Homework Assignment No. 12 Solutions

Problem 1 – (10 points)

Problem 7.5-5. Find the equivalent rms noise voltage of the op amp designed in Example 6.5-2 over a bandwidth of 1Hz to 100kHz. Use the values for *KF* of Example 7.5-1.

Solution

The circuit for this amplifier is shown.

The *W*/*L* ratios in microns are:

$S_1 = S_2 = 12/1$

$S_3 = S_4 = 16/1$

$S_5 = 7/1$

$S_5 = 8.75/1$

$S_6 = S_7 = S_8 = S_{14} = S_{15} = 40/1$

$S_9 = S_{10} = S_{11} = S_{12} = 18.2/1$

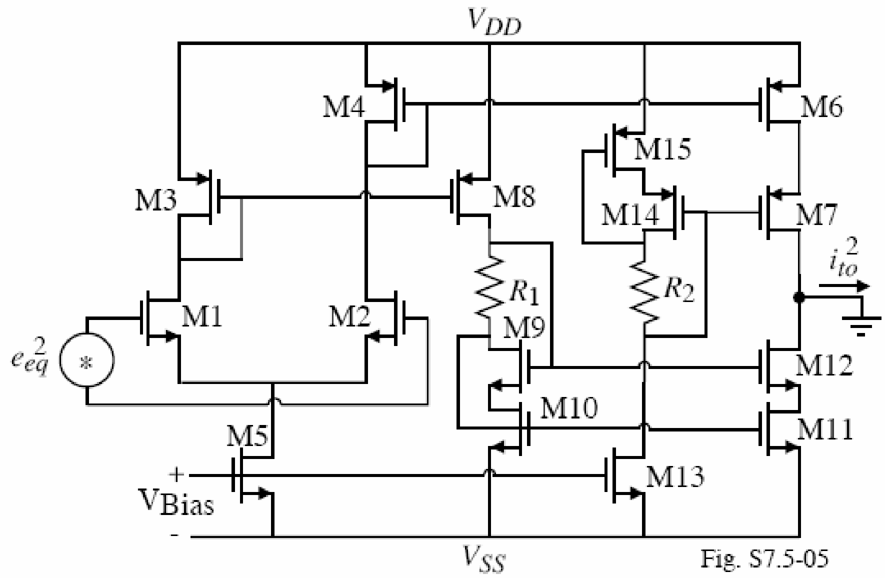


Fig. S7.5-05

Find the short circuit noise current at the output, i_{to}^2, due to each noise-contributing transistor in the circuit (we will not includeM7, M9, M12 and M14 because they are cascodes and their effective g_m is small. The result is,

$$i_{to}^2 = 2g_{m1}^2 e_{n1}^2\left(\frac{g_{m8}^2}{g_{m3}^2}\right) + 2g_{m8}^2 e_{n3}^2 + 2g_{m8}^2 e_{n8}^2 + + 2g_{m11}^2 e_{n10}^2$$

where we have assumed that $g_{m1}=g_{m2}$, $g_{m3}=g_{m4}$, $g_{m6}=g_{m8}$, and $g_{m10}=g_{m11}$ and $e_{n1}=e_{n2}$, $e_{n3}=e_{n4}$, $e_{n6}=e_{n8}$, and $e_{n10}=e_{n11}$. Dividing i_{to}^2 by the transconductance gain gives

$$e_{eq}^2 = \frac{i_{to}^2}{g_{m1}^2 g_{m8}^2/g_{m3}^2} = 2e_{n1}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n3}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n8}^2 + 2\left(\frac{g_{m3}^2 g_{m11}^2}{g_{m1}^2 g_{m8}^2}\right)e_{n10}^2$$

The values of the various parameters are:

$$g_{m1} = 251\mu\text{S},\ g_{m3} = 282.5\mu\text{S},\ g_{m8} = 707\mu\text{S, and } g_{m11} = 707\mu\text{S}.$$

$$\therefore \quad e_{eq}^2 = 2e_{n1}^2\left[1 + 1.266\left(\frac{e_{n3}^2}{e_{n1}^2} + \frac{e_{n8}^2}{e_{n1}^2} + \frac{e_{n10}^2}{e_{n1}^2}\right)\right]$$

Problem 7.5-5 – Continued

1/f Noise:

Using the results of Ex. 7.5-1 we get $B_N = 7.36\text{x}10^{-22}(\text{V}\cdot\text{m})^2$ and $B_p = 2.02\text{x}10^{-22}(\text{V}\cdot\text{m})^2$

$$e_{n1}^2 = \frac{B_N}{fW_1L_1} = \frac{7.36\text{x}10^{-22}}{f\cdot 12\text{x}10^{-12}} = \frac{6.133\text{x}10^{-11}}{f}\ V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N f\cdot W_3L_3} = \frac{B_P\cdot W_1L_1}{B_N W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 16} = 0.2058$$

$$\frac{e_{n8}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N f\cdot W_8L_3} = \frac{B_P\cdot W_1L_1}{B_N W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 40} = 0.0823$$

$$\frac{e_{n10}^2}{e_{n1}^2} = \frac{B_N f\cdot W_1L_1}{B_N f\cdot W_{10}L_{10}} = \frac{B_P\cdot W_1L_1}{B_N W_3L_3} = \frac{12}{18.2} = 0.6593$$

$$\therefore \qquad e_{eq}^2 = 2\,\frac{6.133\text{x}10^{-11}}{f}[1+1.266(0.2058+0.0823+0.6593)] = 2\,\frac{6.133\text{x}10^{-11}}{f}\,2.1995$$

$$e_{eq}^2 = \frac{2.1995\text{x}10^{-10}}{f}\ V^2/\text{Hz}$$

Thermal noise:

$$e_{n1}^2 = \frac{8kT}{3g_{m1}} = \frac{8\cdot 1.38\text{x}10^{-23}\cdot 300}{3\cdot 251\text{x}10^{-6}} = 4.398\text{x}10^{-17}\ V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m3}} = \frac{251}{282.4} = 0.8888 \quad \text{and} \quad \frac{e_{n8}^2}{e_{n1}^2} = \frac{e_{n10}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m8}} = \frac{251}{707} = 0.0.355$$

The corner frequency is $f_c = 2.698\text{x}10^{-10}/2.66\text{x}10^{-16} = 1.01\text{x}10^{6}$ Hz. Therefore in a 1Hz to 100kHz band, the noise is 1/*f*. Solving for the *rms* value gives,

$$V_{eq}^2\,(\text{rms}) = \int_1^{100,000} \frac{2.698\text{x}10^{-10}}{f}df = 2.698\text{x}10^{-10}[\ln(100,000) - \ln(1)]$$

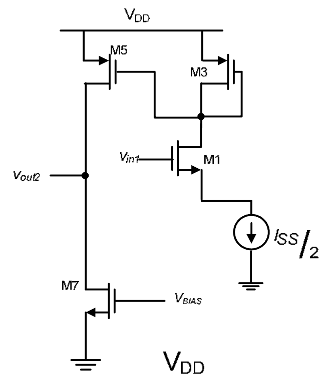
$$= 3.1062\text{x}10^{-9}\ \text{V}^2(\text{rms})$$

$$\therefore \qquad V_{eq}(\text{rms}) = \underline{55.73\mu\text{V(rms)}}$$

Problem 2 – (10 points)

Applying the half-circuit principle, it can be seen that each ½ circuit consists of a cascade of two common-source (CS) stages – the first with a diode connected PMOS load and the other with an NMOS load.

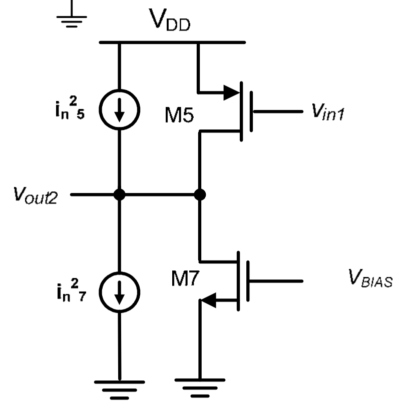
The half circuit representation is shown along-side.



The gain of the first stage is:

$$A_{v1} = G_m R_{out} = \frac{g_{m1}}{g_{m3}}$$

In general for a CS stage with an active load, the primary noise contributors can be represented as shown below (for the second CS stage in the problem). From the figure along-side, we have:



$$\overline{i}_{n5}^2 = 4kT\left(\frac{2}{3}\right)g_{m5} + \frac{K_P g_{m5}^2}{C_{OX}(WL)_5 f}$$

$$\overline{i}_{n7}^2 = 4kT\left(\frac{2}{3}\right)g_{m7} + \frac{K_N g_{m7}^2}{C_{OX}(WL)_7 f}$$

Therefore the input referred noise at the gate of M5 is given by:

$$\overline{v}_{o1}^2 = \frac{i_{n5}^2 + i_{n7}^2}{g_{m5}^2}$$

Similarly, the noise at the gate of M1 due to M1 and diode connected M3 can be expressed as:

$$\overline{v}_{in(1,3)}^2 = \frac{i_{n1}^2 + i_{n3}^2}{g_{m1}^2}$$

Where, $\overline{i}_{n1}^2 = 4kT\left(\frac{2}{3}\right)g_{m1} + \frac{K_N g_{m1}^2}{C_{OX}(WL)_1 f}$ and $\overline{i}_{n3}^2 = 4kT\left(\frac{2}{3}\right)g_{m3} + \frac{K_P g_{m3}^2}{C_{OX}(WL)_3 f}$

Therefore the total noise referred to the input (gate of M1) is:

$\overline{v}_{in}^2 = \overline{v}_{in(1,3)}^2 + \frac{v_{o1}^2}{A_{v1}^2} = \frac{\left(i_{n1}^2 + i_{n3}^2\right)}{g_{m1}^2} + \frac{\left(i_{n5}^2 + i_{n7}^2\right)\times g_{m3}^2}{g_{m5}^2 \times g_{m1}^2}$. Therefore for one half-circuit,

$$\overline{v}_{in}^2 = \frac{g_{m5}^2 \times \left(i_{n1}^2 + i_{n3}^2\right) + \left(i_{n5}^2 + i_{n7}^2\right)\times g_{m3}^2}{g_{m5}^2 \times g_{m1}^2}$$

Considering only the thermal noise, the total input referred noise is:

$$\overline{v}^{2}_{in(THERMAL)} = 2 \times \frac{g_{m5}^{2}\left[4kT\left(\frac{2}{3}\right)(g_{m1}+g_{m3})\right] + g_{m3}^{2}\left[4kT\left(\frac{2}{3}\right)(g_{m5}+g_{m7})\right]}{g_{m5}^{2} \times g_{m1}^{2}}$$

Considering only the flicker noise, the total input referred noise is:

$$\overline{v}^{2}_{in(FLICKER)} = 2 \times \frac{g_{m5}^{2}\left[\frac{K_N g_{m1}^{2}}{C_{OX}(WL)_1 f} + \frac{K_P g_{m3}^{2}}{C_{OX}(WL)_3 f}\right] + g_{m3}^{2}\left[\frac{K_P g_{m5}^{2}}{C_{OX}(WL)_5 f} + \frac{K_N g_{m7}^{2}}{C_{OX}(WL)_7 f}\right]}{g_{m5}^{2} \times g_{m1}^{2}}$$

Equating the thermal noise and flicker noise to find the flicker noise corner frequency (f_C), we have:

$$\left\{g_{m5}^{2}\left[\frac{K_N g_{m1}^{2}}{C_{OX}(WL)_1} + \frac{K_P g_{m3}^{2}}{C_{OX}(WL)_3}\right] + g_{m3}^{2}\left[\frac{K_P g_{m5}^{2}}{C_{OX}(WL)_5} + \frac{K_N g_{m7}^{2}}{C_{OX}(WL)_7}\right]\right\}\frac{1}{f_C} =$$
$$g_{m5}^{2}\left[4kT\left(\frac{2}{3}\right)(g_{m1}+g_{m3})\right] + g_{m3}^{2}\left[4kT\left(\frac{2}{3}\right)(g_{m5}+g_{m7})\right]$$

Numerical Calculations:

All transistors in saturation,
$(W/L)_{1,2} = 50/0.6$, $(W/L)_{3,4} = 10/0.6$, $(W/L)_{5,6} = 20/0.6$ and $(W/L)_{7,8} = 56/0.6$
$\mu_n C_{OX} = 75\ \mu A/V_2$ and $\mu_p C_{OX} = 30\ \mu A/V^2$ and $I_{SS} = 0.5$ mA
Therefore → $I_1 = I_2 = I_3 = I_4 = 0.25$ mA and $I_5 = I_6 = I_7 = I_8 = 0.5$mA

Using $g_m = \sqrt{2\mu C_{OX}\left(\frac{W}{L}\right)I_D}$, we obtain:
$g_{m1} = g_{m2} = 1.768$ mS
$g_{m3} = g_{m4} = 0.5$ mS
$g_{m5} = g_{m6} = 1$ mS
$g_{m7} = g_{m8} = 2.646$ mS

Using the above, we obtain, the following values for the thermal and flicker noise powers

$$\boxed{\overline{v}^{2}_{in(THERMAL)} = 2.247 \times 10^{-17}\ V^2/Hz}$$

Assuming t_{ox}= 100A°, we obtain **C_{OX} = 34.53 x 10^{-4}**. Therefore the total flicker noise is given by:

$$\boxed{\overline{v}^{2}_{in(FLICKER)} = \frac{3.2417 \times 10^{-8}}{f}\ V^2/Hz}$$

Equating the noise powers to find the flicker noise corner frequency, we obtain: **f_C = 1.44 GHz.**

Problem 3 – (10 points)

Assumptions:
- $V_{OD} = V_{GS} - V_{TH}$
- Only thermal noise of drain current considered

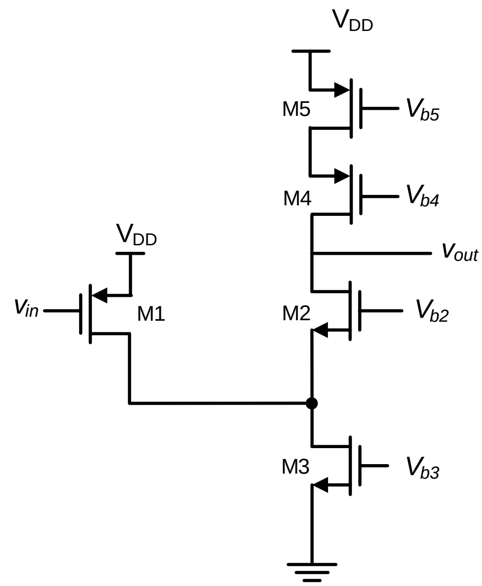
Also, we know for a MOS transistor, we have:

$$g_m = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2I_D}{V_{OD}} \quad \text{and} \quad r_o = \frac{1}{\lambda I_D}$$

Dynamic range of the circuit is defined as:

$$DR = \frac{V_{out-swing}}{V_{noise-out}}$$

where $V_{out\text{-}swing}$ is the maximum output voltage swing of the amplifier and $V_{noise\text{-}out}$ is the total output referred voltage noise.



We know for a folded cascode amplifier,

$$G_m = g_{m1}$$

and

$$R_{out} = g_{m4} r_{o4} r_{05} \| g_{m2} r_{o2} (r_{o1} \| r_{o3}),$$

which on expansion, yields

$$R_{out} = \frac{g_{m2} g_{m4} r_{o1} r_{o2} r_{o3} r_{o4} r_{o5}}{g_{m4} r_{o4} r_{o5} (r_{o1} + r_{o3}) + g_{m2} r_{o1} r_{o2} r_{o3}}$$

Since from the above, we see that ***both*** G_m and R_{out} are dependent on the over-drive voltage, we need to consider the effect of variation of V_{OD} on both. Substituting for g_m and R_{out} in terms of V_{OD}, we obtain the following expressions for G_m and R_{out}

$$G_m = g_{m1} = \frac{2I_D}{V_{OD}}$$

$$Rout = \frac{2}{5} \times \frac{1}{V_{OD} \lambda^2 I_D}$$

Expression for output swing in terms of the over-drive voltage:

Output swing of a folded cascode amplifier:

$$V_{out-swing} = V_{DD} - 4V_{OD}$$

Expression for the output referred noise as a function of over-drive voltage:

The major noise contributors in the folded cascode amplifier are: M1, M3 and M5. Therefore we first obtain the noise contributions of each of these noise sources at the output.

$$\overline{v}^2_{n-out(1)} = \left[4kT\left(\frac{2}{3g_{m1}}\right)\right]\times g^2_{m1}R^2_{out}$$

$$\overline{v}^2_{n-out(3)} = \left[4kT\left(\frac{2}{3g_{m3}}\right)\right]\times g^2_{m3}R^2_{out}$$

$$\overline{v}^2_{n-out(5)} = \left[4kT\left(\frac{2}{3g_{m5}}\right)\right]\times g^2_{m5}R^2_{out}$$

Therefore taking the superposition of these noise sources, we have the output referred noise given by:

$$\overline{v}^2_{n-out} = 4kT\left(\frac{2}{3}\right)\times\left[g_{m1}+g_{m3}+g_{m5}\right]\times R^2_{out}$$

Therefore substituting expressions for g_m and R_{out} into the above, we obtain the total output referred noise power as:

$$\overline{v}^2_{n-out} = 4kT\times\left(\frac{2}{3}\right)\times\left(\frac{32}{25}\right)\times\frac{1}{V^3_{OD}\lambda^4 I_D}$$

Therefore the output referred noise voltage can be expressed as a function of V_{OD} as:

$$v_{n-out} = B\times\frac{1}{(V_{OD})^{3/2}}$$

<u>Dynamic Range calculations:</u>

Initial expression for the dynamic range (DR_1):

$$DR_1 = \frac{(V_{DD}-4V_{OD})(V_{OD})^{3/2}}{B}$$

After the over-drive voltage changes to 75% of its original value, the new dynamic range (DR_2) is given by:

$$DR_2 = \frac{(V_{DD}-3V_{OD})\left(\frac{3}{4}V_{OD}\right)^{3/2}}{B}$$

Therefore, finding the difference between the initial and final dynamic ranges, we can find the variation in the dynamic range caused by 25% reduction in V_{OD}

$$\Delta DR = DR_1 - DR_2 = \frac{(V_{OD})^{3/2}}{B}\left[\frac{V_{DD}-7V_{OD}}{4}\right]$$

Problem 4 – (10 points)

Problem 7.6-1 - If the W and L of all transistor in Fig. 7.6-3 are 100μm and 1μm, respectively, find the lowest supply voltage that gives a zero value of *ICMR* if the dc current in M5 is 100μA.

Solution:

$$I_5 = 100\ \ \mu A, \text{ and } \left(\frac{W}{L}\right) = 100$$

$$V_{IC}(\max) = V_{DD} + V_{T1}(\min) - V_{dsat3}$$

and, $$V_{IC}(\min) = V_{dsat1} + V_{T1}(\max) + V_{dsat5}$$

The input common-mode range is

$$ICMR = V_{IC}(\max) - V_{IC}(\min)$$

For ICMR=0

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\max) - V_{T1}(\min)$$

or, $$V_{DD} = \sqrt{\frac{2I_1}{K_N^{'}S_1}} + \sqrt{\frac{2I_5}{K_N^{'}S_5}} + \sqrt{\frac{2I_3}{K_P^{'}S_3}} + V_{T1}(\max) - V_{T1}(\min)$$

or, $$V_{DD} = \underline{0.671\text{V}}$$

Problem 5 – (10 points)

Problem 7.4-3 - Derive Eq. (17). If A = 2, at what value of v_{in}/nV_t will $i_{out} = 5I_5$ or $5I_b$ if b=1 ?

Solution

Start with the following relationships:

$$i_1 + i_2 = I_5 + A(i_2 - i_1) \qquad \text{Eq. (15)}$$

and $$\frac{i_2}{i_1} = \exp\left(\frac{v_{in}}{nV_t}\right) \qquad \text{Eq. (16)}$$

Defining $i_{out} = b(i_2 - i_1)$ solve for i_2 and I_1.

$$i_1 + i_1 \exp\left(\frac{v_{in}}{nV_t}\right) = I_5 + Ai_1 \exp\left(\frac{v_{in}}{nV_t}\right) - Ai_1$$

or $$i_1[(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)] = I_5 \quad \rightarrow \quad i_1 = \frac{I_5}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

Similarly for i_2,

$$i_1 = \frac{I_5 \exp\left(\frac{v_{in}}{nV_t}\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

$$\therefore \qquad i_{out} = b(i_2 - I_1) = i_{out} = (i_2 - I_1) = \frac{I_5\left(\exp\left(\frac{v_{in}}{nV_t}\right) - 1\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)} \qquad \text{Eq. (17)}$$

Setting $i_{out} = 5I_5$ and solving for $\frac{v_{in}}{nV_t}$ gives,

$$5[3 - \exp\left(\frac{v_{in}}{nV_t}\right)] = \exp\left(\frac{v_{in}}{nV_t}\right) - 1 \quad \rightarrow \quad 16 = 6\exp\left(\frac{v_{in}}{nV_t}\right) \quad \rightarrow \quad \exp\left(\frac{v_{in}}{nV_t}\right) = 2.667$$

$$\therefore \qquad \frac{v_{in}}{nV_t} = \ln(2.667) = \underline{0.9808}$$