IEEE 2004 CUSTOM INTEGRATED CIRCUITS CONFERENCE

Relative Inductance Extraction Method

Kaveh Shakeri and James D. Meindl

Microelectronic Research Center, Georgia Institute of Technology, 791, Atlantic Dr NW, Atlanta, GA, 30332
Email: kaveh@ece.gatech.edu

Abstract

A new relative inductance extraction method is defined to solve massive coupled RLC interconnects. The new relative inductance generates a sparse inductance matrix. Therefore, it enables modeling of large circuits with reasonable speed and accuracy. It maintains accuracy for all frequencies, even for the cases that there are no near return paths. Simulations done for a 16 bit bus with each line divided into 16 segments show that this method is 4 times faster than using the dense inductance matrix.

I. Introduction

As the on-chip wavelength becomes comparable with the length of global interconnects, inductance effects become important for many clock, signal and power interconnects. Since inductance is defined for a loop, the return path should be known in order to calculate inductance. On the other hand, to find the return path, the inductance should be known. To solve this dilemma, partial inductance was [1], [2] defined for each segment assuming that its return path at infinity. Using this method, the inductance can be extracted for each segment without prior knowledge of the return path. This method, however, generates a very dense inductance matrix because there is a considerable coupling between even very far segments. Manipulating such large and dense matrices is quite infeasible due to impractical time and memory requirements. To address this problem, various methods [4]-[7] have been proposed to sparsify inductance matrix, each of which assume that the return path is within a certain radius (window) of the segment and therefore the mutual inductances between far segments are negligible. These methods can be used to sparsify the inductance matrix of circuits with near return paths. However, if the design is such that the return path is not near, then a new sparsification technique is needed. For instance, the return paths of the power/ground wires are not necessarily through the neighboring power/ground wires, therefore, sparsification of the inductance matrix causes errors in the simulation of the on-chip power distribution grid.

The distribution of return current of a signal wire is always such that the overall impedance is minimized. Hence, at low frequencies, that the impedance is largely determined by resistance of the lines, current spreads among many power and ground lines. Therefore, the previous methods [4]-[7], which assume that return current is within a window, are not applicable for low frequencies.

In this paper a new inductance matrix is introduced, in which elements are calculated relative to some arbitrary virtual return paths. It has been proven that this matrix is equivalent to the conventional partial inductance matrix. It, however, is considerably sparser, and solving interconnect problems using this matrix is hence much faster. This technique is accurate for all frequency ranges and all configurations.

In Section II, the partial inductance method and sparsification techniques are briefly discussed. The new relative inductance is introduced in Section III, wherein it is proven to be equivalent to the partial inductance method. In Section IV, it is shown that the relative inductance can be calculated for 3D structures. In Section V, it is illustrated that most of the elements of the new matrix are so small that they can be forced to zero without loosing accuracy. In Section VI, the new approach is applied on some examples and their results are compared with other techniques.

II. Partial Inductance Models

Partial inductance [1] is defined for a segment and does not depend on the return path. Simulating the circuit using the circuit simulator defines the return paths. For each segment, an artificial virtual loop is defined between the segment and infinity (Fig. 1). The mutual partial inductance between two segments is defined as the magnetic flux linkage between one segment (m) and the virtual loop of the other segment (k) when unit current is passing through m (Fig. 2). The total loop inductance can be calculated from the partial inductances of the segments making the loop, using the following equation

$$L_{loop} = \sum_{k} \sum_{m} S_{d_{(k,m)}} L_{P_{(k,m)}} , \tag{1}$$

where $S_{d_{(k,m)}}$ is +1 if the currents in the segments k and m are in the same direction , −1 if their currents are in opposite directions and zero if the segments are orthogonal. The advantage of using the partial inductance method is that inductance can be extracted without prior knowledge of return paths. However, the virtual loop is infinitely large; therefore, the mutual inductance between any two segments is considerably large. As a result the inductance matrix for a large circuit is large and dense. Thereby, simulating large circuits is almost impractical. For instance, a

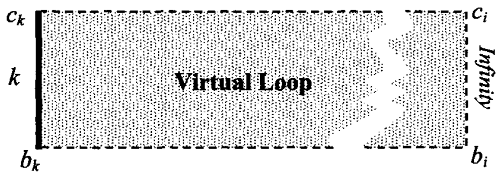

Fig. 1. Artificial virtual loop defined in the partial inductance method.

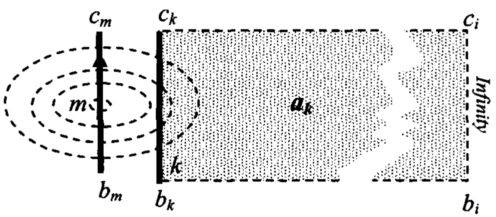

Fig. 2. Mutual partial inductance.

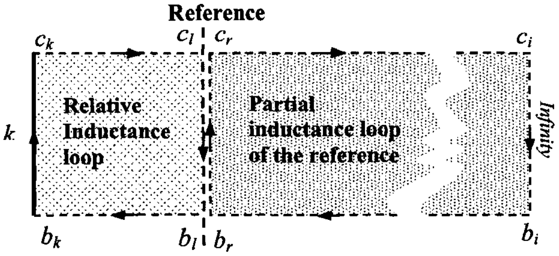

Fig. 3. Relative self inductance. The virtual loop of partial inductance of segment *m* is divided into two loops the relative inductance loop and the partial inductance loop for the reference.

circuit with 1,000 segments will have $1{,}000^2$=1,000,000 elements. Dealing with such a large matrix in a circuit simulator is prohibitively time and memory consuming.

To simulate large circuits like on-chip power distribution networks, a technique is needed to sparsify the inductance matrix. Unfortunately, the sparsification is not simple. Neglecting mutual inductances between far elements can result in a non-positive definite matrix [2]. In the following section, a new definition for inductance is introduced to sparsify the matrix without ignoring the far return paths.

III. Relative Inductance

Partial mutual inductance between two filaments *k* and *m* can be calculated from [2]

$$L_{p_{(k,m)}} = \frac{1}{I_m} \int_{a_k} B_{k,m} \cdot da_k \,, \tag{2}$$

where I_m is the current in segment m , $B_{k,m}$ is flux density created in the virtual loop of segment k due to current passing through segment m, and a_k is the area bounded by the virtual loop (Fig. 2). Magnetic vector potential is defined as

$$B_{k,m} = \nabla \times A_{k,m} \,, \tag{3}$$

Therefore, using Stokes' theorem, (2) can be written as

$$L_{p_{(k,m)}} = \frac{1}{I_m} \int_{a_{k'}} \left(\nabla \times A_{k,m}\right) \cdot da_{k'} = \frac{1}{I_m} \oint_C A_{k,m} \cdot dl_k \,, \tag{4}$$

where C is the loop $b_kc_kc_ib_i$ surrounding the area a_k (Fig. 2). The magnetic vector potential is in the direction of current, therefore, the integral along the perpendicular sides (c_kc_i and b_kb_i) is zero. The magnetic vector potential is zero at infinity; therefore, the integral along the side which is at infinity (b_ic_i) is also zero. As a result partial inductance is equal to

$$L_{p_{(k,m)}} = \frac{1}{I_m} \int_{b_k}^{c_k} A_{k,m} \cdot dl_k \tag{5}$$

The magnetic vector potential can be calculated from

$$A_{k,m} = I_m \frac{\mu}{\pi} \int_{b_m}^{c_m} \frac{dl_m}{r_{k,m}} \,, \tag{6}$$

where μ is the permeability of the medium , dl_m is an element of conductor m along its axis, and

$$r_{k,m} = \left|r_k - r_m\right| \,, \tag{7}$$

where r_k and r_m are the position vectors of segments k and m, respectively. Replacing A_{km} in (5) by (6), we have

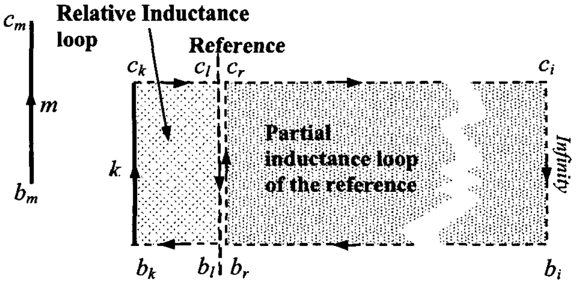

Fig. 4. Relative mutual inductance. The virtual loop of partial inductance of segment *m* is divided into two loops the relative inductance loop and the partial inductance loop for the reference.

$$L_{p_{(k,m)}} = \frac{\mu}{4\pi} \int_{b_k}^{c_k} \int_{b_m}^{c_m} \frac{dl_m \cdot dl_k}{r_{k,m}} \,. \tag{8}$$

The virtual loop of the partial inductance can be divided into two loops (Fig. 3 and Fig. 4), the virtual loop of the partial inductance of the reference ($b_rc_rc_ib_i$) and the *relative* loop defined as the loop ($b_kc_kc_lb_l$), which is between the segment and the reference. The partial inductance of the reference is equal to

$$L_{p_{(r,m)}} = \frac{\mu}{4\pi} \int_{b_r}^{c_r} \int_{b_m}^{c_m} \frac{dl_m \cdot dl_r}{r_{r,m}} \,, \tag{9}$$

and the relative inductance is equal to

$$L_{r_{(k,m)}} = \frac{\mu}{4\pi} \int_{b_k}^{c_k} \int_{b_m}^{c_m} \frac{dl_m \cdot dl_k}{r_{k,m}} - \frac{\mu}{4\pi} \int_{b_l}^{c_l} \int_{b_m}^{c_m} \frac{dl_m \cdot dl_k}{r_{k,m}} \,. \tag{10}$$

Depending on the place of the source (*m*), victim (*k*) and reference (*r*), two different situations might happen. Hence, the partial mutual inductance of two segments can be written as a function of relative mutual inductance and the partial mutual inductance of the segment and the reference as

$$L_{p_{(k,m)}} = S_{l_{(k,m)}} L_{r_{(k,m)}} + L_{p_{(r,m)}} \,, \tag{11}$$

where $S_{l_{(k,m)}}$ is defined by the location of the segments with respect to the reference as

$$S_{l_{(k,m)}} = \begin{cases} +1 & \textit{if } k \textit{ is nearer to } m \\ -1 & \textit{if the reference is nearer to } m \end{cases} \,. \tag{12}$$

A new sign is defined called the *relative sign*

$$S_{r_{(k,m)}} = S_{d_k} S_{d_m} S_{l_{(k,m)}} \,, \tag{13}$$

where S_{d_k} is called the *direction sign*

$$S_{d_k} = \begin{cases} +1 & \textit{if current in k has the same direction} \\ & \textit{as the reference direction} \\ -1 & \textit{if current in k is in the opposite direction} \end{cases} \tag{14}$$

The sign defined in partial inductance method can be written as a function of the *direction sign* for the two segments

$$S_{d_{(k,m)}} = S_{d_k} S_{d_m} \,. \tag{15}$$

IV. Relative Mutual Inductance for 3D structures

The mutual inductance between two orthogonal segments is zero (5). Figure 5 shows a path on the planes orthogonal to the segment *k*. The mutual inductance between the segment and any line on these orthogonal planes is zero. Therefore the path on the

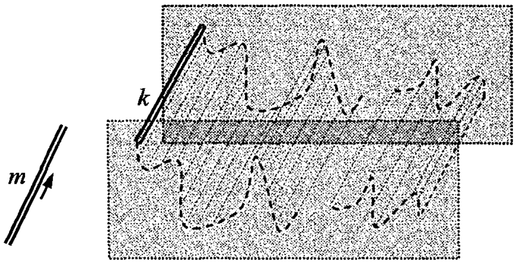

Fig. 5. Any orthogonal path which is on the orthogonal planes to the segments has no coupling with the segment. Therefore any path can be selected on the orthogonal planes for the sides of the virtual loop.

orthogonal planes which is selected for the loop does not change the mutual partial inductance and any path can be selected for the sides of the virtual loop.

Figure 6 shows two virtual loops. One is the partial virtual loop defined by [1]. The second loop is from the second segment (k) to the reference (relative loop) and from the reference to infinity (partial mutual inductance of the segment and the reference). Since the paths selected for the side lines are on the orthogonal planes, the partial inductance of the virtual loops are equal. As a result, we have

$$L_{P(k,m)} = S_{l(k,m)} L_{r(k,m)} + L_{P(r,m)} \cdot \tag{16}$$

Therefore, even if the relative loop and the partial inductance loop of the reference are not on the same plane, the same equation (11), exists between the relative and the partial inductance loop as long as the sides are on the orthogonal planes. As a result, the relative inductance can be used for any 3D structure.

V. Sparsifying the matrix

Magnetic flux is related to current as

$$\boldsymbol{\varphi} = \mathbf{L} \times \mathbf{I}\,, \tag{17}$$

where $\boldsymbol{\varphi}$ is the magnetic flux matrix, $\boldsymbol{I}$ is the current matrix and $\boldsymbol{L}$ is the inductance matrix. Equation (17) can be rewritten as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \varphi_n \end{bmatrix} = \begin{bmatrix} L_{p_{(1,1)}} & S_{d_{(1,2)}} L_{p_{(1,2)}} & S_{d_{(1,3)}} L_{p_{(1,3)}} & \cdots & S_{d_{(1,n)}} L_{p_{(1,n)}} \\ S_{d_{(2,1)}} L_{p_{(2,1)}} & L_{p_{(2,2)}} & S_{d_{(2,3)}} L_{p_{(2,3)}} & \cdots & S_{d_{(2,n)}} L_{p_{(2,n)}} \\ S_{d_{(3,1)}} L_{p_{(3,1)}} & S_{d_{(3,2)}} L_{p_{(3,2)}} & L_{p_{(3,3)}} & & S_{d_{(3,n)}} L_{p_{(3,n)}} \\ \vdots & \vdots & & \ddots & \\ S_{d_{(n,1)}} L_{p_{(n,1)}} & S_{d_{(n,2)}} L_{p_{(n,2)}} & S_{d_{(n,3)}} L_{p_{(n,3)}} & \cdots & S_{d_{(n,n)}} L_{p_{(n,n)}} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix}, \tag{18}$$

where each element of the inductance matrix is the partial inductance defined by [1]. The inductance matrix is symmetric, hence is equal to its transpose:

$$\mathbf{L} = \mathbf{L}^{T} \tag{19}$$

Replacing the partial inductances by the relative inductances described by (11), (19) can be rewritten as

$$L = \begin{bmatrix} S_{d_{(1,1)}}\left(L_{r_{(1,1)}} + L_{p_{(r1,1)}}\right) & S_{d_{(2,1)}}\left(S_{l_{(2,1)}} L_{r_{(2,1)}} + L_{p_{(r1,1)}}\right) & \cdots & S_{d_{(n,1)}}\left(S_{l_{(n,1)}} L_{r_{(n,1)}} + L_{p_{(r1,1)}}\right) \\ S_{d_{(1,2)}}\left(S_{l_{(1,2)}} L_{r_{(1,2)}} + L_{p_{(r1,2)}}\right) & S_{d_{(2,2)}}\left(L_{r_{(2,2)}} + L_{p_{(r1,2)}}\right) & \cdots & S_{d_{(n,2)}}\left(S_{l_{(n,2)}} L_{r_{(n,2)}} + L_{p_{(r1,2)}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ S_{d_{(1,n)}}\left(S_{l_{(1,n)}} L_{r_{(1,n)}} + L_{p_{(r1,n)}}\right) & S_{d_{(2,n)}}\left(S_{l_{(2,n)}} L_{r_{(2,n)}} + L_{p_{(r1,n)}}\right) & & S_{d_{(n,n)}}\left(L_{r_{(n,n)}} + L_{p_{(r1,n)}}\right) \end{bmatrix} \tag{20}$$

Equation (18) can be rewritten with an extra row and column

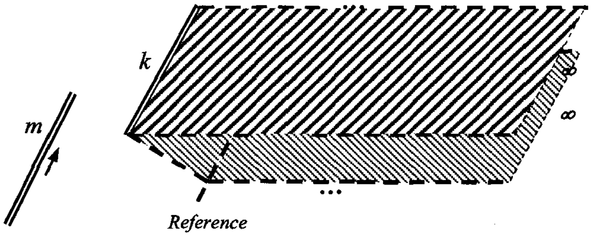

Fig. 6. Two equal virtual loops are shown. The first one is the virtual loop defined by [2] and the second loop is from the second segment to the reference (relative loop) and from the reference to infinity.

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \varphi_n \\ 0 \end{bmatrix} = \begin{bmatrix} L_{r_{(1,1)}} & S_{r_{(2,1)}} L_{r_{(2,1)}} & S_{r_{(3,1)}} L_{r_{(3,1)}} & \cdots & S_{r_{(n,1)}} L_{r_{(n,1)}} & S_{d_1} L_{p_{(r,1)}} \\ S_{r_{(1,2)}} L_{r_{(1,2)}} & L_{r_{(2,2)}} & S_{r_{(3,2)}} L_{r_{(3,2)}} & \cdots & S_{r_{(n,2)}} L_{r_{(n,2)}} & S_{d_2} L_{p_{(r,2)}} \\ S_{r_{(1,3)}} L_{r_{(1,3)}} & S_{r_{(2,3)}} L_{r_{(2,3)}} & L_{r_{(3,3)}} & & S_{r_{(n,3)}} L_{r_{(n,3)}} & S_{d_3} L_{p_{(r,3)}} \\ \vdots & \vdots & & \ddots & & \\ S_{r_{(1,n)}} L_{r_{(1,n)}} & S_{r_{(2,n)}} L_{r_{(2,n)}} & S_{r_{(3,n)}} L_{r_{(3,n)}} & & L_{r_{(n,n)}} & S_{d_n} L_{p_{(r,n)}} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} |i_1| \\ |i_2| \\ |i_3| \\ \vdots \\ |i_n| \\ \sum_{i=1}^{n} S_{d_i} |i_i| \end{bmatrix}, \tag{21}$$

wherein all magnetic flux and current elements are the same as those in (18).This equation describes the magnetic flux as a function of the relative inductances to the reference and the partial inductances of the references. This matrix is still dense. To make a sparse matrix, multiple references should be defined. The relative inductance for each segment is calculated for the nearest reference. An extra row and column should be added to the matrix for each reference. The extra current elements added to the current vector are the summation of the currents in the segments related to that reference. In this case where we have multiple references, (17) can be written as shown in Fig. 7.

The mutual partial inductances in (18) are not negligible because their virtual loops are infinitely large. Figure 8 shows the mutual inductance between two segments versus distance for partial and relative inductance methods. As shown, the relative mutual inductance drops much faster. Hence, the mutual relative inductance of far segments can be truncated which makes the inductance matrix of Fig. 7 sparse. This matrix has some unique features that make it interesting:

1) Although the new inductance matrix has more elements than the original matrix, it is significantly sparser.
2) It is asymmetrical because L_{ij} is not equal to L_{ji}.
3) The summation of the currents at each reference makes the same magnetic flux that those segments make for far segments, and therefore, in this method the mutual inductance between far segments are not neglected.

VI. Implementation

The new methodology is applied to data buses with various numbers of lines. The self and mutual inductances have been extracted for the dense partial inductance and the sparse relative inductance for these buses. The extracted matrices are then incorporated in HSPICE [8] circuit simulations. Mutual inductances entered in HSPICE simulators need to have the following properties:

$$M_{ij} = M_{ji} \tag{22}$$

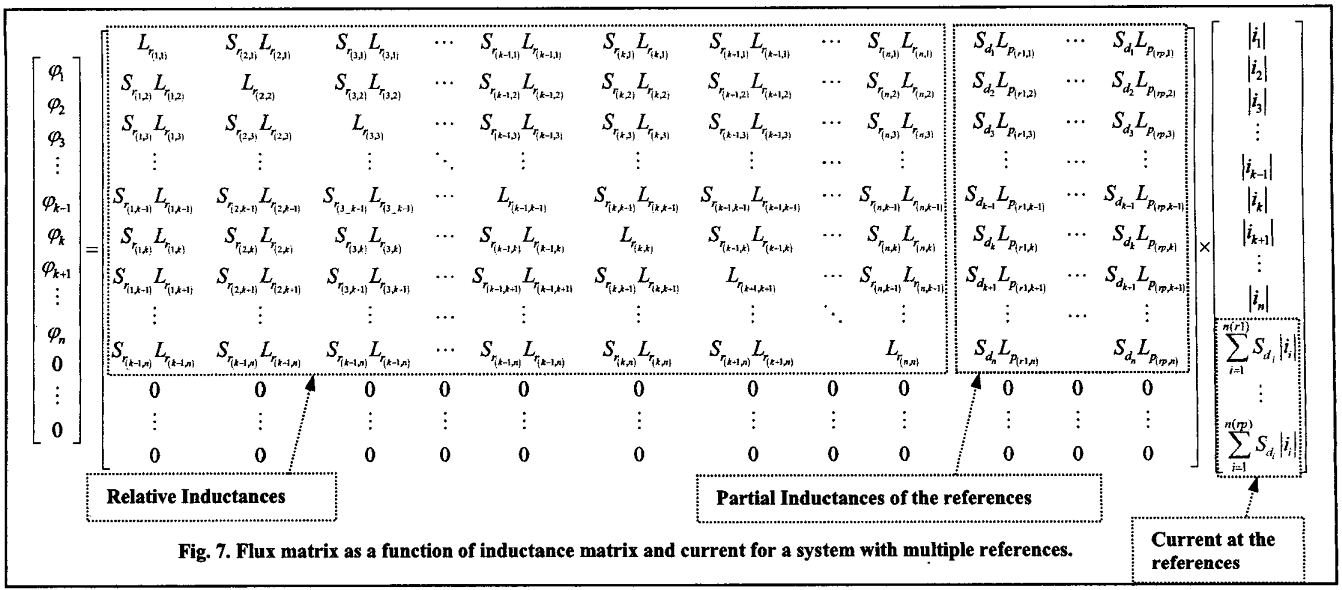

Fig. 7. Flux matrix as a function of inductance matrix and current for a system with multiple references.

and

$$M_{ij} = K\sqrt{L_i \cdot L_j} \qquad 0 < |K| \le 1 \qquad (23)$$

These conditions are not satisfied in the new relative inductance matrix. Thereby, in HSPICE simulations, the relative inductances and the references are implemented as voltage-controlled voltage sources and controlled current sources.

Figure 9 shows simulation results for different cases using the partial inductance method and the relative inductance method. Results show that for a small number of segments the overhead caused by the references will actually increase the simulation time. However, as the number of segments increase, the relative inductance method is much faster because of the sparser inductance matrix.

VII. Conclusion

A new relative inductance matrix is defined for solving massively coupled RLC interconnects. All self and mutual inductance elements are calculated using some virtual return paths that are relatively close to interconnects, and the new inductance matrix is hence very sparse. It has been proven that the relative inductance matrix is equivalent to the conventional partial inductance matrix that calculates all inductance elements assuming return paths are at infinity. The analysis is hence accurate for all frequency ranges and all configurations. Using the new inductance matrix makes the circuit simulations significantly faster without losing accuracy. Simulations for a 16-bit bus show that the new technique is 4 times faster than the conventional partial inductance technique. This approach can be particularly helpful for modeling power distribution networks wherein currents return through distant paths.

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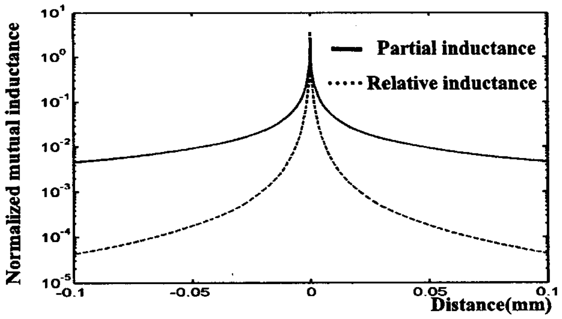

Fig. 8. Mutual inductance versus distance for partial inductance and the relative mutual inductance.

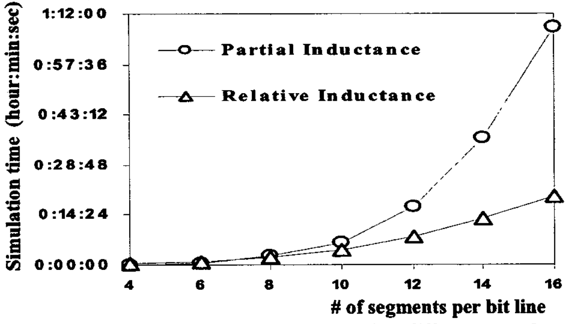

Fig. 9. Simulation time of a 16-bit bus for different number of segments per bit-line with partial inductance and relative inductance.