CONTROL USING CAPACITY CONSTRAINTS FOR INTERFERING MIMO LINKS

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Abstract - This paper considers joint optimization of power and array weights for a set of interfering co-channel links, given the desired capacity for each link. All transmitters and receivers are assumed to have array antennas. An iterative method for determining the link parameters is given. In this method, the desired capacities for the interfering links are the control parameters. It is also shown that the iterative method does not maximize the network throughput. Reducing the number of independent data streams for each link may enhance the network performance.

Keywords - antenna arrays, MIMO systems, capacity, co-channel interference, distributed algorithm, network throughput

I. Introduction

Multiple-input multiple-output (MIMO), or array-to-array, links are well known to provide extremely high spectral efficiency in rich multi-path environments [1]. As the speed and density of integrated circuit technology continue to increase, manufacturers have begun considering putting arrays on all sorts of user platforms that were not previously considered because of their cost, from homes for fixed wireless access to laptops and handhelds for indoor and picocell wireless networks [2, 3].

This paper considers joint optimization of power and array weights for a set of interfering co-channel links, where all transmitters and receivers are assumed to have array antennas. Flat-fading is assumed. Our present goal is to understand the flexibility in such a network in order to concisely quantify link control parameters that might someday be used by resource allocation algorithms.

Many studies consider a single point-to-point link, without co-channel interference [4–6]. The problem of determining the capacity of a single MIMO link with fixed interference is considered in [7, 8].

With its ability to do spatial filtering at both ends, a MIMO link can devote degrees of freedom in both its transmitter and receiver to achieving a high capacity, while limiting the interference caused at the receivers of other links. This functionality requires that the transmit array has channel information. For example, if there are two interfering links with disparate quality of service (QoS) requirements, the one with the lower QoS can be forced to devote more of its transmitter resources to avoid making interference on the high QoS link as well as using more of its receiver resources to suppress the interference caused by the high QoS link.

Some studies have considered joint optimization of sets of co-channel links that interfere with each other. These studies assume that only the base-station has an array antenna; [9] considers the uplink and [10] considers the downlink. In a few of these studies, iterative methods are used to determine optimal power control and weight adaptation [9, 10]. Control over the spatial domain is limited to only one end of the link in these link sets.

The capacity of each link and its optimum transmitter signal vector correlation matrix are functions of the channel and the external interference at the receiver. Since the external interference is affected by the transmitter correlation matrices of the interfering transmitters, the optimum transmission strategies and the capacities of all links are mutually dependent.

In [11], a distributed iterative method was given to determine capacities of interfering links, for given total transmitter powers and numbers of antenna elements. In that paper, it was suggested that once the capacity versus power tables were known, total transmitter powers for the different nodes constituted a set of control parameters. This paper introduces a different iterative method for determining optimal transmitter correlation matrices and receiver transformations for each link in a set of interfering co-channel links. In this method, which is also distributed in nature, the desired capacities for the interfering links are the control parameters.

The rest of this paper is organized as follows: In Section II, we review the MIMO water-filling solution assuming additive Gaussian noise with fixed interference. Section III introduces an iterative algorithm that gives the required transmit powers of a set of interfering MIMO links given the desired capacities. The algorithm is compared with that of [11] and results are illustrated. In Section IV, optimality of the algorithms in terms of total network throughput is discussed. It is shown that controlling the number of independent data streams for each link enhances the network performance. Section VI concludes the paper.

II. MIMO Channel Capacity

In a flat-fading MIMO system, the channel capacity is obtained by maximizing the mutual information between the transmitted and received signal vectors, as

$$C = \max_{\mathbf{P}} \log_2 \left| \mathbf{I} + \mathbf{HPH}' \right|, \tag{1}$$

where $\mathbf{P}$ is the transmitter signal vector correlation matrix, or the power allocation matrix, normalized by the received noise power on a single receive antenna element. $\mathbf{H}$ is the complex channel matrix. Each element $\mathbf{H}(a, b)$ of the channel matrix gives the complex channel gain from antenna element b of the transmitter to antenna element a of the receiver.

In open-loop spatial multiplexing, no channel information is used at the transmitter and each antenna element transmits a different data stream with equal power. Specifically, setting $\mathbf{P} = (P_T/N)\mathbf{I}$, gives the best result [4,12]. Here, P_T is the total transmitted power, normalized by the additive noise power; and N is the number of transmit antennas.

In closed-loop spatial multiplexing (CLSM), channel-dependent matrix transformations in both the transmitter and receiver decompose the matrix channel into a collection of uncoupled parallel channels or "channel modes". The output of each transmit antenna is a linear combination of the multiplexed signals. We use CLSM in this paper, although open-loop and closed-loop give the same result at high signal-to-noise ratio (SNR) [13].

Let the singular-value decomposition of $\mathbf{H}$ be denoted as $\mathbf{H} = \mathbf{USV}'$ and the eigenvalue decomposition of $\mathbf{P}$ as $\mathbf{P} = \mathbf{D\Sigma D}'$. Furthermore, let $\alpha^{(k)}$ and $\lambda^{(k)}$, $k = 1, ...K$ be the non-zero eigenvalues of $\mathbf{P}$ and $\mathbf{HH}'$, respectively. With the choice of $\mathbf{D} = \mathbf{V}$, the expression for the capacity becomes

$$C = \max_{\alpha^{(k)}} \sum_{k=1}^{K} \log_2(1 + \lambda^{(k)}\alpha^{(k)}) \tag{2}$$

With a total transmitted power of P_T, the classical water-filling solution

$$\alpha^{(k)} = \left[\mu - \frac{1}{\lambda^{(k)}}\right]^+, \tag{3}$$

maximizes the sum in (2) where $[\cdot]^+$ indicates that only non-negative values are acceptable, and μ is chosen so that $\sum_{k=1}^{K} \alpha^{(k)} = P_T$.

The water-filling approach can be modified to accommodate fixed interference at the receiver of a link (represented by a covariance matrix, $\mathbf{R}$) by "whitening the channel matrix" first. Applying a spatial whitening transform to the channel yields

$$\widetilde{\mathbf{H}} = [\mathbf{I} + \mathbf{R}]^{-1/2}\, \mathbf{H}, \tag{4}$$

which reduces the capacity relation to the simple form in (1), with a substitution of $\mathbf{H} \rightarrow \widetilde{\mathbf{H}}$ [7].

The whitening modifies the modes and robs gain from them, so that the capacity of the link is less than it would be without the interference. In the infinite interference-to-noise (INR) case, whitening projects a square channel matrix onto a lower-dimensional subspace, thereby simply reducing the number of available modes by the number of interfering data streams [7].

III. Distributed Optimization of Interfering Link Parameters

In a network with multiple interfering links, the interference correlation matrix seen by each receiver array varies with the transmitter correlation matrices of the interfering nodes. The whitened channel matrix, $\widetilde{\mathbf{H}}$, for a given link is a function of the interference, $\mathbf{R}$. The transmission strategy, in turn, is dependent on the whitened channel matrix. As a result, a change in the power allocation matrix of one link induces a change in the optimum power allocation matrix of the other co-channel links. Therefore, the optimum transmission strategies and the capacities of interfering co-channel links are mutually dependent, and cannot be calculated directly.

An iterative method was introduced to determine optimal transmitter correlation matrices and receiver transformations in a network with interfering CLSM links for given total transmitter powers [11]. At each iteration, every transmitter-receiver pair optimizes its link capacity for the measured interference at the receiver, and its respective given total transmitted power. Each link's transmission strategy is determined according to the water-filling solution given in (3) for the current spatially whitened channel. Note that with this iterative algorithm, optimal transmission strategies for a given link

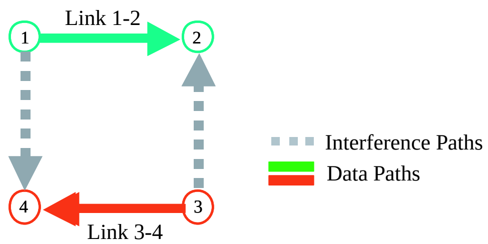


Fig. 1. A simple network with 4 nodes forming two interfering links.

are found based only on the channel gains for that link and the interference correlation matrix observed at the receiver array for that link. The knowledge of the channel gains between other transmitter-receiver pairs is not necessary.

In this paper we assume that the desired capacities are given for the interfering MIMO links and we want to determine optimal transmission strategies and the required transmit powers. The transmitter node of one link tries to minimize a measure of interference it causes at the receiver of another; while aiming for the link's target capacity. With this method, transmitter power of a link is distributed to the channel modes in a way to least degrade the other links. This method also tends to minimize the total transmitter power required to achieve the target capacity. For this method, it is necessary that a transmitter node knows the channel with the receivers that it interferes with, as well as the one that it aims to communicate with.

We use an iterative algorithm to solve our optimization problem. At each iteration, every transmitter-receiver pair optimizes its link capacity for the measured channel and interference conditions. This type of iterative algorithm has been proven to converge for mobile users with omnidirectional antennas and cellular base stations with antenna-arrays [4].

For illustration, we look at the 4-node network of Figure 1. Suppose we would like to get the maximum capacity for link 1-2, and set a target capacity, C_T, for the other link. In other words, link 1-2 maximizes its capacity using the water-filling based method given in [11] for its fixed total noise-normalized transmit power, while link 3-4 uses the interference-minimizing power allocation method to achieve its target capacity. The problem to be solved to obtain the transmission strategy at node 3 can be stated as

$$\min_{\mathbf{P_3}} trace\left[\mathbf{R_{32}}\right] \quad \text{s. t.} \quad \begin{cases} C_{34} = C_T \\ trace\left[\mathbf{P_3}\right] \leq P_{Max} \end{cases} \tag{5}$$

$\mathbf{R_{32}} = \mathbf{H_{32}P_3H'_{32}}$ is the correlation matrix of the interference caused by node 3 on 2, and $\mathbf{P_3}$ is the power allocation matrix of node 3. The subscripts of the channel matrix determine the end nodes. The eigenvectors of the power allocation matrix should be aligned with the right singular-vectors of the whitened channel matrix, $\widetilde{\mathbf{H}}_{34}$. Therefore, with

$$\widetilde{\mathbf{H}}_{34} = \left[\mathbf{I} + \mathbf{R_{14}}\right]^{-1/2} \mathbf{H_{34}} = \widetilde{\mathbf{U}}_{\mathbf{34}} \widetilde{\mathbf{S}}_{\mathbf{34}} \widetilde{\mathbf{V}}'_{\mathbf{34}}, \tag{6}$$

the power allocation matrix should be of the form

$$\mathbf{P_3} = \widetilde{\mathbf{V}}_{\mathbf{34}} \mathbf{\Sigma_3} \widetilde{\mathbf{V}}'_{\mathbf{34}}, \tag{7}$$

where $\mathbf{S_{34}}$ and $\mathbf{\Sigma_3}$ are diagonal. We would like to determine the powers to be allocated to the modes of link 3-4, $[\alpha_3^{(1)}, \alpha_3^{(2)}, \ldots, \alpha_3^{(K)}]$, which are the non-zero elements of $\mathbf{\Sigma_3}$. The eigenvalues of $\widetilde{\mathbf{H}}_{34}\widetilde{\mathbf{H}}'_{34}$ are represented as $\widetilde{\lambda}_{34}^{(k)}$. The trace of the interference correlation matrix can be written in terms of $\alpha_3^{(k)}$ as follows:

$$\begin{aligned} trace\left[\mathbf{R_{32}}\right] &= trace\left[\mathbf{H_{32}}\widetilde{\mathbf{V}}_{\mathbf{34}}\mathbf{\Sigma_3}\widetilde{\mathbf{V}}'_{\mathbf{34}}\mathbf{H}'_{\mathbf{32}}\right] && (8)\\ &= trace\left[\widetilde{\mathbf{V}}'_{34}\mathbf{H}'_{\mathbf{32}}\mathbf{H_{32}}\widetilde{\mathbf{V}}_{\mathbf{34}}\mathbf{\Sigma_3}\right] && (9)\\ &= trace\left[\mathbf{A}\mathbf{\Sigma_3}\right] && (10)\\ &= \sum_{k=1}^{K} a_k \alpha_3^{(k)}, && (11) \end{aligned}$$

where $\mathbf{A} = \widetilde{\mathbf{V}}'_{\mathbf{34}}\mathbf{H}'_{\mathbf{32}}\mathbf{H_{32}}\widetilde{\mathbf{V}}_{\mathbf{34}}$, a_k is the k^{th} diagonal element of $\mathbf{A}$, and we have used $trace\left[\mathbf{AB}\right] = trace\left[\mathbf{BA}\right]$. Therefore, our cost function at node 3 reduces to

$$\min_{\alpha_3^{(k)}} \sum_{k=1}^{K} a_k \alpha_3^{(k)} \quad \text{s. t.} \quad \begin{cases} \sum_{k=1}^{K} \log_2(1 + \widetilde{\lambda}_{34}^{(k)} \alpha_3^{(k)}) = C_T \\ \sum_{k=1}^{K} \alpha_3^{(k)} \leq P_{Max} \end{cases} \tag{12}$$

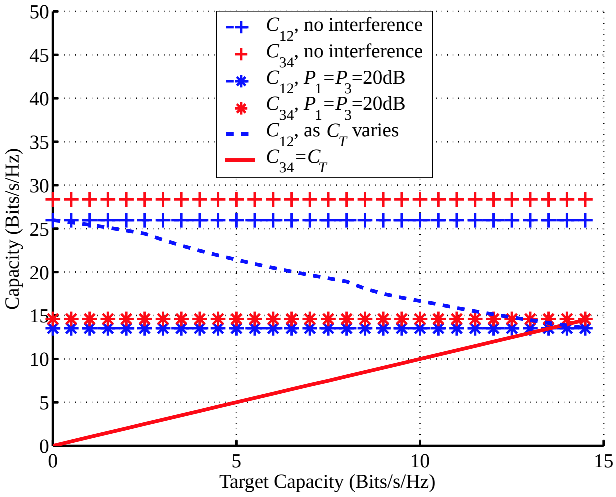
The solution is

$$\alpha_3^{(k)} = \left[\frac{\mu_1}{a_k + \mu_2} - \frac{1}{\widetilde{\lambda}_{34}^{(k)}}\right]^+, \tag{13}$$

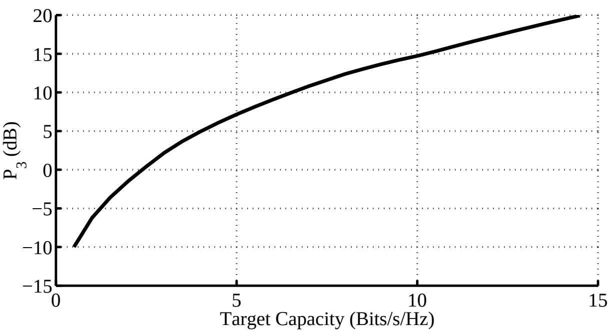
where μ_1 and μ_2 are constants chosen such that the constraints in (12) are satisfied.

Notice the similarity of this expression to the water-filling solution in (3). Whereas we had a fixed maximum level, μ, for all power coefficients of the channel modes in (3), with this solution we have different maximum levels for each $\alpha_3^{(k)}$. Each of these levels depends on the strength of the interference that the corresponding channel mode causes.

Figure 2(a) shows the capacities of the two links for one channel trial, as a function of the target capacity of link 3-4 (horizontal axis). The top two constant lines show



(a) Link capacities vs. target capacity of node 3.



(b) Required total transmit power vs. target capacity of node 3.

Fig. 2. Dependence of the link capacities and required transmit power of node 3 as target capacity of node 3 is varied.

the capacities of the two links for this channel trial in absence of any interference with 20dB noise-normalized total transmit powers. The two constant lines below show the capacities of the links found by the power-controlled algorithm of [11] for equal transmitter powers, when both links are active. The line with the constant positive slope is the target capacity of link 3-4. The decreasing curve is the capacity of link 1-2. Observe that the capacities found by the two methods are the same for the extreme values of C_T. Figure 2(b) shows the total transmit power required to achieve the desired target capacity. It can be seen that as the target capacity of link 3-4 is increased, the transmit power required at node 3 goes up. Consequently, link 1-2 is degraded by more powerful interference streams (by a larger number of streams, in some cases), thus, its capacity decreases.

Although it was illustrated for only link 3-4, the new algorithm can be employed at all links in a network to find optimal transmission strategies and transmitter powers when the desired set of capacities is given.

IV. Number of Transmitted Streams

Both methods introduced in [11] and here are distributed; each link determines its own transmission strategy based on the channel conditions. However, the cost functions considered are not based on the total network throughput, but on the powers and data rates of the individual links. Although these iterative methods yield optimal points that allow multiple co-channel CLSM links to operate simultaneously, they do not maximize the total network throughput. In some cases, limiting the number of streams transmitted, thus limiting the number of interference streams created, may yield better performance in the network level [14]. For independent flat-Rayleigh fading wireless systems, the data streams can be resolved effectively by linear methods if the number of data and interference streams is less than or equal to the number of receive antennas [14, 15].

Figure 3 shows each link's capacity, for the network shown in Figure 1, as the number of allowed streams is varied. The capacities are obtained using the method in [11], averaged over 1000 channel trials. 4-antenna arrays are used at each node. The total transmit powers of both links are 20dB. The highest capacity is reached when the number of streams for each transmitter is limited to 2. For this case, each link transmits through at most the 2 strongest channel modes. Each receiver receives at most two streams of data and two streams of interference.

Similarly, with the capacity-control method introduced in this paper, the required powers to obtain certain target capacities are lower when the number of streams is limited to 2.

Both capacity allocation algorithms can be extended to limit the number of channel modes used at each link based on the number of transmitting nodes in the neighborhood.

V. Conclusion

An iterative algorithm is presented to jointly optimize the link parameters for a number of interfering MIMO flat-fading links. Whereas in [11] transmit power is used as the control parameter, with this approach the desired capacities for the interfering links are the control parameters. It is shown that the two iterative algorithms can be used jointly in a network to meet different data flow requirements for each link.

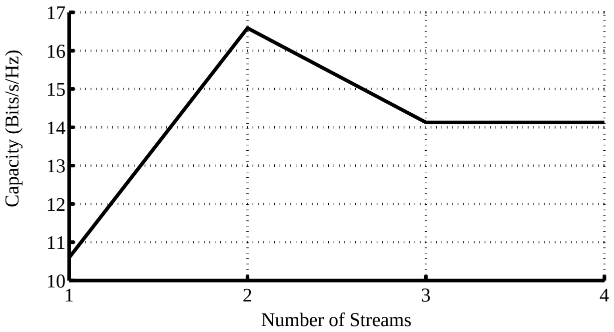


Fig. 3. Capacity vs. number of streams used at each link.

Although these iterative algorithms give link-optimal parameters for interfering MIMO links, the network throughput is not maximized. It is shown that reducing the number of independent data streams for each link may enhance the network performance.

References

[1] G. G. Raleigh and V. K. Jones, "Multivariate modulation and coding for wireless communication," *IEEE Journal on Selected Areas in Communications*, vol. 17, pp. 851–866, May 1999.

[2] "http://www.cisco.com/warp/public/cc/pd/witc/wt2700/2750/wt275_ds.htm".

[3] H. Zheng and D. Samardzija, "Performance evaluation of indoor wireless systems using BLAST testbed," *Vehicular Technology Conference*, vol. 2, Oct. 2001.

[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[5] G. G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Transactions on Communications*, vol. 46, Mar. 1998.

[6] F. R. Farrokhi, A. Lozano, G. J. Foschini, and R. A. Valenzuela, "Spectral efficiency of wireless systems with multiple transmit and receive antennas," *PIMRC*, 2000.

[7] D. Bliss, K. W. Forsythe, A. O. Hero, and A. L. Swindlehurst, "MIMO environmental capacity sensitivity," *Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers*, vol. 1, Oct. 2000.

[8] F. R. Farrokhi, G. J. Foschini, A. Lazano, and R. A. Valenzuela, "Link-optimal BLAST processing with multiple-access interference," *IEEE Vehicular Technology Conference*, vol. 1, pp. 87–91, Oct. 2000.

[9] F. R. Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1437–1450, Oct. 1998.

[10] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, pp. 641–646, Nov. 1993.

[11] M. F. Demirkol and M. A. Ingram, "Power-controlled capacity for interfering MIMO links," *Vehicular Technology Conference*, vol. 1, Oct. 2001.

[12] E. Telatar, "Capacity of multi-antenna gaussian channels," *Technical Report, AT&T Bell Labs*, June 1995.

[13] A. M. Sengupta and P. P. Mitra, "Capacity of multivariate channels with multiplicative noise: I. random matrix techniques and large-n expansions for full transfer matrices," *Bell Labs Technical Report*, Apr. 2001.

[14] R. S. Blum, J. H. Winters, and N. R. Sollenberger, "On the capacity of cellular systems with MIMO," *Vehicular Technology Conference*, vol. 2, Oct. 2001.

[15] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Transactions on Communications*, vol. 42, no. 2, 1994.