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Convergence Property of Transmit Time Pre-Synchronization for Concurrent Cooperative Communication

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***Abstract*—This paper proposes a preamble detection-based cooperative transmit time pre-synchronization technique specially for Concurrent Cooperative Transmission (CCT). CCT enables a collection of power-constrained radios to transmit as a group and achieve a transmit range that is much greater than the range of a single device. In the absence of a GPS system or a globally synchronized clock, each cooperating node can derive its transmit time autonomously by detecting the preamble from a received signal. The paper first compares two methods for estimating the start of packet (SOP) received in the presence of a carrier frequency offset (CFO): the peak method, which is optimal in a single diversity channel without CFO, and a mean method, which outperforms peak with large CFO. Next, the SOP is estimated by combining all diversity channels using an approximated Best Linear Unbiased Estimator (BLUE). Using theory and measurement, we show that the time error statistics are convergent as the number of hop increases.**

I. Introduction

Cooperative transmission (CT) is a physical layer wireless communication scheme in which spatially separated sensor nodes collaborate to transmit the same source message so that the transmission power required for each node can be reduced or the range can be extended [1][2]. Collaborating nodes can be chosen in a centralized or distributed manner and grouped into a cluster. Therefore, we can consider cooperative communication to be cluster-to-cluster transmission (MIMO) or cluster-to-single-node transmission (MISO). Concurrent CT (CCT) is a special form of CT which allows cooperating nodes transmit the diversity versions of the message at the approximately same time. Any relative synchronization errors in the transmit times of the cooperators roughly add to the delay spread of the multipath channel, so our aim is for the root mean square (RMS) transmit time spread to be at most comparable to the channel delay spread.

This paper considers how synchronization time errors evolve over *consecutive* CCT hops without single source redistribution within a cluster. We show by both measurement in a typical indoor office environment as well as theoretical analysis that the error statistics are convergent with the number of hops. The measurement data shows that the error spread is comparable to delay spreads of the indoor environment.

The impairment caused by time synchronization errors on the performance of cooperative MIMO was theoretically treated in [3]. The authors also showed that jitters as large as 10% of the bit duration do not have much effect on the BER performance of the system. Using a globally synchronized clock by network time protocol or a GPS has been proposed for CCT [4]. These approaches require additional devices or a convergence period for accurate time synchronization.

In this paper, we show experimental results for CCT, using binary frequency shift keying (BFSK), with non-coherent demodulation. Orthogonality is achieved in the frequency domain, by having different cooperating radios transmit on different orthogonal carriers. Because CCT is not supported by any off-the-shelf radios, we use software-defined radios (SDRs) to implement our approach.

II. System Model and Preamble Detection

Without using a globally synchronized clock, cooperating nodes can decide their cooperative transmission time by adding fixed amount of period to the reception time of the message to be relayed. It can be achieved by locating the preamble or sync word embedded in the packet.

Suppose that $s(t)$ is the modulated baseband signal of a known preamble which has energy $\int |s(t)|^2 dt = 1$. The received signal $r(t)$ has a frequency offset $\triangle f$ which is introduced by carrier frequency difference between the transmitter and the receiver. Assuming a quasi-static fading channel with a channel coefficient α by

$$r(t) = h \cdot s(t-\tau) \cdot e^{j2\pi \triangle f t} + w(t) \tag{1}$$

where τ is a propagation delay and $w(t)$ is zero-mean complex additive white Gaussian noise (AWGN) with a power spectral density of N_0.

If the preamble is known a priori at the receiver, the beginning of the preamble can be found by cross-correlating the received signal $r(t)$ with a preamble signal $s(t)$ and finding a threshold or peak. Mathematically, the cross-correlation of the preamble-bearing signals $s(t)$ and the received signals $r(t)$ which has encountered a frequency offset $\triangle f$ can be described as

$$d(t,\tau,\triangle f) = h \cdot q(t,\tau,\triangle f) + z(t) \tag{2}$$

where $q(t,\tau,\triangle f) = \int_{-\infty}^{\infty} s(x-\tau)s^*(t+x)e^{j2\pi\triangle f(x-\tau)}dx$ and $z(t) = \int_{-\infty}^{\infty} w(x)s^*(t+x)dx$. $z(t)$ is zero-mean complex AWGN with a variance N_0. Under the assumption of high signal-to-noise ratio (SNR) and $\triangle f = 0$ [5] the estimation of

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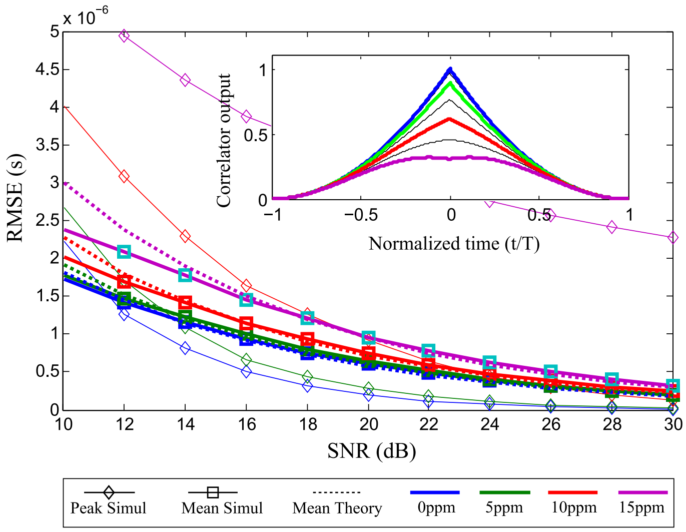

Fig. 1. Decision function $\Lambda(t)$ of a single preamble in the presence of carrier frequency offset (CFO) and the accuracy of peak and mean estimator in the presence of CFO

SOP time can be achieved by choosing t to maximize $\Lambda(t) = \sum_{i=0}^{P-1} |d(t-iT)|^2$ where P is the length of preamble.

The accuracy of correlation rule is proportional to the SNR and sidelobe suppression of the correlation output [6]. With a carrier frequency offset (CFO), however, peak searching suffers performance degradation because CFO makes the filter mismatched. Example filter output shown in in-set figure in Figure 1, which is the decision function $\Lambda(t)$ over $(-T, T)$ where $h = 1$, $P = 1$, $\tau = 0$, and noise-free $z(t) = 0$. It is shown that as CFO increases, signal strength diminishes.

A. Mean Estimation

Instead of searching a peak of correlation output to determine a SOP time, we propose a mean (center-of-mass) estimation. Without loss of generality, let us assume $\tau = 0$ which implies that the truth time of SOP is 0. We also assume that the soft-valued signal $\Lambda(t)$ is sampled with an oversampling rate $1/T_s$. The finding a mean of $\Lambda(t)$ can be realized as follows:

$$\omega = T_s \frac{\sum_{-K}^{K} k \cdot \Lambda[k]}{\sum_{-K}^{K} \Lambda[k]}. \quad (3)$$

In Appendix A, we show that the estimation error ω has an approximately zero mean Gaussian distribution in high SNR and its variance σ_ω^2 can be approximated by

$$\sigma_\omega^2 = \frac{N_0 K T s^2}{|h|^2} \quad (4)$$

where

$$K = \frac{2\sum_i \sum_j ij|q[i, \triangle f]||q[j]|(s \star s)^2[i-j]}{(\sum_i q^2[i])^2}. \quad (5)$$

Figure 1 shows a MATLAB simulation of the root-mean-square error (RMSE) of peak and mean estimator in the presence of CFO $= \{0, 5, 10, 15\}$ppm. For this simulation, 1MHz sampling rate $T_s = 10^{-6}$ and a BFSK waveform with a rectangular shaping pulse are used. It also shows the theoretical RMSE treated in Equation 4 by using a dotted line. While peak estimation outperforms mean estimation in the absence of or relatively small CFO, mean estimation gives better performance in the presence of relatively large CFO. Intriguingly, it is shown that RMSE of peak estimator at 15ppm CFO rises rapidly because the output of correlator $\Lambda(t)$ becomes blunt and finally it becomes bimodal as shown in the in-set figure in Figure 1.

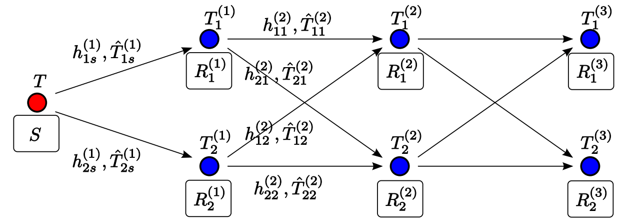

Fig. 2. Illustration of the multiple Concurrent Cooperative Transmission (CCT) hops

III. Cooperative Transmit Time Synchronization

In this section, we analyze the Start of Packet (SOP) time estimation error at each relay in a sequence of relay clusters, such that each cluster has N relays and does concurrent cooperative transmission (CCT). We assume that the packet is originally transmitted by a single-antenna source (S) at time T, as shown in Figure 2 for $N = 2$. To simplify the analysis, we assume that the N nodes in each cluster are co-located (not shown this way in the figure) and that the clusters are arranged on a line with equal spacing. This implies that the propagation distances between nodes in different clusters are equal and deterministic. We assume that each relay in a cluster transmits a preamble that is orthogonal to the other $N-1$ preambles being transmitted in the cluster. These orthogonal preambles define the N diversity channels for synchronization, and are transmitted simultaneously. We assume that each receiver has a bank of N correlators, with one correlator for each diversity channel. We assume independent Rayleigh fading in all diversity channels.

For all terms used in this paper, a superscript and subscript represent the cluster and node index, respectively. For example, $T_k^{(j)}$ is a time of transmission of Relay k in Cluster j. For example of two subscripts case, $h_{21}^{(j)}$ is a channel between Relay 1 in Cluster $j-1$ and Relay 2 in Cluster j. $\hat{T}_{kn}^{(j)}$ is an estimate of $T_n^{(j-1)}$ based only on correlation in channel $h_{kn}^{(j)}$ at the Node k in the Cluster j. $\omega_{kn}^{(j)}$ is the "correlator estimation error" of $\hat{T}_{kn}^{(j)}$ transmitted from the Node n of previous cluster. $\hat{T}_k^{(j)}$ is an approximated Best Linear Unbiased Estimate (BLUE) of T without clock error, which is a linear combination of $\hat{T}_{kn}^{(j)}$'s. $\xi_k^{(j)}$ is a clock compensation error of the Node k in the Cluster j.

Each relay autonomously estimates the time of the SOP. Let $R_k^{(j)}$ be the k_{th} relay in the j_{th} cluster, and let $\hat{T}_k^{(j)}$ be its estimate of the SOP. The details on how that estimate is formed are explained below. Next, *ideally*, each relay determines its transmit time by adding an universally known constant, T_{proc}, to $\hat{T}_k^{(j)}$, to allow enough time for the signal processing on all relays in a cluster to be completed [7]. However, because each relay has a slightly different oscillator frequency from the other relays, some clocks run fast and some run slow; this is "clock skew." Each relay will attempt to compensate its clock skew, but the compensation is not perfect, so some relays will still have T_{proc} expire faster than others. The deviation in T_{proc} expiration on Node k of Cluster j from an imaginary universal reference clock is designated as the "clock error," $\xi_k^{(j)}$. $\xi_k^{(j)}$ is assumed to be zero mean and Gaussian random variable with a variance σ_ξ^2. Therefore, the transmit time of the k_{th} relay in the j_{th} cluster is $T_k^{(j)} = \hat{T}_k^{(j)} + \xi_k^{(j)}$.

To compute $\hat{T}_k^{(j)}$, the estimate of the SOP at the k_{th} node in the j_{th} cluster, the node first creates N estimates, $\hat{T}_{kn}^{(j)}$, $n = 1, 2, \ldots, N$, of the SOP in each of its N diversity channels, respectively, by defining each estimate to be a centroid of the correlator output as discussed in Section II. Since only one of the previous cluster nodes transmits in a given diversity channel, $\hat{T}_{kn}^{(j)}$ is actually an estimate of the SOP of the packet received from $R_n^{(j-1)}$, i.e. $\hat{T}_{kn}^{(j)} = T_n^{(j-1)} + \omega_{kn}^{(j)}$.

A. Approximated Linear Unbiased Estimation

The performance of cooperative transmit time synchronization can be assessed by a covariance of transmission time $\hat{T}_k^{(j)}$. In this subsection, we design an approximated linear unbiased estimator to minimize the variance of transmission time $\hat{T}_k^{(j)}$ where $\hat{T}_k^{(j)}$ is a linear combination of $\hat{T}_{kn}^{(j)}$ for $1 \le n \le N$. Let $a_{kn}^{(j)}$ denote a coefficient of combiner. By best linear unbiased estimator (BLUE) and under the assumption that the average transmission time errors of previous cluster nodes are same, the coefficient vector $\boldsymbol{a}_k^{(j)} = [a_{k1}^{(j)}, a_{k1}^{(j)}, \ldots, a_{kN}^{(j)}]^T = (\mathbf{V}^{-1}\mathbf{1})/(\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1})$ where $\mathbf{V}$ is a covariance matrix of $\hat{\boldsymbol{T}}_k^{(j)} = [\hat{T}_{k1}^{(j)}, \hat{T}_{k2}^{(j)}, \ldots, \hat{T}_{kN}^{(j)}]^T$ and $\mathbf{1}$ is the column vector of N ones. Since single correlation errors $\omega_{kn}^{(j)}$ are uncorrelated over index n, the covariance matrix becomes a diagonal matrix $\mathbf{V} = diag[\sigma_{\omega_1}^2, \sigma_{\omega_2}^2, \ldots, \sigma_{\omega_N}^2]$. Therefore, the estimator coefficient becomes $a_{kn}^{(j)} = |h_{kn}^{(j)}|^2 / \left(\sum_{i=1}^N |h_{ki}^{(j)}|^2 \right)$.

To simplify analysis, we assume that all deterministic times are equal to zero; this includes the propagation time and T_{proc}. This allows us to focus only on the behavior of time estimation *errors*. Since all the errors are zero mean, we have that all estimators are trying to estimate the same thing, T, which is the original transmit time of the source (S).

B. Markov Process Model

Let the vector $\boldsymbol{T}^{(j)} = [T_1^{(j)}, T_2^{(j)}, \ldots, T_N^{(j)}]^T$ represent the transmit times of each relay in Cluster j (or "hop j"). From the arguments above, each element of $\mathbf{T}^{(j)}$ is an unbiased estimate of the original packet transmit time, or $E\{\mathbf{T}^{(j)}\} = T \cdot \mathbf{1}$. Let the estimation error vector of $\boldsymbol{T}^{(j)}$ be denoted as $\boldsymbol{e}^{(j)} = [e_1^{(j)}, e_2^{(j)}, \ldots, e_N^{(j)}]^T$. Our objective is to get a recursion for the covariance of transmit time error $\mathbf{C}^{(j)} = E\{\mathbf{e}^{(j)}\mathbf{e}^{(j)^T}\}$, from which we derive the sample variance.

Because our estimator coefficients at Cluster j do not depend on the multi-path channels of previous hops, we can see that the statistics of the errors at Cluster j are independent of the errors of past hops, assuming $\boldsymbol{T}^{(j-1)}$ is given. In other words, the conditional joint probability density function (PDF) of $\boldsymbol{T}^{(j)}$, given the entire past transmit times $\{\boldsymbol{T}^{(j-1)}, \boldsymbol{T}^{(j-2)}, \ldots, \boldsymbol{T}^{(1)}\}$, is the same as the conditional joint probability density function (PDF) of $\boldsymbol{T}^{(j)}$, given only the latest vector of transmit times $\boldsymbol{T}^{(j-1)}$. This makes $\boldsymbol{T}^{(j)}$, $j = 1, 2, \ldots, N$, a vector-valued, continuous-state, discrete-time *Markov Process*. $\boldsymbol{e}_k^{(j)}$ with substitution of $\hat{T}_k^{(j)}$ and $\hat{T}_{kn}^{(j)}$ can be written as

$$\begin{aligned}\boldsymbol{e}_k^{(j)} &= \hat{T}_k^{(j)} + \xi_k^{(j)} - T = \sum_{n=1}^N \left(a_{kn}^{(j)} \hat{T}_{kn}^{(j)} \right) - T + \xi_k^{(j)} \\ &= \sum_{n=1}^N a_{kn}^{(j)} e_n^{(j-1)} + \sum_{n=1}^N a_{kn}^{(j)} \omega_{kn}^{(j)} + \xi_k^{(j)}\end{aligned}$$

noting that $\sum_{n=1}^N a_{kn}^{(j)} = 1$ for all j and k. Let $\mathbf{A}^{(j)}$ and $\boldsymbol{\omega}^{(j)}$ be a estimator coefficient and correlator error matrix whose elements of k_{th} column and n_{th} row are $a_{kn}^{(j)}$ and $\omega_{kn}^{(j)}$ respectively, and $\boldsymbol{\xi}^{(j)} = [\xi_1^{(j)}, \xi_2^{(j)}, \ldots, \xi_N^{(j)}]^T$. The covariance matrix $\mathbf{C}^{(j)}$ has an iterative form as

$$\begin{aligned}\mathbf{C}^{(j)} &= E\{\boldsymbol{e}^{(j)}\boldsymbol{e}^{(j)^T}\} = E\{\mathbf{A}^{(j)}\mathbf{C}^{(j-1)}\mathbf{A}^{(j)^T}\} \\ &+ E\left\{ \left[[\mathbf{A}^{(j)} \circ \boldsymbol{\omega}^{(j)}]\mathbf{1} + \boldsymbol{\xi}^{(j)} \right] \left[[\mathbf{A}^{(j)} \circ \boldsymbol{\omega}^{(j)}]\mathbf{1} + \boldsymbol{\xi}^{(j)} \right]^T \right\} \quad (6)\end{aligned}$$

where $\circ$ denotes an element-wise product operator. In the following sections, we show that the expected value of the sample variance of estimation error $\boldsymbol{e}_k^{(j)}$ is convergent even though the absolute covariance matrix is not.

C. Convergence Property

The performance of transmit time pre-synchronization can be assessed by relative transmit time error of relay nodes within a cluster. The relative transmit time error of j_{th} cluster's relay nodes is basically equivalent to the sample variance of estimation error $\boldsymbol{e}^{(j)}$. To simplify analysis of a convergence property of the covariance matrix $\mathbf{C}^{(j)}$, we assume that the statistics of all channels, clocks and noises do not vary with relay index within a cluster. These assumptions yield that the variance and the covariance of the estimation error are not functions of a node index. Therefore, the diagonal terms of the matrix $\mathbf{C}^{(j)}$, denoted as $C_d^{(j)}$, are identical. Also, the off-diagonal terms, denoted as $C_o^{(j)}$, are identical. In Appendix B, we show that $C_d^{(j)}$ and $C_o^{(j)}$ can be written as

$$\begin{aligned}C_d^{(j)} &= \frac{2}{N+1} C_d^{(j-1)} + \frac{N-1}{N+1} C_o^{(j-1)} + H + N\sigma_\xi^2 \\ C_o^{(j)} &= \frac{1}{N} C_d^{(j-1)} + \frac{N-1}{N} C_o^{(j-1)}\end{aligned}$$

where $H = \sum_{n=1}^N E\{a_{gn}^{(j)} \omega_{gn}^{(j)}\}$.

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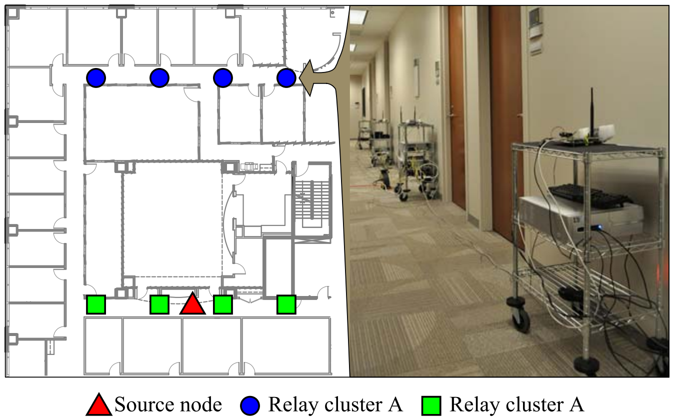
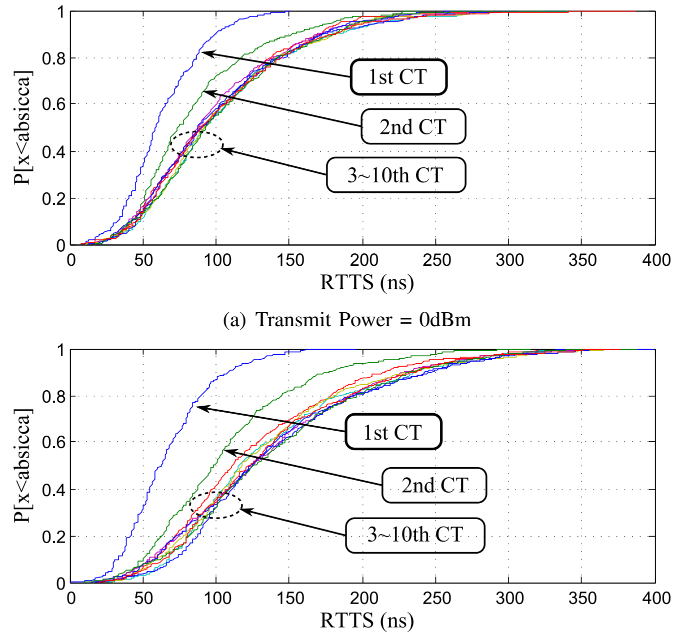


Fig. 3. A floor map and photo of the experiment which had conducted on the fifth floor of the Centergy Building at the Georgia Institute of Technology



(a) Transmit Power = 0dBm

(b) Transmit Power = -5dBm

Fig. 4. Measured RTTS of "ping-pong" experiment

In [8], the expected value of sample variance in which the samples are not mutually independent and pairwise covariances are constant γ, is given by $E\{\sigma_s^2\} = \sigma^2 - \gamma$ where s^2 is the sample variance and σ^2 is the variance. From this relationship, we can get the linear difference equation form of the expected value of the sample variance at j_{th} cluster as

$$E\{\sigma_s^{2(j)}\} = \frac{N-1}{N(N+1)} E\{\sigma_s^{2(j-1)}\} + H + N\sigma_\xi^2.$$

The expected value of the sample variance is convergent to

$$E\{\sigma_s^2\} = \frac{N(N+1)}{N^2+1}(H + N\sigma_\xi^2) \text{ as } j \to \infty.$$

IV. Experimental Setup and Results

The synchronization method we designed in Sections II and III was evaluated in a Software Defined Radio (SDR) testbed. Each wireless node in this experiment is composed of a RF-daughterboard (RFX-2400), an Universal Software Radio Peripheral (USRP1) board, a personal computer (PC), and the GNU radio software. The USRP1 board has an ADC/DAC and a FPGA to convert passband signal to baseband signal and vice versa. All baseband processing is done on the PC.

Binary frequency shift keying (BFSK) with non-coherent envelope detection was used for the experiments. Orthogonal preambles were achieved by choosing orthogonal center frequencies. We used 64 kbps bit-rate with 1 Mhz sampling-rate. The total length of a packet is 24 bytes consisting of 4 bytes preamble, 6 bytes header, 10 bytes data and 2 bytes CRC. The source packet is scrambled at every transmission in which the index of scrambler is included in the header. All baseband processing for non-coherent BFSK modulator/demodulator and the transmit time synchronization schemes are programmed by using C++ and Python languages.

The rms transmit time spread (RTTS), $\sigma_s^{(j)} = \sqrt{\frac{\sum_i^N (T_i^{(j)} - \bar{T}^{(j)})^2}{N-1}}$ of cooperative relay nodes in j_{th} CT was used to evaluate the performance where $\bar{T}^{(j)}$ is the sample mean of transmission time of all relay nodes at Cluster j. To simulate multiple consecutive cluster hops in a large, we designed the "ping-pong" experiment, where two groups of cooperating nodes transmit the source message back and forth up to 10 hops (or "CT"s). Figure 3 shows a floor map and photo of the "ping-pong" experiment was conducted on the fifth floor of the Centergy Building at the Georgia Institute of Technology. The experiment was repeated 500 times to get 500 trials of $\sigma_s^{(j)}$. USRP1 boards are programmed to generate a trigger pulse at the time of each transmission and the trigger pulse was captured by a customized FPGA board by wire.

Figure 4 shows the empirical RTTS of the cooperative nodes with two different transmit powers. Each curve represents an empirical cumulative density function (CDF) of RTTS of each CT. We note that we measured sample variance of cooperative nodes so that the CDF does not follow the CDF of the error statistic we derived in Section III. As shown in the figure, the CDFs corresponding to -5dBm transmit power are shifted by approximately 20% compared to the CDFs corresponding to 0dBm. This was predicted in the simulation result in Figure 1. We also observe that RTTS tends to be converged after the third hop. This provides strong evidence that our analysis of convergence property is correct. Moreover 90% of the RTTS is less than 300ns in both transmit powers, which indicates that CCT can support up to 300 kbps data rate in narrow-band waveforms without significant ISI degradation [3].

V. Conclusion

In this paper, we proposed a preamble detection-based transmit time synchronization scheme for CCT. We designed a mean estimation method for non-coherent BFSK demodulation to estimate the start of packet (SOP) time. We showed that the mean estimation method outperforms the peak searching method in the presence of CFO by both analysis and simulation. Diversity channels are formed by having each cooperating node transmit on an orthogonal carrier frequency. The estimates of the SOP times in each diversity channel are combined using an approximate BLUE method, to produce an overall estimate of the SOP, which is the reference for the retransmit times. We show that the transmit time error statistics are convergent over consecutive CCT hops based on measurement in an SDR testbed in a typical indoor office environment, as well as by theoretical analysis. Our experiment and implementation show that CCT without using single-source redistribution within clusters and without using a globally synchronized clock is practical for the indoor environment.

APPENDIX A

Let us consider a single bit preamble $P = 1$. At any index k, $\Lambda[k]$ can be considered as an absolute square of a non-zero mean complex Gaussian random variable whose normalized value has a non-central chi-square distribution with two degrees of freedom and non-centrality parameter $\lambda = \mu^2/\sigma^2$ where μ and σ^2 are mean and variance of unnormalized complex Gaussian random variable. The mean and variance of normalized non-central chi-square distribution with two degrees of freedom are $2+\lambda$ and $4(1+\lambda)$ respectively. Therefore, the mean and variance of $\Lambda[k]$ at any index k are

$$\mu_{\Lambda[k]} = N_0 + |h \cdot q[k]|^2$$
$$\sigma^2_{\Lambda[k]} = N_0^2 + 2N_0|h \cdot q[k]|^2$$

where $q[k]$ is a sampled signal of $q(t)$. Similarly, the mean and variance of $k \cdot \Lambda[k]$ are

$$\mu_{k\cdot\Lambda[k]} = k(N_0 + |h \cdot q[k]|^2)$$
$$\sigma^2_{k\cdot\Lambda[k]} = k^2(N_0^2 + 2N_0|h \cdot q[k]|^2)$$

By central limit theorem (CLT), a probability distribution of $\sum k \cdot \Lambda[k]$ and $\sum \Lambda[k]$ can be approximately Gaussian distribution. Let X and Y be denoted as $\sum k\cdot\Lambda[k]$ and $\sum \Lambda[k]$ respectively and define $Z = X/Y$. In Equation 2, $z[k]$, a sampled version of $z(t)$, is a filtered white Gaussian noise so called a *colored* Gaussian noise . Its auto-covariance is $\text{Cov}(z[i], z[j]) = N_0 \cdot (s \star s)[i - j]$. The mean and variance of X and Y are

$$\mu_x = 0, \mu_y = \sum_i \mu_{\Lambda[i]}$$
$$\sigma_x^2 = \sum_i \sum_j \sigma_{i\cdot\Lambda[i]}\sigma_{i\cdot\Lambda[j]}(s \star s)^2[i - j]$$
$$\sigma_y^2 = \sum_i \sum_j \sigma_{\Lambda[i]}\sigma_{\Lambda[j]}(s \star s)^2[i - j].$$

By Geary-Hinkley transformation [9], a random variable ω can be transformed to $\omega = (T_s\sqrt{KN_0}/|h|)u$ where u is a zero-mean unit variance Gaussian random variable.

APPENDIX B

From Equation 6, $C_d^{(j)}$ and $C_o^{(j)}$ can be written as

$$C_d^{(j)} = C_d^{(j-1)} \sum_{n=1}^{N} E\{(a_{gn}^{(j)})^2\} + C_o^{(j-1)} \sum_{\substack{x=1 \\ x\neq y}}^{N} \sum_{y=1}^{N} E\{a_{gx}^{(j)} a_{gy}^{(j)}\} + \sum_{n=1}^{N} E\{a_{gn}^{(j)}\omega_{gn}^{(j)}\} + \sum_{n=1}^{N} E\{\xi_n^{(j)}\}$$

$$C_o^{(j)} = C_d^{(j-1)} \sum_{n=1}^{N} (E\{a_{gn}^{(j)}\})^2 + C_o^{(j-1)} \sum_{\substack{x=1 \\ x\neq y}}^{N} \sum_{y=1}^{N} E\{a_{gx}^{(j)}\} E\{a_{gy}^{(j)}\}$$

It can be shown that $E\{a_{gn}^{(j)}\} = 1/N$ regardless any indexes [10]. Assuming that $X = |h_{gn}^{(j)}|^2$ and $Y = \sum_{k=1}^{N} |h_{gk}^{(j)}|^2$, $E\{(a_{gn}^{(j)})^2\}$ can be written as $E\{X^2\}/E\{Y^2\}$. Since X and Y have exponential and gamma distribution respectively, $E\{a_{gn}^{(j)}\}$ can be simplified to $\frac{2}{N(N+1)}$. Also we can get

$$\sum_{\substack{x=1 \\ x\neq y}}^{N} \sum_{y=1}^{N} E\{a_{gx}^{(j)} a_{gy}^{(j)}\} = E\{(\sum_{n=1}^{N} a_{gn}^{(j)})^2\} - E\{\sum_{n=1}^{N} (a_{gn}^{(j)})^2\}$$
$$= 1 - E\{\sum_{n=1}^{N} (a_{gn}^{(j)})^2\} = \frac{N-1}{N+1}$$

It is noted that the third and fourth term of $C_d^{(j)}$ do not depend on the index j because we assume that the statistics of all channels, clocks and noises do not vary.

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