link's capacity when all three transmitters are on, with a noise-normalized-power of 20dB. Because of the symmetry of the hexagon, this CDF is also the same for all links.

The curve on the right is the CDF of the total capacity when the three links are active at the same time. The difference between the right and the middle curves is about 3 bits/s/Hz at the median; this shows that approximately a 12.4% capacity gain is obtained when the links are allowed to operate simultaneously, as opposed to a system where nodes take turns to transmit.

The curves in Figure 4 are very close to CDFs of normal distributions. For a single-input single-output (SISO) link, the capacity is given by

$$C_{SISO} = \log_2(1 + \lambda p) \tag{7}$$

where λ is a chi-squared random variable when the channel coefficient is Rayleigh [2], and p is the noise-normalized transmission power. For nodes with 4 antennas, assuming the channel matrix has full rank, the capacity given by equation (3) is the sum of 4 terms similar to the expression in equation (7). Thus, the capacity is closer to a normal random variable.

Figure 5 shows the CDFs of capacity for 1×1, 3×3, 4×4 and 8×8 MIMO configurations. In these graphs, the vertical axis is distorted in such a way that the CDF for a normal random variable appears as a straight line. The plots in Figure 5 show that as the number of antennas increases, the capacity distribution becomes more normal. For each plot, 500 sets of channel samples are used. For 4 antennas the plot is linear, indicating that the capacity can be modeled by a normal distribution.

VI. Conclusions

An iterative algorithm is presented to jointly optimize the capacities for a number of interfering MIMO flat-fading links. Transmit power is used as the control parameter to allocate capacities as desired to each of the interfering links. Results of a Monte-Carlo simulation are given for an example three-link geometry. The statistical variation in the capacity is small, suggesting that the relation between the control parameters and the mean capacities may be sufficient for network performance analysis. The improvement in channel utilization acquired when co-channel links are allowed instead of time-division multiplexing is found to be about 12.4% for our three-link example. It is found that the distribution of a MIMO link capacity can be modeled as normal when the array size is 4 or more.

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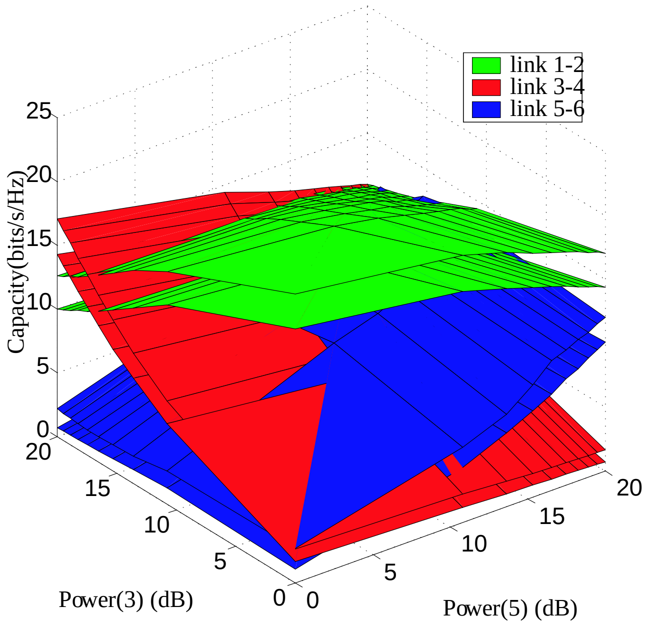
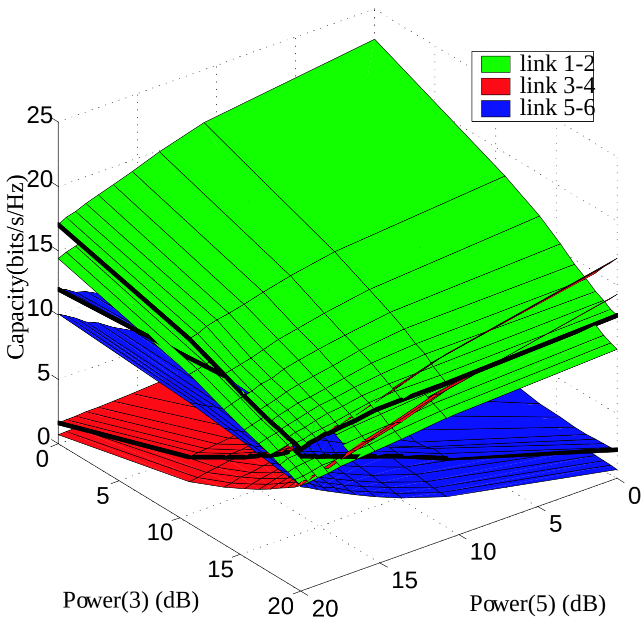


(a)



(b)

Fig. 3. Ninety percent confidence intervals for the capacities of the links from two different viewpoints.

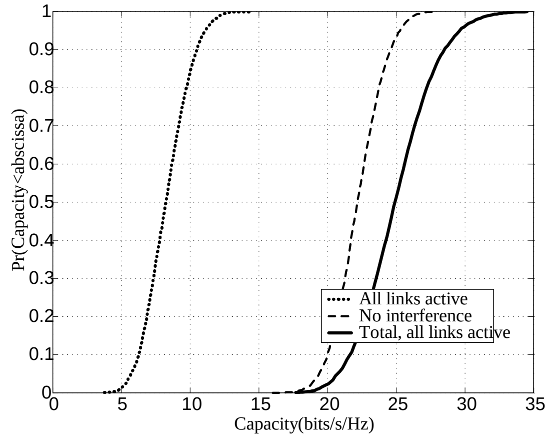
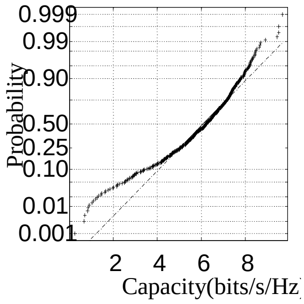
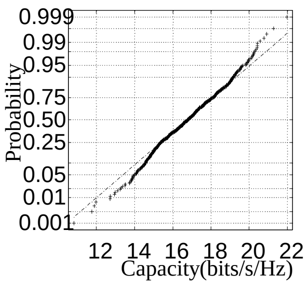


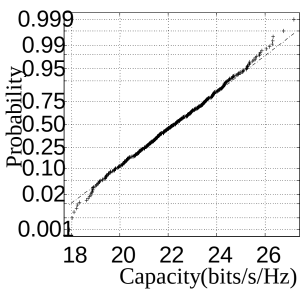
Fig. 4. CDFs of the capacities. The curve in the middle is the for one link when there is no interference, the curve on the left shows the capacity of one link when all three transmitters are on, the one on the right is the total capacity when the three links are active at the same time.



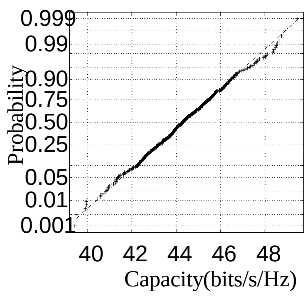
(a) 1 antenna



(b) 3 antennas



(c) 4 antennas



(d) 8 antennas

Fig. 5. The normal probability plots of the capacities for 1, 3, 4, and 8 antenna configurations.