A Compact Substrate Spreading Resistance Model for SoC

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Abstract

Mixed signal SoC IC's are increasingly being used in commercial products. However, the analog circuits are sensitive to noise which is produced by the digital circuits and propagated by the substrate. In this paper, compact physical 2D and 3D models have been derived for the spreading resistance between two contacts for two different substrates. These models can be used to estimate substrate noise.

I. Introduction

To model the impact of the substrate noise of the digital circuits on the analog circuits in a SoC, a model is needed for the substrate impedance. Previous models are either empirical [1], [2], [3] or are too complicated with many approximations [4], [5]. In this paper, new compact physical models are introduced which enable modeling of the spreading resistance between two contacts as a function of the substrate doping, size of the contacts and the distance between them for two kinds of substrates. The p- substrate has uniform doping, and the p+ substrate is a low doping epitaxial layer on a high doping substrate.

The calculation of the spreading resistance between two contacts using the capacitance between them is described in Section II. The models for 2D spreading resistances of the p- and p+ substrate are described in sections III and IV respectively. The models for the 3D spreading resistances are described in sections V and VI.

II. Spreading Resistance Calculation

A medium is called homogenous if the dielectric constant and the conductivity of the medium have the same space dependence (independent of space coordinates). The resistance and capacitance between two conductors in a homogenous medium are related as

$$RC = \frac{\varepsilon}{\sigma}\ , \tag{1}$$

where R and C are the resistance and capacitance between the conductors, and ε and σ are the dielectric constant and the conductivity of the medium respectively. Therefore, by calculating the capacitance between two conductors the resistance between them can be calculated. The capacitance between two conductors can be calculated by calculating the voltage potential between them because of their charges

$$C = \frac{q}{V}\ , \tag{2}$$

where V is the voltage difference between the conductors because of the charge q on one of the conductors and $-q$ on the other. Substituting for C in (1) by (2), the resistance can be calculated as

$$R = \frac{\varepsilon \cdot V}{\sigma \cdot q}\ . \tag{3}$$

Fig. 1a and Fig 1b show the contacts connected to the p- and the p+ substrates. If their mirror is added to them, the fields are not changed (Fig.2) however their capacitances are increases by two times. As a result the contact resistances shown in Fig. 1 is twice the resistance calculated for Fig. 2

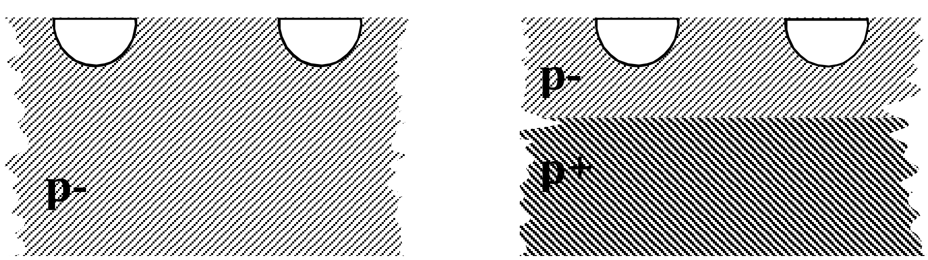
$$R_{Contact} = \frac{2 \cdot \varepsilon \cdot V}{\sigma \cdot q}\ . \tag{4}$$

Hence, the contact resistance can be calculated by calculating the voltage difference between the conductors shown in Fig. 2 because of a charge $+q$ on one of them and a $-q$ on the other.

III. 2D Spreading Resistance for p- Substrate

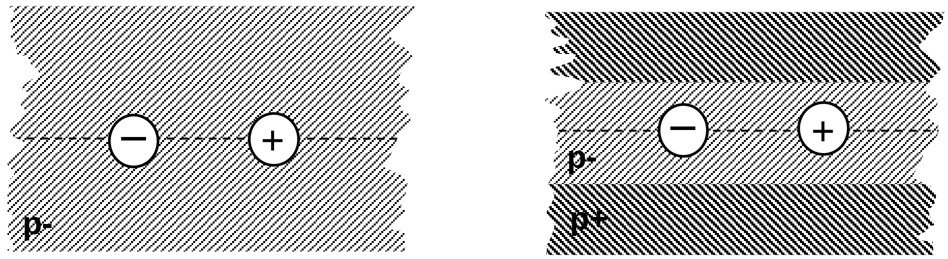
The p- substrate is shown in Fig.1a and its equivalent is shown in Fig2a. By using the image method used in electromagnetics, the insulator can be omitted by placing a charge with the same charge and same distance at the other side of the insulator. If there are two insulators at both sides then they should be replaced by an infinite number of charges as shown in Fig. 3.

The next step is to calculate the voltage difference because of the charges. The voltage potential between these nodes should be calculated by adding the voltages produced by each of the charges in Fig. 3. Calculating the voltage difference produced by the charges and simplifying it results



(a) (b)

Fig. 1. Contacts to two different substrates (a) p- substrate (b) p+ substrate



(a) (b)

Fig. 2. Contacts to two different substrates. The resistance calculated for this case is half the resistance calculated for Fig.1. (a) p- substrate (b) p+ substrate

in:

$$V=\frac{\rho_\ell}{\pi\varepsilon}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)+\ln\left[\prod_{k=1}^{\infty}\left(1+\frac{d^2}{4k^2T^2}\right)\right]\right]\quad . \tag{5}$$

where d is the distance between the contacts, r is the radius of the contacts , T is the thickness of the substrate and ρ_ℓ is the charge per unit length of the cylinder.

$$\rho_\ell=\frac{q}{\ell}\ . \tag{6}$$

where ℓ is the length of the cylinder. The contact resistance can be calculated using (4) and (5)

$$R_{Contact}=\frac{2}{\pi\sigma\ell}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)+\ln\left[\prod_{k=1}^{\infty}\left(1+\frac{d^2}{4k^2T^2}\right)\right]\right]\quad . \tag{7}$$

The series in the above equation can be simplified as

$$\left(\frac{\pi\cdot d}{2T}\right)\cdot\prod_{k=1}^{\infty}\left(1+\frac{d^2}{4k^2T^2}\right)=\sinh\left(\frac{\pi\cdot d}{2T}\right). \tag{8}$$

As a result the resistance between two cylindrical contacts of length ℓ can be calculated

$$R_{Contacts}=\frac{2}{\pi\sigma\ell}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)+\ln\left[\frac{\sinh\left(\frac{\pi\cdot d}{2T}\right)}{\left(\frac{\pi\cdot d}{2T}\right)}\right]\right]. \tag{9}$$

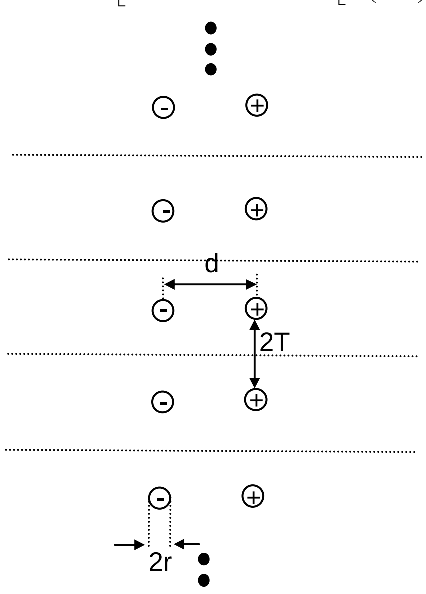


Fig. 3. Infinite series of charges replacing the two insulators on both sides of Fig 2a.

Fig. 4 shows a comparison between the model and simulation done by Raphael. As shown the model has less than 1% error with the simulations done by Raphael.

The model shows that for short distances, the resistance increases very rapidly. The rate of increase of the resistance is reduced for longer distances. Therefore, in this case the noise is not only a function of the area of the noise source but also a function of the distance from the noise source.

IV. 2D Spreading Resistance for p+ Substrate

The p+ substrate is made by a low doping epitaxial layer on a high doping substrate. The substrate in this case has a very low resistance compared to the resistance of the epitaxial layer; therefore it can be modeled as a good conductor. Using the mirror technique the conductors can be replaced by an infinite number of charges as shown in Fig. 5. Calculating and simplifying the voltage difference between the contacts because of the charges results in

$$V=\frac{\rho_\ell}{\pi\varepsilon}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)-\ln\left[\prod_{k=1}^{\infty}\left(\frac{1+\frac{d^2}{4(2k-1)^2T^2}}{1+\frac{d^2}{16k^2T^2}}\right)\right]\right]\ , \tag{10}$$

where T is the thickness of the epitaxial layer. Substituting (10) in (5) the contacts spreading resistance can be

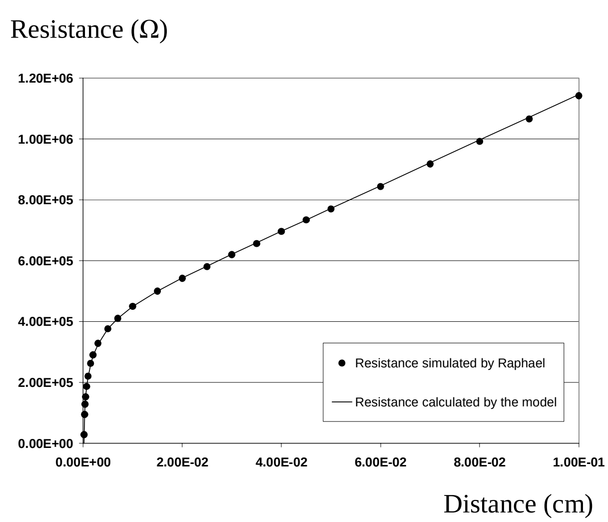


Fig. 4. Substrate resistance between two cylinders 1 μm long as a function of the distance between them in a p- substrate. (*r=1μm , T=200μm, σ=0.067(Ω-cm)*$^{-1}$)

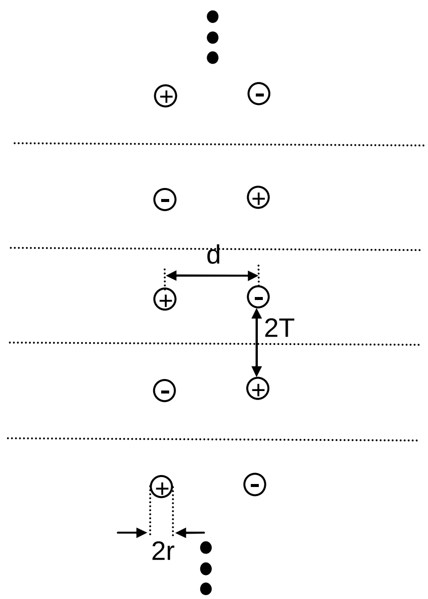


Fig. 5. Infinite series of charges replacing the two conductors on both sides of Fig 2b.

calculated

$$R_{Contact}=\frac{2}{\pi\sigma\ell}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)-\ln\left[\prod_{k=1}^{\infty}\left(\frac{1+\frac{d^2}{4(2k-1)^2T^2}}{1+\frac{d^2}{16k^2T^2}}\right)\right]\right]. \quad (11)$$

The series can be simplified as

$$\prod_{k=1}^{\infty}\left(\frac{1+\frac{d^2}{4(2k-1)^2T^2}}{1+\frac{d^2}{16k^2T^2}}\right)=\frac{\pi\cdot d}{4T}\coth\left(\frac{\pi\cdot d}{4T}\right). \quad (12)$$

Replacing it in (11) the spreading resistance of two cylindrical contacts with length ℓ can be calculated

$$R_{Contacts}=\frac{2}{\pi\sigma\ell}\left[\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)-\ln\left[\frac{\pi\cdot d}{4T}\coth\left(\frac{\pi\cdot d}{4T}\right)\right]\right]. \quad (13)$$

Fig. 6 shows the resistances calculated from simulation using the Raphael and from the model. As seen in Fig. 6 error using the model is less than 1%.

Results show that the resistance increases as the distance between the contacts is increased until it saturates. For long distances, the resistance is constant and is not a function of the distances between the contacts, because in this case most of the current passes straight down to the low resistance substrate and then passes through the low resistance substrate. Therefore because of the low resistivity of the substrate the resistance increases a little, as the distance between the contacts increase. Noise for p+ substrate is proportional to the area of the noise source and not the distance between the noise source and the victim. One technique to reduce noise is to add ground contacts to the substrate. The model shows that the noise reduction because of the ground connections to the substrate is proportional to the area of the connections and not to the distance between the connection and the noise source unless the distance between them is too small.

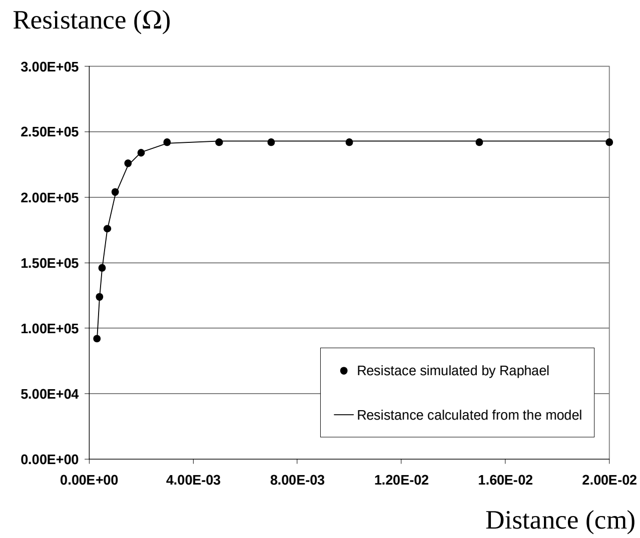


Fig. 6. Substrate resistance between two cylinders 1 μm long as a function of the distance between them in a p+ substrate. ($r=1\mu m$, $T=10\mu m$, $\sigma=0.067(\Omega\text{-cm})^{-1}$)

V. 3D Spreading Resistance for p- Substrate

The 3D spreading resistance modeled in this part is the resistance between two half spheres on a p- substrate. Using the same mirror technique which was used for the 2D case the insulators can be removed by replacing an infinite number of charges.

The resistance between the two spheres in an infinite space can be calculated from [6]

$$R_{Sph}=\frac{1}{2\pi\sigma r\left(1+\alpha+\frac{\alpha^2}{1-\alpha}+\frac{\alpha^2}{(1-\alpha)\left(1-\frac{\alpha^2}{1-\alpha^2}\right)}+...\right)}, \quad (14)$$

where

$$\alpha=\frac{r}{d}\cdot \quad (15)$$

The voltage caused by the infinite number of charges can be calculated from

$$V=\frac{q}{2\pi\varepsilon}\left[\frac{1}{T}\sum_{k=1}^{\infty}\frac{1}{k}-2\sum_{k=1}^{\infty}\frac{1}{\sqrt{d^2+4k^2T^2}}\right]. \quad (16)$$

Therefore that part of resistance which is because of the infinite charges is

$$R_{series}=\frac{1}{\pi\sigma}\left[\frac{1}{T}\sum_{k=1}^{\infty}\frac{1}{k}-2\sum_{k=1}^{\infty}\frac{1}{\sqrt{d^2+4k^2T^2}}\right]. \quad (17)$$

The total resistance can be calculated by adding (14) and (17).

$$R_{Contact}=R_{Sph}+\frac{1}{\pi\sigma}\left[\frac{1}{T}\sum_{k=1}^{\infty}\frac{1}{k}-2\sum_{k=1}^{\infty}\frac{1}{\sqrt{d^2+4k^2T^2}}\right] \quad (18)$$

Fig. 7 shows the comparison between this model and simulation done by Raphael as shown in the Fig. the error between the model and the simulations is less than 5%. Despite the 2D model of the p- substrate where the resistance increased as the distance between the contacts increased, the resistance in this case gets constant and does not increase after a certain distance. Therefore increasing the distance to the digital circuits does not reduce the noise caused by them.

VI. 3D Spreading Resistance for p+ Substrate

The 3D spreading resistance model derived in this part is the resistance between two half spherical contacts on a p+ substrate. The resistance can be calculated by the same mirror technique used for the 2d case the only difference is that in this case there are infinite charges instead of infinite line charges.

The resistance between the spheres is the same (14). The voltage between the two contacts because of the infinite series is

$$V=\frac{q}{2\pi\varepsilon}\left[\frac{1}{T}\sum_{k=1}^{\infty}\left(\frac{1}{2k}-\frac{1}{2k-1}\right)-2\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{d^2+4(2k-1)^2T^2}}-\frac{1}{\sqrt{d^2+16k^2T^2}}\right)\right]. \tag{19}$$

Therefore the part of the resistance which is because of the infinite series is

$$R_{series}=\frac{1}{\pi\sigma}\left[\frac{1}{T}\sum_{k=1}^{\infty}\left(\frac{1}{2k}-\frac{1}{2k-1}\right)-2\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{d^2+4(2k-1)^2T^2}}-\frac{1}{\sqrt{d^2+16k^2T^2}}\right)\right]. \tag{20}$$

Therefore the total resistance can be calculated from

$$R_{Contact}=R_{Sph}+\frac{1}{\pi\sigma}\left[\frac{1}{T}\sum_{k=1}^{\infty}\left(\frac{1}{2k}-\frac{1}{2k-1}\right)-2\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{d^2+4(2k-1)^2T^2}}-\frac{1}{\sqrt{d^2+16k^2T^2}}\right)\right]. \tag{21}$$

Fig. 8 shows the resistance between two contacts on a p+ substrate versus the distance between them. As shown in the Figure the results have less than 5% error. It also shows that the resistance saturates as the distance between the contacts increases. Therefore for long distances the noise is proportional to the area of the source noise not to the distance from source for long distances.

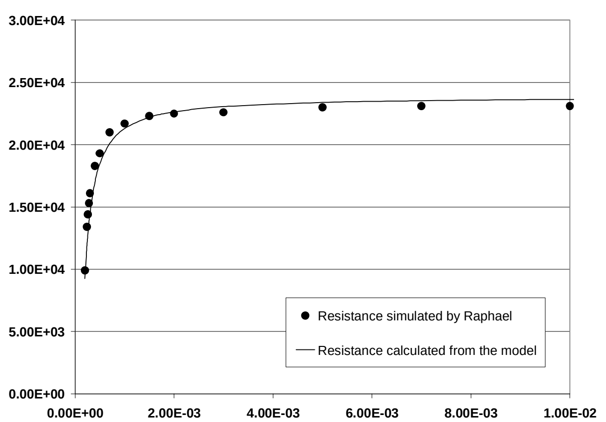


Fig. 7. Substrate resistance as a function of the distance between the contacts for two spheres in a p- substrate. (*r=1μm, T=200μm, σ=0.067(Ω-cm)$^{-1}$*)

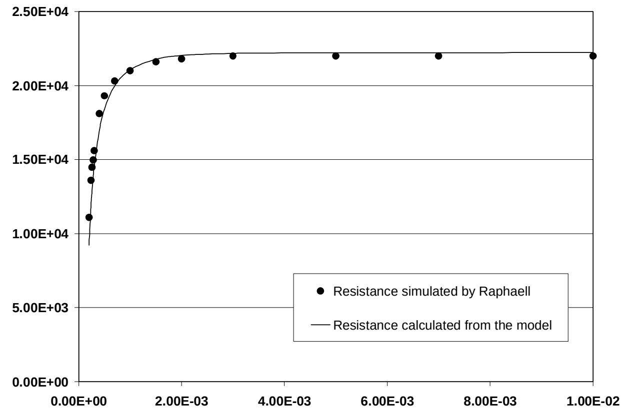


Fig. 8. Substrate resistance as a function of the distance between the contacts for two spheres in a p+ substrate. (*r=1μm, T=10μm, σ=0.067(Ω-cm)$^{-1}$*)

VII. Conclusion

VLSI technology enables us to fabricate cheap mixed signal SoC ICs; however, the analog part of the chip suffers from noise generated by the digital part. Most of the noise produced by the digital circuits is propagated through the substrate. In this paper new compact physical models for 2D and 3D spreading resistances are introduced to calculate the substrate resistance between two contacts for two different substrates. Results show that there is less than 5% error between the model and the simulation done with Raphael.

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