Prediction of Interconnect Fan-out Distribution Using Rent's Rule

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Abstract

Based on Rent's rule, a well-established empirical relationship, a rigorous derivation of the interconnect fan-out distribution for random logic networks is performed. Through comparison with actual product data, it is shown that the model successfully predicts the fan-out distribution of a random logic network. Using the closed form expression for the fan-out distribution, its application to predict the global level netlist information in a system-on-a-chip is presented.

Keywords

Fan-out distribution, interconnect network prediction, Rent's rule.

1. Introduction

Multilevel interconnect modeling provides primary information used to elucidate the limitations and the opportunities for future gigascale integration (GSI) products. For instance, the various applications of the wire-length distribution such as the prediction of clock frequency, dynamic power dissipation, and chip size are described in [1]. Likewise, the interconnect fan-out distribution can be used in multilevel interconnect modeling. Primary information extracted from the interconnect fan-out distribution is the average fan-out that is used in wire-length distribution models [2]. Also, the fan-out distribution model presented in this paper provides an estimation method for the Rent's exponent from the average fan-out. Moreover, the netlist information at global level in a system-on-a-chip [3] can be also predicted by using the fan-out distribution model presented here. Note that the fan-out distribution model presented in this paper is an extension of the work presented in [8].

In Section 2, the assumptions and derivation algorithm of the fan-out distribution are examined. Also, to validate this derivation, the model is compared to an experimental result from a real chip. In Section 3, the closed form expressions for the average fan-out, total number of nets, and maximum fan-out are presented. Some applications of the fan-out distribution such as the average fan-out prediction, Rent's exponent estimation, prediction of the global level netlist information in a system-on-a-chip, and other potential applications are described in Section 4.

2. Derivation of Fan-out Distribution

In this section the complete derivation of the closed-form model for fan-out distribution is described.

2.1 Assumptions

The underlying assumption for the fan-out distribution model is that the Rent's rule holds throughout an entire system. Rent's rule is a simple power-law relationship between the number of I/O terminals for a logic block, T, and the number of gates contained in that block, N. Specifically, it is shown as [5]-[6]:

$$T = kN^{p} \qquad (1)$$

where k and p are empirical parameters. The fan-out distribution is derived by using the Rent's rule recursively in a collection of gates.

2.2 Derivation

For simplicity, first assume a boundary containing two gates A and B, as shown in Fig. 1. Conserving all I/O's of A and B, inside and outside of the boundary, gives:

$$T_{Internal} = T_A + T_B - T_{External} \qquad (2)$$

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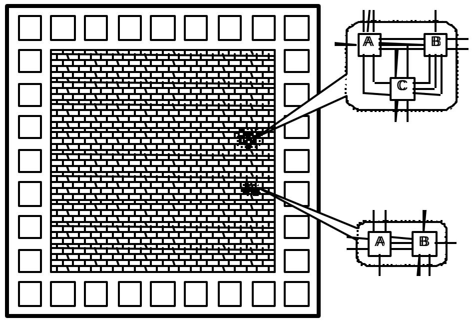


Figure 1. A collection of two and three gates in a random logic network

where $T_{Internal}$ is the total number of terminals of the gates inside of the boundary and $T_{External}$ is the total number of terminals of the gates outside of the boundary.

The same equation as (2), is also valid for the boundary containing m gates as below:

$$T_{Internal} = \left(\sum_{i=1}^{m} T_i \right) - T_{External} \tag{3}$$

where T_i is the number of terminals of each single gate, on average, in the boundary. Now applying Rent's rule to (3), gives:

$$T_{Internal} = \left(\sum_{i=1}^{m} k \cdot 1^p \right) - k \cdot m^p = k(m - m^p) \tag{4}$$

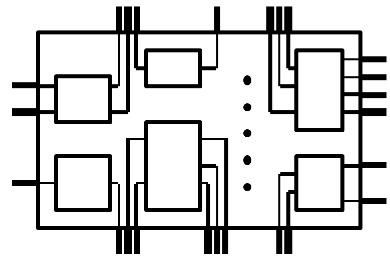
where k and p are the Rent's rule parameters of the network. Normalizing the total number of internal terminals to the number of gates gives, on average, the number of terminals that each gate shares with the other gates in the boundary

$$\frac{T_{Internal}}{m} = k(1 - m^{p-1}). \tag{5}$$

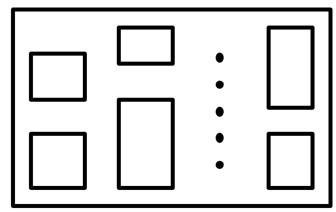
Figure 1 shows the number of shared terminals for each gate in a boundary of three gates, consists of two parts: 1) the number of terminals shared through "two-point-nets", and 2) the number of terminals shared through "three-point-nets". In general case, the number of shared terminals for each gate in a boundary of m gates is:

$$\sum_{i=2}^{m} T_{Net}(i) = k(1 - m^{p-1}) \tag{6}$$

where $T_{Net}(i)$ is the number of terminals shared through an "i-point-net" of each gate in the boundary of m gates. $T_{Net}(m)$ can be computed recursively from (6) as below:



(a) a case with *p*=1



(b) a case with *k*=0

Figure 2. Example of systems in two extreme cases

$$\sum_{i=2}^{m} T_{Net}(i) - \sum_{i=2}^{m-1} T_{Net}(i) = k(1 - m^{p-1}) - k(1 - (m-1)^{p-1})$$

$$\Rightarrow \quad T_{Net}(m) = k((m-1)^{p-1} - m^{p-1}). \tag{7}$$

Since there are N_g gates in the entire system, the overall number of terminals connected through "m-point-nets" is $N_g \times T_{Net}(m)$. Therefore the total number of "m-point-nets" is simply $N_g \times T_{Net}(m)/m$. This yields the following equation from (7)

$$Net(m) = \frac{kN_g((m-1)^{p-1} - m^{p-1})}{m} \tag{8}$$

where $Net(m)$ is the total number of "m-point-nets" in the entire system. Substituting m by Fo+1, where Fo is fan-out, gives

$$Net(Fo) = \frac{kN_g(Fo^{p-1} - (Fo+1)^{p-1})}{Fo+1}. \tag{9}$$

Equation (9) describes the stochastic fan-out distribution in a random logic network of N_g gates and Rent's parameters of k and p.

Expression (9) can be verified with examining the two extreme conditions shown in Fig. 2. For the case of p=1, as shown in Fig. 2-a, all pins of the gates come out of the system with no internal connections. Hence $Net(Fo)$=0 for all Fo. For the case of k=0, as shown in Fig 2-b, there are no pins in the system and hence $Net(Fo)$=0 for all Fo. Both conditions verify (9) since p=1, or k=0 result in $Net(Fo)$=0 for all Fo.

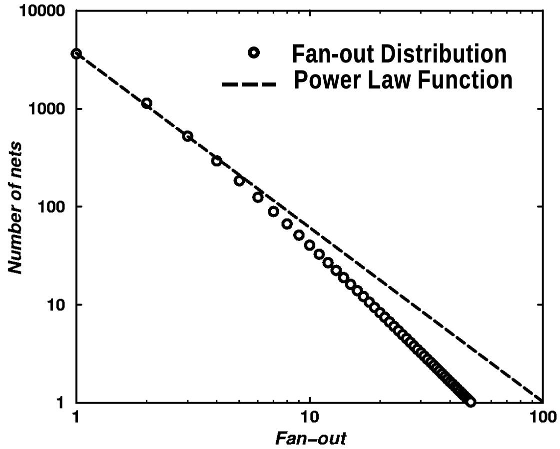


Figure 3. The fan-out distribution model compared to a power law function for N_g=15000, k=2.0, and p=0.6

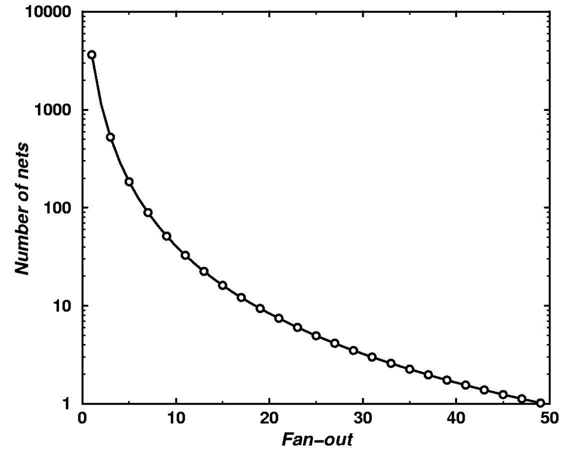


Figure 4. The fan-out distribution model in a semi-log plot of data for N_g=15000, k=2.0, and p=0.6

2.3 An Example of a Fan-out Distribution

Figure 3 illustrates an example of a fan-out distribution model using (9) compared to a power law function in a log-log plot. As shown in Fig. 3, the fan-out distribution model follows a power law function and then falls down in the end. This behavior of fan-out distribution has been also observed previously by Stroobandt et al [8]. This observation implies that the closed-form expression for the fan-out distribution given in (9) captures the behavior of multi-terminal nets in a large range of the fan-outs.

Since the range of the fan-out in a system is often limited, one can use a semi-log plot of data to illustrate the fan-out distribution. For instance, although in the example of Fig. 3 the number of gates in the system is 15000, the maximum fan-out is limited to 50. The example of Fig. 3 is depicted in Fig. 4 in a semi-log plot. In this paper both representations for the fan-out distribution are considered.

3. Characterization of Multi-terminal Nets

In this section by using the fan-out distribution model derived in Section 2, the closed-form expressions for maximum fan-out, total number of nets, and average fan-out are derived.

3.1 Maximum Fan-out

An approximation for the maximum fan-out can be computed by the following equation:

$$\frac{kN_g(Fo_{Max}^{p-1}-(Fo_{Max}+1)^{p-1})}{Fo_{Max}+1}\equiv 1 \qquad (10)$$

Assuming Fo_{Max}>>1, the maximum fan-out can be computed as follows:

$$kN_g Fo_{Max}^{p-1}[1-(1+\frac{1}{Fo_{Max}})^{p-1}] = Fo_{Max}+1$$

$$kN_g Fo_{Max}^{p-1}[1-(1+\frac{p-1}{Fo_{Max}}+....)] \approx Fo_{Max}$$

$$Fo_{Max} \approx \left(kN_g(1-p)\right)^{\frac{1}{3-p}} \qquad (11)$$

3.2 Total Number of Nets

The total number of nets in the system can be easily calculated by using (9) and (11).

$$Net_{Tot} = \sum_{Fo=1}^{Fo_{Max}} \frac{kN_g(Fo^{p-1}-(Fo+1)^{p-1})}{Fo+1} \qquad (12)$$

Using series expansion, (12) can be simplified as:

$$Net_{Tot} = kN_g\left\lfloor 1-(Fo_{Max}+1)^{p-2}-\Psi(p,Fo_{Max})\right\rfloor \qquad (13)$$

where:

$$\Psi(p,Fo_{Max}) = \sum_{n=1}^{Fo_{Max}} \frac{n^p}{n^2(n+1)} \qquad (14)$$

3.3 Average Fan-out

An average fan-out can be readily derived from (9) and the following definition.

$$Fo_{avg} = \frac{\sum_{Fo=1}^{Fo_{Max}} Fo \cdot Net(Fo)}{\sum_{Fo=1}^{Fo_{Max}} Net(Fo)} \quad (15)$$

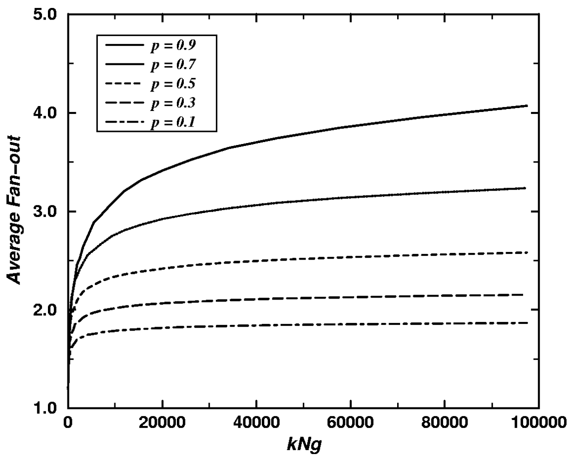


Figure 5. Variation of the average fan-out as a function of *p*, *k*, and N_g

Using series expansion, (15) can be simplified as:

$$Fo_{avg} = \frac{1-(Fo_{Max}+1)^{p-1}}{1-(Fo_{Max}+1)^{p-2} - \Psi(p, Fo_{Max})} - 1 \quad (16)$$

where $\psi(p, Fo_{Max})$ is the summation defined by (14).

Figure 5 illustrates the variation of average fan-out against p and kN_g. As seen in Fig.5, the average fan-out for the variety of designs, $0.1<p<0.9$ and $kN_g<100{,}000$, in a homogeneous logic network is roughly in the range of $2.0<Fo_{avg}<4.0$.

The above result verifies that, in most cases, a large amount of the nets are 2- or 3-terminal nets and the remaining nets normally do not have high fan-outs as implied in [6].

4. Applications of Fan-out Distribution Model

The closed-form expression for the fan-out distribution model presented in this paper can be used as a simple and effective tool for different applications as described in the following.

4.1 Prediction of Fan-out Distribution

In the absence of the actual netlist of a design, one can utilize the fan-out distribution model to characterize the interconnect structure for the design in the future technology generations. For instance, with the minimum knowledge of the number of gates and the Rent's rule parameters, all of the netlist information such as the total number of nets, the average fan-out and the maximum fan-out can be predicted by using the compact models presented in Section 3.

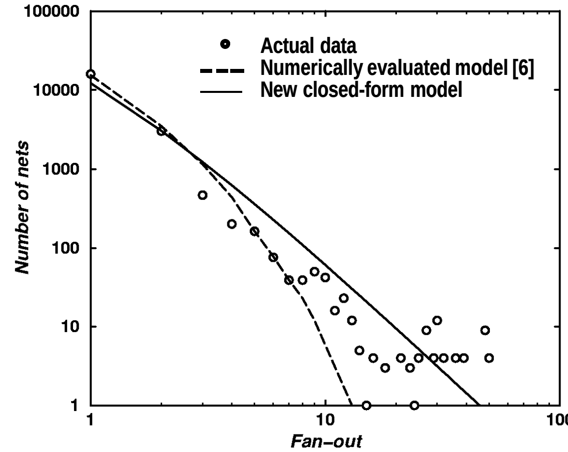


Figure 6. Comparison of the new closed-form fan-out distribution model to actual data and the previous model

In order to validate the closed-form model for fan-out distribution, some examples of the ISCAS benchmark designs are examined as presented in Table 1. The measured average fan-out and the Rent's rule parameters of the benchmarks are taken from [6] and the predicted values are computed by using the closed-form expressions given in Section 3. As shown, the estimated average fan-out clearly follows the measured one.

Moreover, in order to compare the new closed-form fan-out distribution model with the previously existing numerically evaluated model by Stroobandt et al [6], a plot of fan-out distribution for a design in [6] is depicted in Fig. 6. The design parameters used for this plot are N_g=23815 gates, with k=2.41 and p=0.28.

As seen in Fig. 6, the new closed-form model for the fan-out distribution captures the behavior of multi-terminal nets in the larger range of fan-outs than that of the previous model.

Table 1. The ISCAS benchmark designs selected from [6]

Design	Design Parameters			Actual	Predicted		
Name	N_g	k	p	Fo_{av}	Fo_{av}	Net_{tot}	Fo_{Max}
S208.1	112	2.69	0.39	1.557	1.71	80	7.4
S298	133	2.94	0.42	1.941	1.78	102	8.2
S344	175	2.62	0.34	1.603	1.72	129	8.6
S382	179	3.83	0.34	1.830	1.73	142	8.9
S832	293	2.65	0.58	2.558	2.08	229	12
S953	424	2.82	0.68	1.807	2.18	210	13
S1196	547	2.88	0.64	1.856	2.22	304	15

S1423	731	2.69	0.38	1.662	1.96	544	15

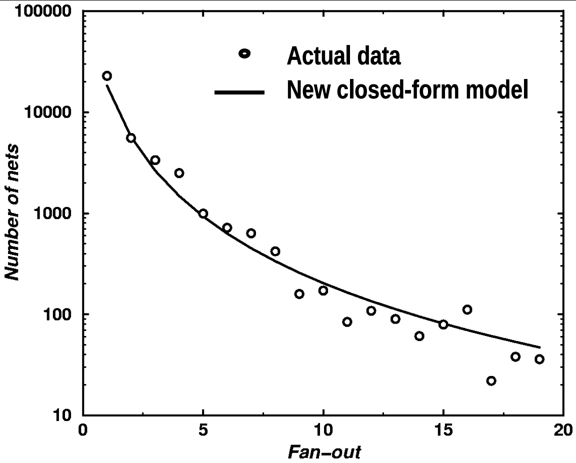


Figure 7. Estimation of the Rent's exponent by utilizing the fan-out distribution model

4.2 Estimation of Rent's Exponent

The estimation process for the Rent's exponent is often computationally expensive. The conventional method of the Rent's exponent evaluation is based on the partitioning of the design into several hierarchical sub-blocks as described in [4]. This partitioning process is often not available in commercial CAD tools. On the other hand, the statistical net data can be easily obtained from the netlist file. Therefore, the closed-form fan-out distribution models as well as the easy accessibility of fan-out distribution data from the CAD tools enable us to estimate the Rent's exponent with an easier way than that of conventional partitioning process.

For instance, one can use (13) and the total number of nets, which is a basic data from the netlist file, to compute the Rent's exponent. A more accurate Rent's exponent can be achieved by fitting the fan-out distribution data, extracted from the netlist, to the closed-form fan-out distribution model in (9).

An example of the Rent's exponent estimation by using the fan-out distribution model is illustrated in Fig. 7. The data is from a random logic part of an ASIC design with N_g=44803 gates with k=3.36. As shown, the best fit to the data is achieved when the Rent's exponent is p=0.6, which is similar to the Rent's exponent obtained from the partitioning process.

4.3 Heterogeneous Netlist Information

Netlist information is one of the components of the global net-length distribution model in a heterogeneous system-on-a-chip as described in [3]. The closed-form expression for the fan-out distribution presented here, gives the first order approximation for the netlist information.

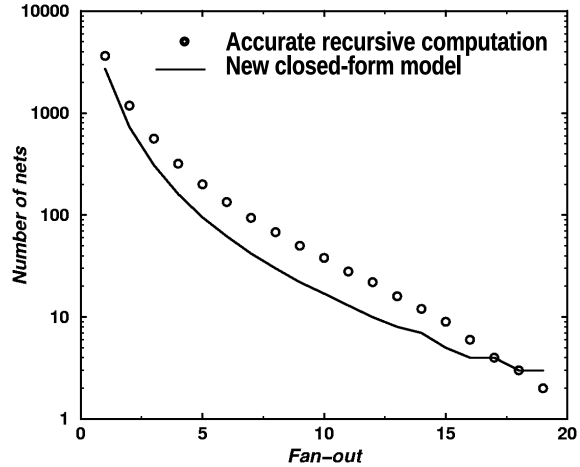


Figure 8. The fan-out distribution of a heterogeneous system using the recursive computation and the closed-form model

The netlist information is a generic fan-out distribution model that defines the connectivity between megacells. In a system with N_{Meg} megacells, the netlist information is computed recursively by examining all possible combinations of 2,3,…,m out of N_{Meg} megacells. For a system with a large number of megacells, this task requires excessive computation in order to extract the netlist information.

In this case, the number of global nets with fan-out of Fo in a system with N_{Meg} megacells can be approximated from (9) as

$$N_{net}(Fo) = \frac{\tilde{K} N_{Meg}\left(Fo^{\tilde{P}-1} - (Fo+1)^{\tilde{P}-1}\right)}{Fo+1} \qquad (17)$$

Table 2. The Rent's parameters of a heterogeneous system [3]

Megacell's Name	k	N	p	Megacell's Name	k	N	p
Instruction Cache	4.12	380000	0.20	Instr. Fetch Address	3.20	16500	0.60
Instruction Cache Tags	3.80	18000	0.47	Instr. Fetch Data Path	3.20	13800	0.60
Data Cache	4.12	350000	0.20	Instr. Fetch Control	3.20	9500	0.60
Data Cache Tags	3.80	25500	0.47	Address Queue	3.20	22000	0.60
TLB	3.80	22400	0.35	Inst. Decode & Reg. Ren.	3.20	45300	0.60
Secondary Cache Ctrl.	3.20	15700	0.60	Integer Data Path	3.20	43800	0.60
External Interface	3.20	18400	0.60	Integer Queue	3.20	19700	0.60
Sys. Interf. Buffers	3.20	22600	0.60	Floating Point Data Path	3.20	32600	0.60
Free List	3.20	9800	0.60	Floating Point Queue	3.20	51000	0.60

Graduation unit	3.20	26300	0.60	Floating Point Multiplier	3.20	19300	0.60

where $\tilde{K}$ and $\tilde{P}$ are the "equivalent megacells" Rent's rule parameters. Note that the equivalent megacells Rent's rule parameters can be estimated from the Rent's rule parameters of each megacell by using the heterogeneous Rent's rule [7], as

$$\begin{cases} \tilde{K} = \left(\prod_{i=1}^{N_{Meg}} k_i N_i^{p_i} \right)^{\frac{1}{N_{Meg}}} \\ \tilde{P} = \dfrac{\sum_{i=1}^{N_{Meg}} p_i N_i}{N_{Meg}} \end{cases} \tag{18}$$

where k_i, and p_i are the Rent's rule parameters of the ith megacell and N_{Meg} is the total number of megacells in the heterogeneous system-on-a-chip. The closed-form fan-out distribution of (17) and (18) significantly reduces the computation time for the netlist information in large systems.

As an example, Fig. 8 illustrates the fan-out distribution in a heterogeneous system with 20 megacells described in Table 2 using the recursive computation as well as the closed-form approximation with $\tilde{K}$=738.4 and $\tilde{P}$=0.34. Although the closed-form model provides a first order approximation of the netlist information, it takes only less than 0.1 second where the recursive computation takes about 90 seconds for the simulation program.

5. Conclusion

Based upon Rent's rule, a closed-form model for fan-out distribution in a random logic network is rigorously derived. The new model is verified through comparison with actual data from different ISCAS benchmark designs. Some important applications of the closed-form fan-out distribution are presented. The described applications are the following: (1) prediction of the fan-out distribution in the absence of the netlist of the future technology generations, (2) estimation of the Rent's exponent from a given fan-out distribution, and (3) approximation of the global netlist information in a heterogeneous system-on-a-chip.

6. Acknowledgments

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