Homework Assignment No. 9 Solutions

Problem 1 – (10 points)

8.1(a)

$a = 100{,}000$

$f = 10^{-3}$

$$A = \frac{a}{1+af} = \frac{10^5}{1+10^2} = 990.1$$

$$\frac{dA}{da} = \frac{1}{(1+af)^2}$$

$$da = \frac{1}{10} \times 10^5 = 10^4$$

$$dA = \frac{10^4}{(1+10^5 \times 10^{-3})^2} = 0.98 \rightarrow \frac{dA}{A} = \frac{0.98}{990.1} = \% 0.099$$

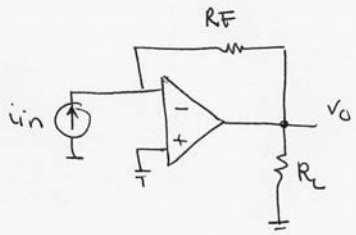
8.1(b) $f = 0.1$

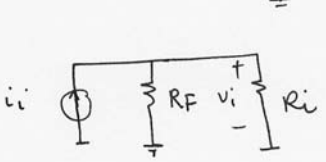
$$\frac{\delta A}{A} = \frac{\frac{\delta a}{a}}{1+af} = \frac{0.1}{1+10^5 \times 0.1} = 9.99 \times 10^{-4} \%$$

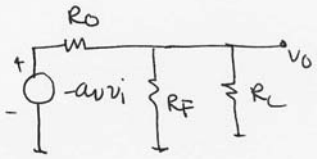
Problem 2 – (10 points)

8.6a

$R_F = 150k$
$R_L = 110k$
$R_i = 800k$
$R_o = 200\Omega$
$a_v = 75{,}000$







$f = \frac{-1}{R_f}$

$a = \frac{v_o}{i_{in}} = \underbrace{\frac{-R_F R_i}{R_F + R_i}} \times a_v \times \frac{R_L \| R_F}{(R_L \| R_F) + R_o}$

$= \underbrace{-(150k \| 800k)}_{126.3k} \times 75 \times 10^3 \times \frac{(110k \| 150k)}{\underbrace{(110k \| 150k)}_{63k} + 200\Omega} = -9.4 G\Omega$

$\rightarrow T = af = 63{,}000$

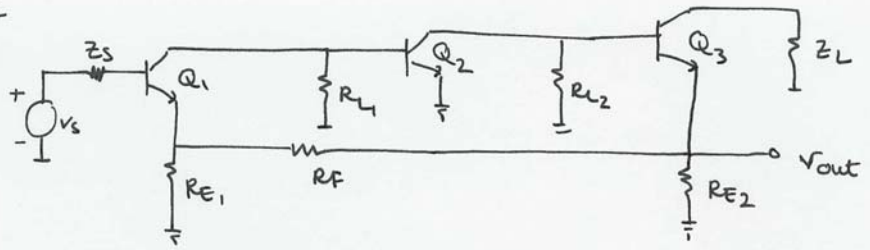
$Z_{in} = \frac{Z_i}{1+af} = \frac{R_F \| R_i}{63000} = 2\Omega$

$Z_{out} = \frac{Z_o}{1+af} = \frac{R_o \| R_F \| R_L}{63000} = 0.013$

$A = \frac{a}{1+af} = \frac{-9.4G}{63000} = -149 k\Omega \simeq -R_F$

Problem 3 – (10 points)

8.12



Z_{in} , Z_{out} , A , T

$R_{E1} = R_{E2} = 290\,\Omega$

$R_F = 1.9\,k\Omega$

$R_{L1} = 10.6\,k\Omega$

$R_{L2} = 6\,k$

$I_{C1} = 0.5\,mA$

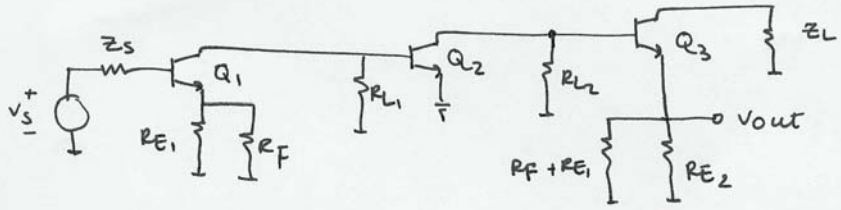
$I_{C2} = 0.77\,mA$

$I_{C3} = 0.73\,mA$

$\beta = 120$, $V_A = 40\,V$

$r_b = 0$

Series – Shunt feed back



$$\frac{V_o}{V_s} = A = \frac{a}{1+af}$$

$$f = \frac{R_{E1}}{R_{E1}+R_F} = \frac{0.29}{1.9+0.29} = 0.132$$

$$\frac{V_{C1}}{V_s} = \frac{-g_{m1} \times \left(R_{L1} \| r_{\pi 2} \| r_{o1}(1+g_{m1}(R_{E1} \| R_F))\right)}{1+g_{m1}(R_{E1} \| R_F)} \times \frac{r_{\pi 1}(1+g_{m1}(R_E \| R_F))}{r_{\pi 1}(1+g_{m1}(R_E \| R_E)) + Z_S}$$

ignoring $Z_S \rightarrow \frac{V_{C1}}{V_s} = \frac{-g_{m1} R_{out1}}{1+g_{m1}(R_{E1} \| R_F)} = -9.4$ 1

$g_{m1} = \frac{0.5mA}{25mV} = 0.02$ ℧

$R_{E1} \| R_F = 251\,\Omega$

$R_{L2} = 3.896k$

$R_{out1} = 2.83k$

$r_{o1} = 80k$

8.12 cont.

$$\frac{V_{C2}}{V_{C1}} = \left[R_{L2} \| r_{O2} \| r_{\pi 3}\left(1 + g_{m3}\left[R_{E2} \| (R_F + R_{E1})\right]\right)\right] \times (-g_{m2})$$

$g_{m2} = 0.031$

$r_{O2} = 51.95\,k\Omega$

$r_{\pi 3} = 4.1\,k\Omega$

$g_{m3} = 0.029$

$R_{E2} \| (R_F + R_{E1}) = 256\,\Omega$

$$\rightarrow \frac{V_{C2}}{V_{C1}} = -4.64k \times 0.031 = -143.8$$

$$\frac{V_{out}}{V_{C2}} = \frac{g_{m3} \times 256}{1 + g_{m3} 256} = 0.88$$

$$\rightarrow \frac{V_{out}}{V_S} = (-9.4)(-143.8)(0.88) = \underline{1190} = a$$

$$\rightarrow A = \frac{a}{1+af} = \frac{1190}{1 + \frac{1190 \times 0.29}{2.19}} = \underline{7.52} \qquad T = 15\cancel{7}$$

$$Z_{in} = \frac{Z_i}{(1+af)^{-1}} = Z_i(1+af)$$

$$Z_i = r_{\pi 1}\left[1 + \underbrace{g_{m1}(R_{E1} \| R_F)}_{5.02}\right] = 6k \times 6.02 = 36.12\,k$$

$$\rightarrow Z_{in} = 36.12k \times 15\cancel{7} = \underline{5.67}\,M\Omega$$

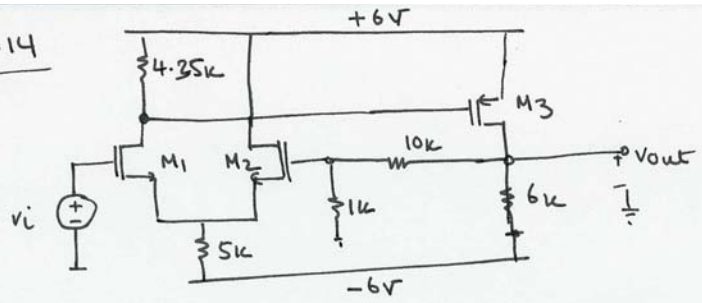
$$Z_{out} = \frac{Z_O}{1+af}$$

$$Z_O = \frac{1}{g_{m3}} \| 256\Omega = 30.2\,\Omega$$

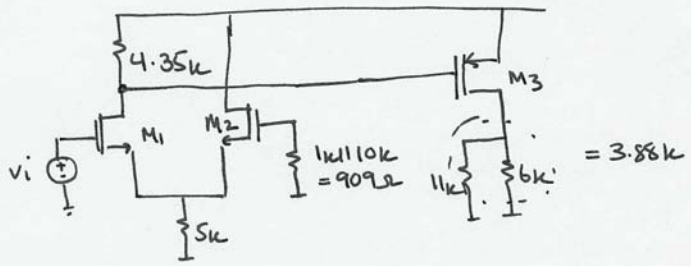
$$\rightarrow Z_{out} = \frac{30.2}{15\cancel{7}} = 0.19\,\Omega$$

Problem 4 – (10 points)

8.14



Series - shunt



$I_{D1} = \frac{V_{SG_3}}{4.35K}$

$I_{D3} = \frac{6 - V_{olDC}}{6K} = 1mA$

$I_{D3} = \frac{K'_P}{2}\left(\frac{W}{L}\right)_3 (|V_{GS}| - |V_{TP}|)^2 \rightarrow |V_{GS3}| = |V_T| = 2V$

$\rightarrow I_{D1} = 0.459\ mA$

$$\begin{cases} 5K \times (I_{D2} + I_{D1}) = 6 - V_{GS1} \\ V_{GS1} = 1.39V \end{cases} \rightarrow I_{D1} + I_{D2} = 0.92 \rightarrow I_{D2} = 0.46 \rightarrow \begin{cases} I_{D1} = 0.46\ mA \\ I_{D2} = 0.46\ mA \end{cases}$$

$a = \frac{V_o}{V_i} = A_{V1} \times A_{V2}$

$A_{V1} = \frac{V_{D1}}{V_i} = -4.35K \times \frac{g_{m1}}{1 + g_{m1}(5K \| \underbrace{\frac{1}{g_{m2}}}_{425\Omega})} = -5.32$

$g_{m1} = \sqrt{2K'_N \frac{W}{L} I_{D1}} = 2.35\ mS$

$g_{m2} = \sqrt{2K'_N \frac{W}{L} I_{D2}} = g_{m1} = 2.35$

8.14
Cont.

$$A_{V_2} = \frac{V_O}{V_{D_1}} = -g_{m3} \times 3.88k = 7.76$$

$$g_{m3} = \sqrt{2k'_p \times \frac{W}{L} \times I_{D3}} = 2mS$$

$$\rightarrow a = 7.76 \times 5.32 = 41.27$$

$$f = \frac{1}{1+10} = \frac{1}{11}$$

$$T = af = 3.75$$

$$\rightarrow \frac{V_O}{V_i} = A = \frac{41.27}{1+3.75} = 8.68$$

$$Z_{in} = \frac{Z_i}{(1+T)^{-1}} = Z_i(1+T)$$

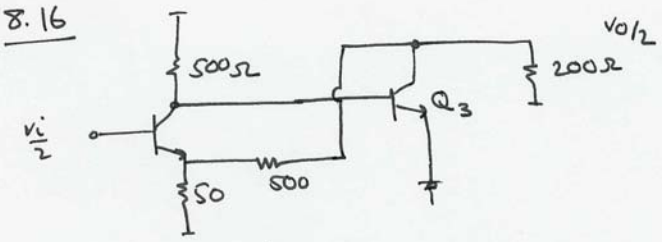
$$Z_i = \infty \rightarrow Z_{in} = \infty$$

$$Z_{out} = \frac{Z_O}{1+T} \qquad \rightarrow Z_{out} = 816.8\ \Omega$$

$$Z_O = 3.88k$$

Problem 5 – (10 points)

8.16



$$f = \frac{50}{50+500} = \frac{1}{11}$$

$$a = \frac{g_{m1}}{1+g_{m1} \times 45\Omega} \times (500\Omega \,||\, r_{\pi 3}) \times g_{m3} \times (200\Omega \,||\, 550\Omega)$$

$45\Omega \downarrow 50||500$

$T = af$

(a) if the 50Ω resistor increases → f increases
a slightly decreases (if g_m is not too large)
→ T: increases (if g_m is too large: T stays almost constant)

(b) if 500Ω feed back resistor increases → f decreases
a slightly decreases
→ T: decreases

(c) if 200 resistor increases → f: unchanged
a: increases
→ T: increases

Problem 6 – (10 points)

8.23

$$\frac{V_{DD}-V_{GS2}}{10k} = I_{D2} = \frac{30\mu}{2}20(V_{GS2}-0.8)^2$$

$$5-a = 3(a-0.8)^2$$

$$= 3a^2 - 4.8a + 1.92$$

$$0 = 3a^2 - 3.8a - 3.08$$

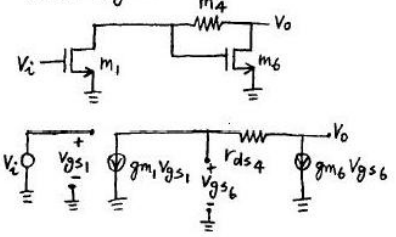
$$a = \frac{+3.8 \pm \sqrt{3.8^2 - 4(3)(-3.08)}}{6}$$

$$V_{GS2} = 1.83\,V$$

$$I_{D2} = \frac{30\mu}{2}20(1.83-0.8)^2$$

$$= 317\mu A = I_{D1} = I_{D5} = I_{D6}$$

small signal



$$V_o = V_{gs6} - g_{m6}V_{gs6}r_{ds4}$$

$$g_{m1} = g_{m6}$$

$$-g_{m1}V_{gs1} = g_{m6}V_{gs6}$$

$$-V_{gs1} = V_{gs6} = -V_i$$

$$V_o = -V_i + g_{m6}V_i r_{ds4}$$

$$\frac{V_o}{V_i} = -1 + g_{m6}r_{ds4}$$

$$g_{m6} = \sqrt{2(317\mu)(60\mu)(20)}$$

$$= 872\,\mu A/V$$

$$g_{ds4} = \mu_n C_{ox}\frac{W}{L}(V_{GS4}-V_{t4})$$

$$= 150\mu(V_c - 1.53 - 1.14)$$

$$= 150\mu(V_c - 2.67)$$

$$V_{t4} = V_{t_o} + \gamma\left(\sqrt{V_{SB_4}+2\phi_f} - \sqrt{2\phi_f}\right)$$

$$= 0.8 + 0.5(\sqrt{1.53+0.6} - \sqrt{0.6})$$

$$= 1.14\,V$$

$$V_{SB_4} = V_{GS6} = V_{t_6} + \sqrt{\frac{2I_{D6}}{\mu C_{ox}\frac{W}{L}_6}}$$

$$= 0.8 + \sqrt{\frac{2(317\mu)}{60\mu(20)}} = 1.53\,V$$

$V_c = 3V$

$$g_{ds4} = 150\mu(3-2.67)$$

$$= 49.5\mu$$

$$r_{ds4} = 20.2\,k$$

$$\frac{V_o}{V_i} = -1 + (872\mu)(20.2k) = 16.6$$

$V_c = 4\,V$

$$g_{ds4} = 150\mu(4-2.67)$$

$$= 200\mu$$

$$r_{ds4} = 5.01\,k$$

$$\frac{V_o}{V_i} = -1 + (872\mu)(5.01k)$$

$$= 3.37$$

$$R_o = \frac{1}{g_{m6}} = \frac{1}{872\mu} = 1.15\,k\Omega$$

Problem 7 – (10 points)

8.24

$I_{D_1} = I_{D_2} = \frac{1}{2} I_{D_6} = 100\mu A = I_{D_3}$

$I_{D_8} = 200\mu A$

$a = \frac{1}{2} g_{m_1} (2r_{o_2} \| r_{o_3}) \frac{g_{m8}(21k)}{1 + g_{m8}(21k)}$

$= \frac{1}{2}(693\mu)(222k)(0.954)$

$= 73.4$

$g_{m_1} = \sqrt{2(100\mu)(60\mu)40}$

$= 693\mu A/V$

$r_{o_2} = r_{o_3} = \frac{1}{\lambda I_D} = \frac{1}{0.03(100\mu)}$

$= 333k$

$g_{m8} = \sqrt{2(200\mu)(60\mu)40}$

$= 980\mu A/V$

$f = \frac{1k}{1k + 20k} = \frac{1}{21}$

$T = af = 3.5$

$A = \frac{a}{1+T} = \frac{73.4}{4.5} = 16.3 = \frac{V_o}{V_i}$

$R_o = \frac{r_{out}}{1+T} = \frac{973}{4.5}$

$= 216\Omega$

$r_{out} = 21k \| \frac{1}{g_{m8}}$

$= 21k \| 1.02k$

$= 973\Omega$

```
VARIABLE GAIN CMOS AMP
 VDD 1 0 5V
 M1 3 4 0 0 NMOS W=20U L=1U
 M2 2 2 1 1 PMOS W=20U L=1U
 R1 2 0 10K
 M3 3 2 1 1 PMOS W=20U L=1U
 M5 5 2 1 1 PMOS W=20U L=1U
 M6 5 3 0 0 NMOS W=20U L=1U
 M4 5 6 3 0 NMOS W=5U L=2U
 VC 6 0 3V
 VI 4 0 1.527V
 .DC VI 1.47 1.6 0.005
 .PLOT DC V(5)
 .TF V(5) VI
 .NODESET V(2)=3.17V V(3)=1.53V V(5)=1.53V
 .MODEL NMOS NMOS KP=60U VTO=0.8 GAMMA=0.5
 .MODEL PMOS PMOS KP=30U VTO=-0.8 GAMMA=0.5
 .OPTIONS NOPAGE NOMOD
 .WIDTH OUT=80
 .OP
 .END
0****    DC TRANSFER CURVES
  VI        V(5)
            1.0D+00   2.0D+00   3.0D+00   4.0D+00  5.0D+00
            ----------------------------------------
1.470D+00  1.065D+00 .*
1.475D+00  1.092D+00 .*
1.480D+00  1.119D+00 .*
1.485D+00  1.149D+00 .*
1.490D+00  1.180D+00 .*
1.495D+00  1.213D+00 .*
1.500D+00  1.248D+00 .*
1.505D+00  1.287D+00 .*
1.510D+00  1.329D+00 .*
1.515D+00  1.376D+00 .*
1.520D+00  1.430D+00 .*
1.525D+00  1.495D+00 .*
1.530D+00  1.580D+00 .*
1.535D+00  1.707D+00 .*
1.540D+00  4.292D+00 .                                  *
1.545D+00  4.463D+00 .                                   *
1.550D+00  4.563D+00 .                                    *
1.555D+00  4.632D+00 .                                    *
1.560D+00  4.683D+00 .                                    *
1.565D+00  4.722D+00 .                                    *
1.570D+00  4.753D+00 .                                    *
1.575D+00  4.777D+00 .                                    *
1.580D+00  4.796D+00 .                                    *
1.585D+00  4.811D+00 .                                    *
1.590D+00  4.822D+00 .                                    *
1.595D+00  4.830D+00 .                                    *
1.600D+00  4.836D+00 .                                    *
            ----------------------------------------

0****    SMALL SIGNAL BIAS SOLUTION
 NODE   VOLTAGE
( 1)  5.0000  ( 2)  3.1718  ( 3)  1.5271  ( 4)  1.5270
( 5)  1.5259  ( 6)  3.0000

0**** MOSFETS
0            M1         M2         M3         M5         M6         M4
0MODEL     NMOS       PMOS       PMOS       PMOS       NMOS       NMOS
 ID     3.17E-04  -3.17E-04  -3.17E-04  -3.17E-04   3.17E-04  -5.96E-08
 VGS       1.527     -1.828     -1.828     -1.828      1.527      1.473
 VDS       1.527     -1.828     -3.473     -3.474      1.526     -0.001
 VBS       0.000      0.000      0.000      0.000      0.000     -1.527
 VTH       0.800     -0.800     -0.800     -0.800      0.800      1.142
 VDSAT     0.727     -1.028     -1.028     -1.028      0.727      0.332
 GM     8.72E-04   6.17E-04   6.17E-04   6.17E-04   8.73E-04   1.80E-07
 GDS    0.00E+0    0.00E+0    0.00E+0    0.00E+0    0.00E+0    4.97E-05
 GMB    2.82E-04   1.99E-04   1.99E-04   1.99E-04   2.82E-04   3.08E-08

0****    SMALL-SIGNAL CHARACTERISTICS
0    V(5)/VI                          = 1.649D+01
0    INPUT RESISTANCE AT VI           = 1.000D+20
0    OUTPUT RESISTANCE AT V(5)        = 1.141D+03
```

8.24 - Cont'd

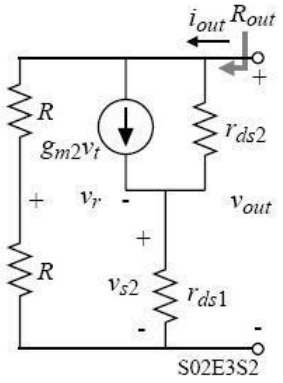
```
VARIABLE GAIN CMOS AMP
 VDD 1 0 5V
 M1 3 4 0 0 NMOS W=20U L=1U
 M2 2 2 1 1 PMOS W=20U L=1U
 R1 2 0 10K
 M3 3 2 1 1 PMOS W=20U L=1U
 M5 5 2 1 1 PMOS W=20U L=1U
 M6 5 3 0 0 NMOS W=20U L=1U
 M4 5 6 3 0 NMOS W=5U L=2U
 VC 6 0 4V
 VI 4 0 1.527V
 .DC VI 1.0 1.8 0.05
 .PLOT DC V(5)
 .TF V(5) VI
 .NODESET V(2)=3.17V V(3)=1.53V V(5)=1.53V
 .MODEL NMOS NMOS KP=60U VTO=0.8 GAMMA=0.5
 .MODEL PMOS PMOS KP=30U VTO=-0.8 GAMMA=0.5
 .OPTIONS NOPAGE NOMOD
 .WIDTH OUT=80
 .OP
 .END
0****     DC TRANSFER CURVES
  VI        V(5)
            0.0D+00    2.0D+00    4.0D+00    6.0D+00  8.0D+00
            ---------------------------------
1.000D+00 7.255D-01 .  *          .          .          .          .
1.050D+00 7.496D-01 .  *          .          .          .          .
1.100D+00 7.806D-01 .  *          .          .          .          .
1.150D+00 8.197D-01 .   *         .          .          .          .
1.200D+00 8.687D-01 .   *         .          .          .          .
1.250D+00 9.299D-01 .    *        .          .          .          .
1.300D+00 1.003D+00 .     *       .          .          .          .
1.350D+00 1.089D+00 .      *      .          .          .          .
1.400D+00 1.188D+00 .      *      .          .          .          .
1.450D+00 1.303D+00 .       *     .          .          .          .
1.500D+00 1.441D+00 .        *    .          .          .          .
1.550D+00 1.607D+00 .         *   .          .          .          .
1.600D+00 1.803D+00 .          *  .          .          .          .
1.650D+00 2.038D+00 .           * .          .          .          .
1.700D+00 2.339D+00 .            *.          .          .          .
1.750D+00 4.054D+00 .             .          *          .          .
1.800D+00 4.132D+00 .             .          .*         .          .
            ---------------------------------

0****     SMALL SIGNAL BIAS SOLUTION
 NODE   VOLTAGE
( 1)  5.0000  ( 2)  3.1718  ( 3)  1.5271  ( 4)  1.5270
( 5)  1.5268  ( 6)  4.0000

0**** MOSFETS
0          M1        M2        M3        M5        M6        M4
0MODEL    NMOS      PMOS      PMOS      PMOS      NMOS      NMOS
 ID     3.17E-04 -3.17E-04 -3.17E-04 -3.17E-04  3.17E-04 -5.96E-08
 VGS       1.527    -1.828    -1.828    -1.828     1.527     2.473
 VDS       1.527    -1.828    -3.473    -3.473     1.527     0.000
 VBS       0.000     0.000     0.000     0.000     0.000    -1.527
 VTH       0.800    -0.800    -0.800    -0.800     0.800     1.142
 VDSAT     0.727    -1.028    -1.028    -1.028     0.727     1.331
 GM     8.72E-04  6.17E-04  6.17E-04  6.17E-04  8.73E-04  4.48E-08
 GDS    0.00E+0   0.00E+0   0.00E+0   0.00E+0   0.00E+0   2.00E-04
 GMB    2.82E-04  1.99E-04  1.99E-04  1.99E-04  2.82E-04  7.68E-09

0****     SMALL-SIGNAL CHARACTERISTICS
0    V(5)/VI                    =  3.369D+00
0    INPUT RESISTANCE AT VI             =  1.000D+20
0    OUTPUT RESISTANCE AT V(5)          =  1.146D+03
```

```
CMOS FEEDBACK AMP
 VDD 1 0 5V
 VSS 9 0 -5V
 M4 2 2 1 1 PMOS W=20U L=1U
 I1 2 3 100UA
 M5 3 3 9 9 NMOS W=20U L=1U
 M6 6 3 9 9 NMOS W=40U L=1U
 M1 1 4 6 9 NMOS W=40U L=1U
 M2 5 7 6 9 NMOS W=40U L=1U
 M3 5 2 1 1 PMOS W=20U L=1U
 M8 1 5 8 9 NMOS W=40U L=1U
 M7 8 3 9 9 NMOS W=40U L=1U
 RF 8 7 20K
 RE 7 0 1K
 VI 4 0 0V
 .TF V(8) VI
 .MODEL NMOS NMOS KP=60U VTO=0.8 LAMBDA=0.03
 .MODEL PMOS PMOS KP=30U VTO=-0.8 LAMBDA=0.03
 .OPTIONS NOPAGE NOMOD
 .WIDTH OUT=80
 .OP
 .END
0****     SMALL SIGNAL BIAS SOLUTION
 NODE   VOLTAGE
( 1)  5.0000  ( 2)  3.6341  ( 3)  -3.7989  ( 4)  -0.0000
( 5)  1.4134  ( 6)  -1.0776  ( 7)  0.0096  ( 8)  0.2016
( 9)  -5.0000

0**** MOSFETS
0          M4        M5        M6        M1        M2        M3        M8
0MODEL    PMOS      NMOS      NMOS      NMOS      NMOS      PMOS      NMOS
 ID     -1.00E-04  1.00E-04  2.16E-04  1.09E-04  1.06E-04 -1.06E-04  2.33E-04
 VGS       -1.366     1.201     1.201     1.078     1.087    -1.366     1.212
 VDS       -1.366     1.201     3.922     6.078     2.491    -3.587     4.798
 VBS        0.000     0.000     0.000    -3.922    -3.922     0.000     5.202
 VTH       -0.800     0.800     0.800     0.800     0.800    -0.800     0.800
 VDSAT     -0.566     0.401     0.401     0.278     0.287    -0.566     0.412
 GM     3.54E-04  4.99E-04  1.08E-03  7.88E-04  7.41E-04  3.76E-04  1.13E-03
 GDS    2.88E-06  2.90E-06  5.79E-06  2.77E-06  2.97E-06  2.88E-06  6.10E-06

0          M7
0MODEL    NMOS
 ID     2.23E-04
 VGS       1.201
 VDS       5.202
 VBS       0.000
 VTH       0.800
 VDSAT     0.401
 GM     1.11E-03
 GDS    5.79E-06

0****     SMALL-SIGNAL CHARACTERISTICS
0    V(8)/VI                    =  1.662D+01
0    INPUT RESISTANCE AT VI             =  1.000D+20
0    OUTPUT RESISTANCE AT V(8)          =  1.707D+02
```

Problem 8 – (10 points)

Solution

$$R_{out}\,(g_{m2}=0) = 2R\|(r_{ds1}+r_{ds2}) = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}$$



RR(port shorted) = ?

$$v_r = 0 - v_{s2} = -\,g_{m2}v_t\left(\frac{r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}\right)$$

$$\Rightarrow \qquad RR(\text{port shorted}) = \frac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}$$

RR(port open) = ?

$$v_r = -\,g_{m2}v_t\left(\frac{r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}\right)$$

$$\Rightarrow \qquad RR(\text{port open}) = \frac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}$$

$$\therefore \qquad R_{out} = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}\left[\frac{1+\dfrac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}}{1+\dfrac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}}\right] = 2R\left(\frac{r_{ds1}+r_{ds2}+g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}+2R+g_{m2}r_{ds2}(r_{ds1}+R)}\right)$$

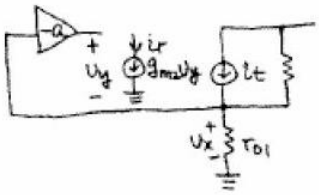
Using the assumptions of $g_m > g_{ds} > (1/R)$ we can simplify R_{out} as

$$R_{out} \approx 2R\left(\frac{g_{m2}r_{ds1}r_{ds2}}{g_{m2}r_{ds2}R}\right) = \underline{\underline{2r_{ds1}}}$$

What is the insight? Well there are two feedback loops, one a series (the normal cascode) and one a shunt. Apparently they are working against each other and the effective output resistance is pretty much what it would be if there were two transistors in series without any feedback.

Problem 9– (10 points)

(a) g_{m2} is the controlled source.



$R_{out}(g_{m2}=0) = r_{o1}+r_{o2}$

$R(\text{short}) = (r_{o1}\|r_{o2})(a+1)g_{m2}$

$R(\text{open}) = 0$

$$R_{out} = R_{out}(g_{m2}=0)\frac{1+R(\text{short})}{1+R(\text{open})}$$

$$= (r_{o1}+r_{o2})\frac{1+(r_{o1}\|r_{o2})(a+1)g_{m2}}{1+0}$$

$$= r_{o1}+r_{o2}+(a+1)g_{m2}r_{o1}r_{o2}$$

$$\approx a g_{m2} r_{o1} r_{o2}$$

$$\frac{v_x}{r_{o1}} = g_{m2}v_x + \frac{v_x - v}{r_{o2}}$$

$$R_{out}(a=0) = \frac{v}{i} = \frac{v}{v_x/r_{o1}}$$

$$= g_{m2}r_{o1}r_{o2} + r_{o1} - r_{o2}$$

$$\approx g_{m2}r_{o1}r_{o2}$$

The output is short

$$v_x = g_{m2}(v_t - v_x)(r_{o1}\|r_{o2})$$

$$v_x = \frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}v_t$$

$$R(\text{short}) = a\frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}$$

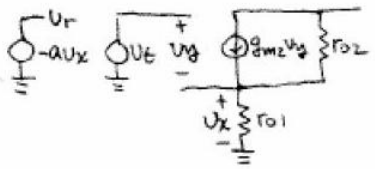
$R(\text{open}) = 0$ ($v_x = 0$ when the output is open)

$$R_{out} = R_{out}(a=0)\frac{1+R(\text{short})}{1+R(\text{open})}$$

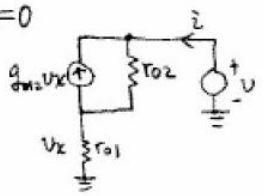
$$\approx g_{m2}r_{o1}r_{o2}\frac{1+a\frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}}{1+0}$$

$$\approx a g_{m2} r_{o1} r_{o2}$$

(b) a is the controlled source.



$a=0$



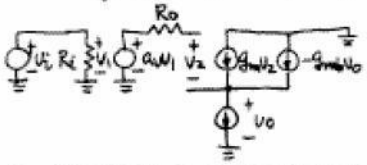
(c) The results are the same, as they should be, even though the terms $R_{out}(k=0)$, $R(\text{open})$, and $R(\text{short})$ differ in (a) and (b).

Problem 10– (10 points)

8.30

(a)

The basic amplifier without the feedback signal inserted at the inverting input of the opamp



$$g_m=\sqrt{2k'\frac{W}{L}I_D}=\sqrt{2\times 80\times10^{-6}\times100\times0.5\times10^{-3}}$$
$$=4.2\times10^{-3}\,A/V$$

$$g_{mb}=\frac{\gamma}{2\sqrt{2\phi_f+V_{SB}}}g_m=\frac{\gamma}{2\sqrt{2\phi_f}}g_m$$
$$=\frac{0.3}{2\sqrt{2\times0.3}}4.2\times10^{-3}=8.1\times10^{-4}\,A/V$$

$$V_o=g_m(a_vV_i-V_o)\frac{1}{g_{mb}}$$

$$a=\frac{V_o}{V_i}=a_v\frac{g_m}{g_m+g_{mb}}$$

$$f=1$$

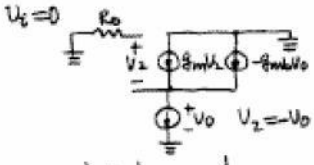
$$af=a_v\frac{g_m}{g_m+g_{mb}}=1000\frac{4.2}{4.2+0.81}=838$$

$$A=\frac{a}{1+af}=\frac{838}{1+838}=0.999$$

$$r_{ia}=R_i$$

$$R_{in}=r_{ia}(1+af)=R_i(1+af)$$
$$=1M(1+838)=839\,M\Omega$$

$V_i=0$

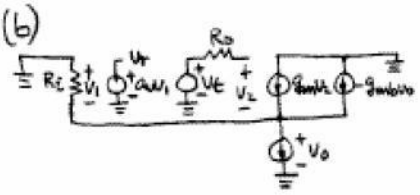


$$r_{oa}=\frac{1}{g_m}\|\frac{1}{g_{mb}}=\frac{1}{g_m+g_{mb}}$$

$$R_{out}=\frac{r_{oa}}{1+af}=\frac{1}{g_m+g_{mb}}\frac{1}{1+af}$$

$$=\frac{1}{4.2\times10^{-3}+8.1\times10^{-4}}\frac{1}{1+838}$$
$$=0.238\,\Omega$$

(b)



$$V_o=g_m(V_t-V_o)\left(\frac{1}{g_{mb}}\|R_i\right)$$

$$V_o=\frac{g_m}{\frac{1}{R_i}+g_m+g_{mb}}$$

$$R=a_v\frac{g_m}{\frac{1}{R_i}+g_m+g_{mb}}$$
$$\approx a_v\frac{g_m}{g_m+g_{mb}}$$
$$=838$$

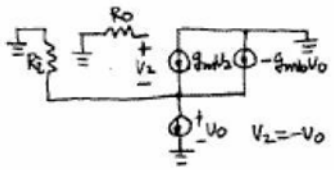
$$A_{oo}=\frac{V_o}{V_i}\Big|_{a_v=\infty}=1\quad(V_1=0\text{ and }V_o=V_i)$$

$$d=\frac{V_o}{V_i}\Big|_{a_v=0}=\frac{\frac{1}{g_m}\|\frac{1}{g_{mb}}}{R_i+\frac{1}{g_m}\|\frac{1}{g_{mb}}}$$
$$=\frac{\frac{1}{g_m+g_{mb}}}{R_i+\frac{1}{g_m+g_{mb}}}$$
$$\approx\frac{1}{(g_m+g_{mb})R_i}$$
$$=\frac{1}{(4.2\times10^{-3}+8.1\times10^{-4})10^6}$$
$$=2.00\times10^{-4}$$

$$A=A_{oo}\frac{R}{1+R}+\frac{d}{1+R}$$
$$=1\,\frac{838}{1+838}+\frac{2.00\times10^{-4}}{1+838}$$
$$=0.999$$

8-30 cont.

$a_v = 0$



$$R_{in}(a_v=0) = R_i + \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx R_i = 1M\Omega$$

$$R(\text{short}) = R = 838$$

$$R(\text{open}) = 0 \quad (v_1 = 0)$$

$$R_{in} = R_{in}(a_v=0)\frac{1+R(\text{short})}{1+R(\text{open})} \approx R_i \frac{1+R}{1+0}$$

$$= R_i(1+R) = 1M(1+838) = 839M\Omega$$

$$R_{out}(a_v=0) = R_i \| \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx \frac{1}{g_m} \| \frac{1}{g_{mb}}$$

$$= \frac{1}{g_m + g_{mb}} = \frac{1}{4.2\times10^{-3} + 8.1\times10^{-4}} = 200\Omega$$

$$R(\text{short}) = 0 \quad (v_o = 0)$$

$$R(\text{open}) = R$$

$$R_{out} = R_{out}(a_v=0)\frac{1+R(\text{short})}{1+R(\text{open})}$$

$$\approx \frac{1}{g_m + g_{mb}} \frac{1+0}{1+R} = \frac{1}{g_m + g_{mb}} \frac{1}{1+R}$$

$$= 200\frac{1}{1+838} = 0.238\Omega$$