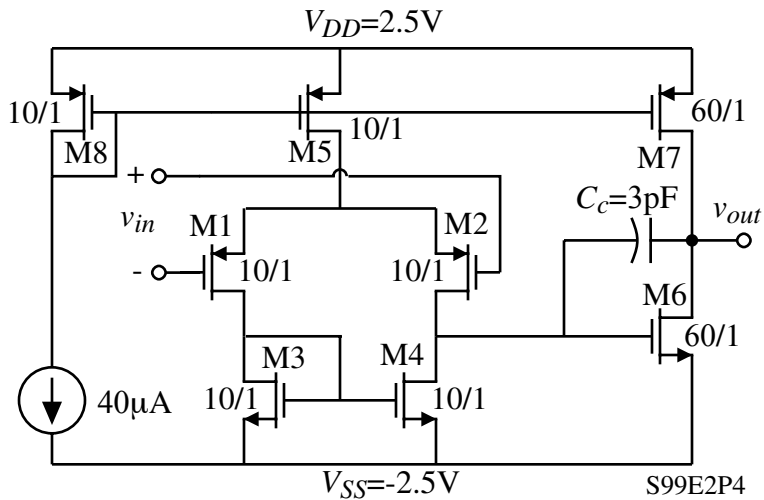


Homework Assignment No. 6 - Solutions

Problem 1 - (10 points)

For the CMOS op amp shown, find the following quantities.

- 1.) Slew rate (V/sec.)
- 2.) Positive and negative output voltage limits (all transistors remain in saturation)
- 3.) Positive and negative input common voltage limits (all transistors remain in saturation and use nominal parameter values)
- 4.) Small signal voltage gain
- 5.) Unity-gainbandwidth (MHz) and 6.) Power dissipation (mW).



Solution

$$1.) \quad SR = \frac{I_5}{C_c} = \frac{40\mu A}{3pF} = 1.1 \times 10^7 \text{ V/second} \quad \Rightarrow \quad \boxed{SR = 1.1 \times 10^7 \text{ V/sec}}$$

$$2.) \quad V_{SD7} = \sqrt{\frac{2I_7}{K_P(W/L)}} = \sqrt{\frac{480\mu A}{50 \cdot 60}} = 0.4 \text{ V} \quad \text{and} \quad V_{DS6} = \sqrt{\frac{480\mu A}{110 \cdot 60}} = 0.2697 \text{ V}$$

$$\therefore \quad \boxed{V_{out}(\text{max}) = 2.5 - 0.4 = 2.1 \text{ V}} \quad \&$$

$$\boxed{V_{out}(\text{min}) = -2.5 \text{ V} + 0.2697 \text{ V} = -2.230 \text{ V}}$$

$$3.) \quad ICM(\text{min}) = -2.5 \text{ V} + V_{GS3} - |V_{TP}| = -2.5 \text{ V} + \sqrt{\frac{2 \cdot 20}{110 \cdot 10}} + 0.7 \text{ V} - 0.7 \text{ V}$$

$$\therefore \quad ICM(\text{min}) = -2.5 + 0.191 = -2.309 \text{ V} \quad \Rightarrow \quad \boxed{ICM(\text{min}) = -2.309 \text{ V}}$$

$$ICM(\text{max}) = ? \quad V_{SD5}(\text{sat}) = \sqrt{\frac{2 \cdot 40}{50 \cdot 10}} = 0.4 \text{ V} \quad \text{and} \quad V_{SG1} = \sqrt{\frac{2 \cdot 20}{50 \cdot 10}} + 0.7 = 0.983 \text{ V}$$

$$\therefore \quad ICM(\text{max}) = 2.5 - V_{SD5}(\text{sat}) - V_{SG1} = 2.5 - 0.4 - 0.983 = 1.117 \text{ V}$$

$$\boxed{ICM(\text{max}) = 1.117 \text{ V}}$$

$$4.) \quad A_v = \frac{g_{m1}g_{m6}}{(g_{sd2} + g_{ds4})(g_{ds6} + g_{sd7})} \quad g_{m1} = \sqrt{\frac{2K_P W_1 I_1}{L_1}} = \sqrt{2 \cdot 50 \cdot 10 \cdot 20} = 141 \mu S$$

$$g_{m6} = \sqrt{\frac{2K_P W_6 I_6}{L_6}} = \sqrt{2 \cdot 110 \cdot 60 \cdot 240} = 1779 \mu S$$

$$G_I = 0.09 \cdot 20 \mu A = 1.8 \mu S$$

$$\text{and} \quad G_{II} = 0.09 \cdot 240 \mu A = 21.6 \mu S$$

$$\therefore \quad A_v = \frac{141 \times 1779}{1.8 \times 21.6} = 6,452 \text{ V/V}$$

$$\boxed{A_v = 6,452 \text{ V/V}}$$

$$5.) \quad GB = \frac{g_{m1}}{C_c} = \frac{141 \mu S}{3pF} = 47 \text{ Mrads/sec}$$

$$\Rightarrow \quad \boxed{GB = 7.48 \text{ MHz}}$$

$$6.) \quad P_{diss} = 5 \times 320 \mu A = 1.6 \text{ mW}$$

$$\Rightarrow \quad \boxed{P_{diss} = 1.6 \text{ mW}}$$

Problem 2 - (10 points)

Bias current calculation:

$$V_{T8} + V_{ON8} + I_8 \cdot R_S = V_{dd} - V_{ss} \text{ or, } V_{T8} + \sqrt{\frac{2 \cdot I_8}{3 \cdot K'_p}} = 5 - I_8 \cdot R_S \text{ .} \quad (1)$$

Solving for I_8 quadratically would give, $I_8 = \underline{36\mu A}$, $I_5 = \underline{36\mu A}$, and $I_7 = \underline{60\mu A}$ Using the formula, $g_m = \sqrt{2 \cdot K'_p \frac{W}{L} I}$ and $g_{ds} = \lambda I$ we get,

$$g_{m2} = 60\mu S, \quad g_{ds2} = 0.9\mu S, \quad g_{ds4} = 0.72\mu S \quad (2)$$

$$g_{m6} = 363\mu S, \quad g_{ds6} = 3\mu S, \quad g_{ds7} = 2.4\mu S \quad (3)$$

Small-signal open-loop gain:

The small-signal voltage gain can be expressed as,

$$A_{v1} = \frac{-g_{m2}}{(g_{ds2} + g_{ds4})} = -37 \quad \text{and} \quad A_{v2} = \frac{-g_{m6}}{(g_{ds6} + g_{ds7})} = -67$$

Thus, total open-loop gain is,

$$A_v = A_{v1} \cdot A_{v2} = \underline{2489V/V} \quad (3)$$

Output resistance:

$$R_{out} = \frac{1}{(g_{ds6} + g_{ds7})} = 185K\Omega \quad (5)$$

Power dissipation:

$$P_{diss} = 5(36 + 36 + 60)\mu W = 660\mu W \quad (6)$$

ICMR:

$$V_{in,max} = 2.5 - V_{T1} - V_{ON1} - V_{ON5} = 0.51V \quad (7)$$

$$V_{in,min} = -2.5 - V_{T1} + V_{T3} + V_{ON3} = -2.21V \quad (8)$$

Output voltage swing:

$$V_{0,max} = 2.5 - V_{ON7} = 1.81V \quad (9)$$

Slew Rate:

Slew rate under no load condition can be given as,

$$SR = \frac{I_5}{C_c} = 6V / \mu s$$

In presence of a load capacitor of 20 pF, slew rate would be,

$$SR = \min\left[\frac{I_5}{C_c}, \frac{I_7}{C_L}\right]$$

Problem 6.3-7 - Continued

CMRR:

Under perfectly balanced condition where $I_1 = I_2$, if a small signal common-mode variation occurs at the two input terminals, the small signal currents $i_1 = i_2 = i_3 = i_4$ and the differential output current at node (7) is zero. So, ideally, common-mode gain would be zero and the value for CMRR would be infinity.

GBW:

Let us design M9 and M10 first. Both these transistors would operate in triode region and will carry zero dc current. Thus, $V_{ds9} = V_{ds10} \cong 0$. The equation of drain current in triode region is given as,

$$I_D \cong K' \frac{W}{L} (V_{GS} - V_T) V_{DS} .$$

The on resistance of the MOS transistor in triode region of operation would be,

$$R_{ON} = K' \frac{W}{L} (V_{GS} - V_T) .$$

It is intended to make the effective resistance of M9 and M10 equal to $\frac{1}{g_{m6}}$.

$$\text{So, } K'_9 \left(\frac{W_9}{L_9} \right) (V_{GS9} - V_{T9}) + K'_{10} \left(\frac{W_{10}}{L_{10}} \right) (V_{GS10} - V_{T10}) = g_{m6} \quad (11)$$

$$V_{D4} = V_{D3} = -2.5 + V_{T3} + V_{ON3} = -1.51V$$

Thus,

$$V_{GS9} \cong 4V \quad \text{and} \quad V_{GS10} \cong -1V .$$

Putting the appropriate values in (11), we can solve for the aspect ratios of M9 and M10. One of the solutions could be,

$$K'_9 \left(\frac{W_9}{L_9} \right) = \frac{1}{1} \quad \text{and} \quad K'_{10} \left(\frac{W_{10}}{L_{10}} \right) = \text{very small} \quad (12)$$

The dominant pole could be calculated as,

$$p_1 = \frac{-(g_{ds4} + g_{ds2})}{2\pi \cdot A_{V2} \cdot C_C} = 640\text{HZ} .$$

And the load pole would be,

$$p_2 = \frac{-g_{m6}}{2\pi \cdot C_L} = -2.8\text{MHz} . \quad \text{for a 20 pF load.}$$

It can be noted that in this problem, the product of the open-loop gain and the dominant pole is approximately equal to the load pole. Thus, the gain bandwidth is approximately equal to 2.8 MHz and the phase margin would be close to 45 degrees.

Problem 6.3-7 - Continued

PSRR:

If a small ripple v_s is applied at the V_{dd} terminal, then the gain of this ripple from this terminal to the output can be expressed as,

$$\frac{v_o}{v_s} = \frac{\left(1 - \frac{R_S}{R_S + (1/g_{m8})}\right) g_{m7}}{g_{ds6} + g_{ds7}} = 2.8V/V$$

Thus, PSRR due to variations in V_{dd} would be, $A_V \mu.8 = 2489 / 2.8 = 889$.

SPICE file:

```
.model nmos nmos vto=0.7 lambda=0.04 kp=110u
.model pmos pmos vto=-0.8 lambda=0.05 kp=50u

vdd 1 0 dc 2.5 ac 0
vss 10 0 dc -2.5 ac 0
vinp 5 0 dc 0 ac 1
*vinn 4 0 dc 0 ac 0

m8 2 2 1 1 pmos w=3u l=1u
rs 2 10 100k
m5 3 2 1 1 pmos w=3u l=1u
m1 6 8 3 3 pmos w=2u l=1u
m2 7 5 3 3 pmos w=2u l=1u
m3 6 6 10 10 nmos w=4u l=1u
m4 7 6 10 10 nmos w=4u l=1u
m7 8 2 1 1 pmos w=5u l=1u
m6 8 7 10 10 nmos w=10u l=1u
cc 7 9 6p
cl 8 0 20p
m9 8 1 9 9 nmos w=1u l=1u
m10 8 10 9 9 pmos w=1u l=100u

.op
.ac dec 10 1 100meg
.option post
.end

Operating points:

**** mosfets

subckt
element 0:m8      0:m5      0:m1      0:m2      0:m3      0:m4
model      0:pmos      0:pmos      0:pmos      0:pmos      0:nmos      0:nmos
region      Cutoff      Cutoff      Cutoff      Cutoff      Saturati    Saturati
id         -35.3708u    -34.8506u    -17.4107u    -17.4399u    17.4107u    17.4399u
ibs         0.          0.          0.          0.          0.          0.
ibd         14.6292f    11.4726f    28.7676f    28.3314f    -9.7598f    -10.1959f
```

Problem 6.3-7 - Continued

```

vgs      -1.4629   -1.4629   -1.3517   -1.3527   975.9818m  975.9818m
vds      -1.4629   -1.1473   -2.8768   -2.8331   975.9818m  1.0196
vbs       0.         0.         0.         0.         0.         0.
vth      -800.0000m -800.0000m -800.0000m -800.0000m 700.0000m
700.0000m
vdsat    -662.9217m -662.9217m -551.7476m -552.7377m 275.9818m
275.9818m
beta     160.9719u  158.6045u  114.3838u  114.1657u  457.1773u  457.9449u
gam eff  527.6252m  527.6252m  527.6252m  527.6252m  527.6252m  527.6252m
gm       106.7118u  105.1423u  63.1110u   63.1037u   126.1726u  126.3844u
gds      1.6480u    1.6480u    761.0636n  763.7975n  670.2604n  670.2604n
gmb      36.9704u    36.4266u    21.8648u   21.8623u   43.7126u   43.7860u
cdtot    2.021e-18   1.585e-18   2.649e-18   2.609e-18   1.797e-18   1.878e-18
cgtot    7.005e-16   7.000e-16   4.693e-16   4.692e-16   9.467e-16   9.467e-16
cstot    6.906e-16   6.906e-16   4.604e-16   4.604e-16   9.208e-16   9.208e-16
cbtot    7.806e-18   7.806e-18   6.216e-18   6.205e-18   2.402e-17   2.402e-17
cgs      6.906e-16   6.906e-16   4.604e-16   4.604e-16   9.208e-16   9.208e-16
cgd      2.021e-18   1.585e-18   2.649e-18   2.609e-18   1.797e-18   1.878e-18

```

```

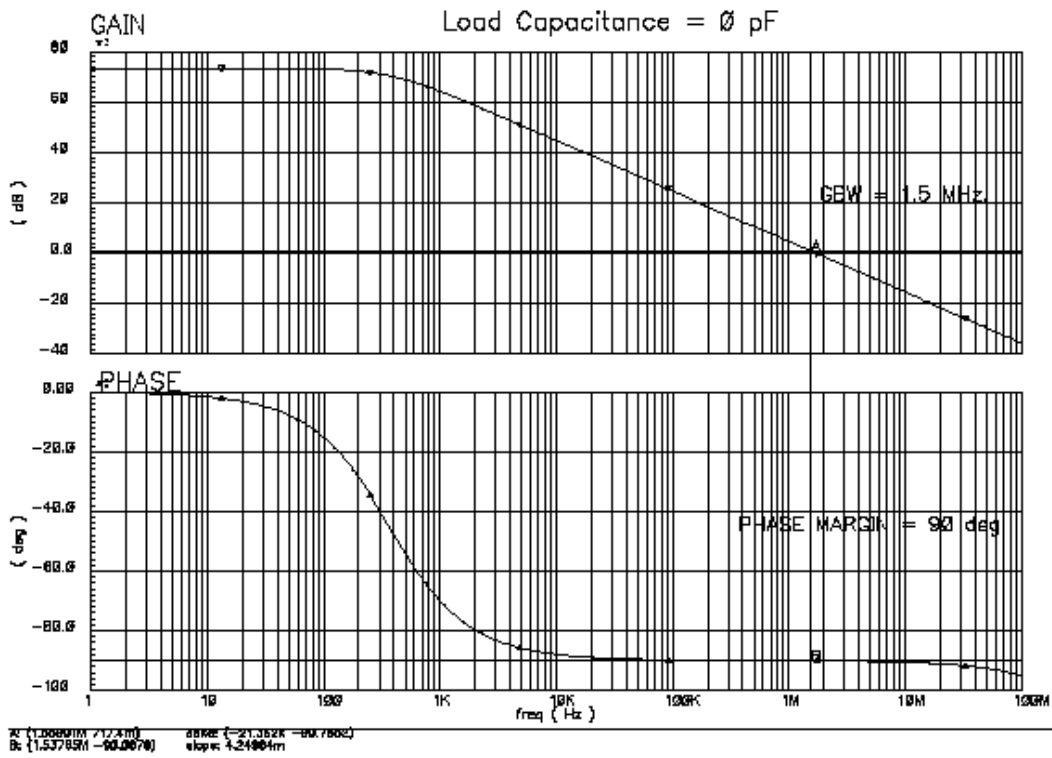
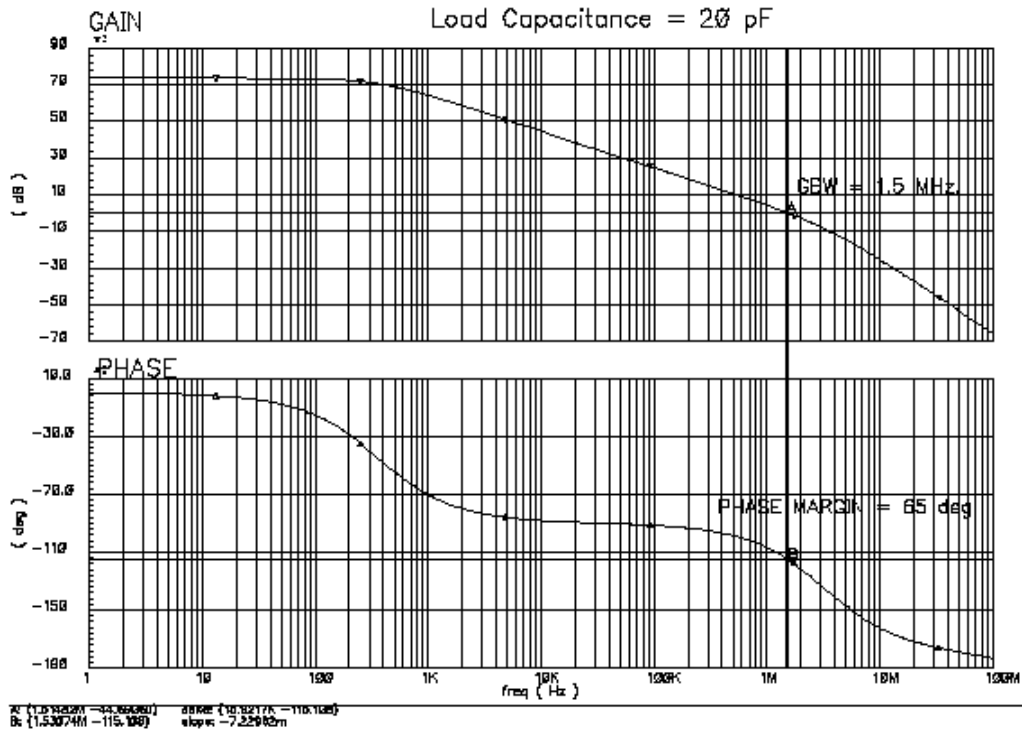
subckt
element 0:m7      0:m6      0:m9      0:m10
model   0:pmos    0:nmos    0:nmos    0:pmos
region  Cutoff    Saturati  Linear    Cutoff
id      -61.7971u  61.7971u  0.         0.
ibs     0.         0.         0.         0.
ibd     24.9901f  -25.0099f  0.         0.
vgs     -1.4629    1.0196    2.4990    -2.5010
vds     -2.4990    2.5010    0.         0.
vbs     0.         0.         0.         0.
vth     -800.0000m 700.0000m 700.0000m -800.0000m
vdsat   -662.9217m 319.5939m 0.         0.
beta    281.2376u  1.2100m  110.0000u  500.0000n
gam eff  527.6252m  527.6252m  527.6252m  527.6252m
gm       186.4385u  386.7225u  0.         0.
gds      2.7467u    2.2471u   197.8911u  850.4951n
gmb      64.5917u   133.9802u  0.         0.
cdtot    5.753e-18   1.152e-17  1.727e-16  17.2658f
cgtot    1.1698f    2.3660f   3.463e-16  34.6349f
cstot    1.1511f    2.3021f   1.727e-16  17.2658f
cbtot    1.301e-17  5.233e-17  9.769e-19  1.033e-16
cgs      1.1511f    2.3021f   1.727e-16  17.2658f
cgd      5.753e-18  1.152e-17  1.727e-16  17.2658f

```

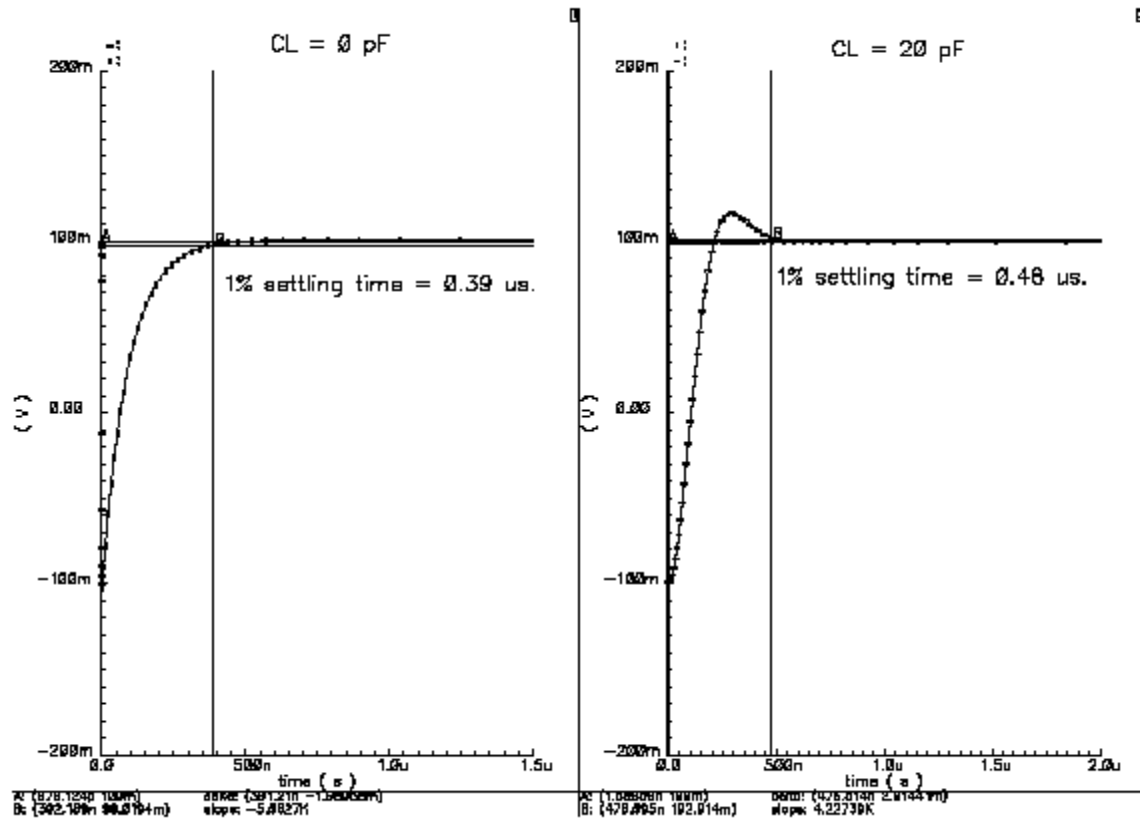
Results from SPICE simulation:

- i. Unloaded output (load capacitor = 0)
 - GBW = 1.5 MHz., Phase Margin = 90 deg, 1% settling time = 0.39 us.
- ii. Loaded output (load capacitor = 20 pF)
 - GBW = 1.5 MHz., Phase Margin = 65 deg, 1% settling time = 0.48 us.

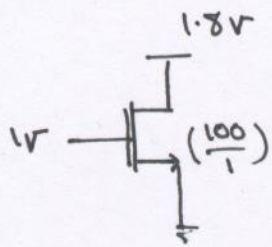
Problem 6.3-7 - Continued



Problem 6.3-7 - Continued



Problem 4 - Design Problem 2 (50 points)

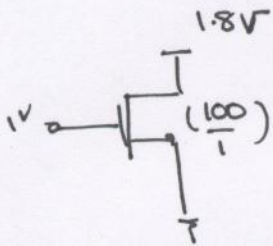


2.916 mA (from spice)
if level is changed to level = 7

$$V_{th} = 0.45 \text{ V}$$

$$I_D = 2.916 \text{ mA} = \frac{\mu_n}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$$\rightarrow 2.916 \text{ mA} = \frac{\mu_n}{2} \times 100 \times 0.55^2 \Rightarrow \mu_n = 192.8 \frac{\mu\text{A}}{\text{V}^2}$$



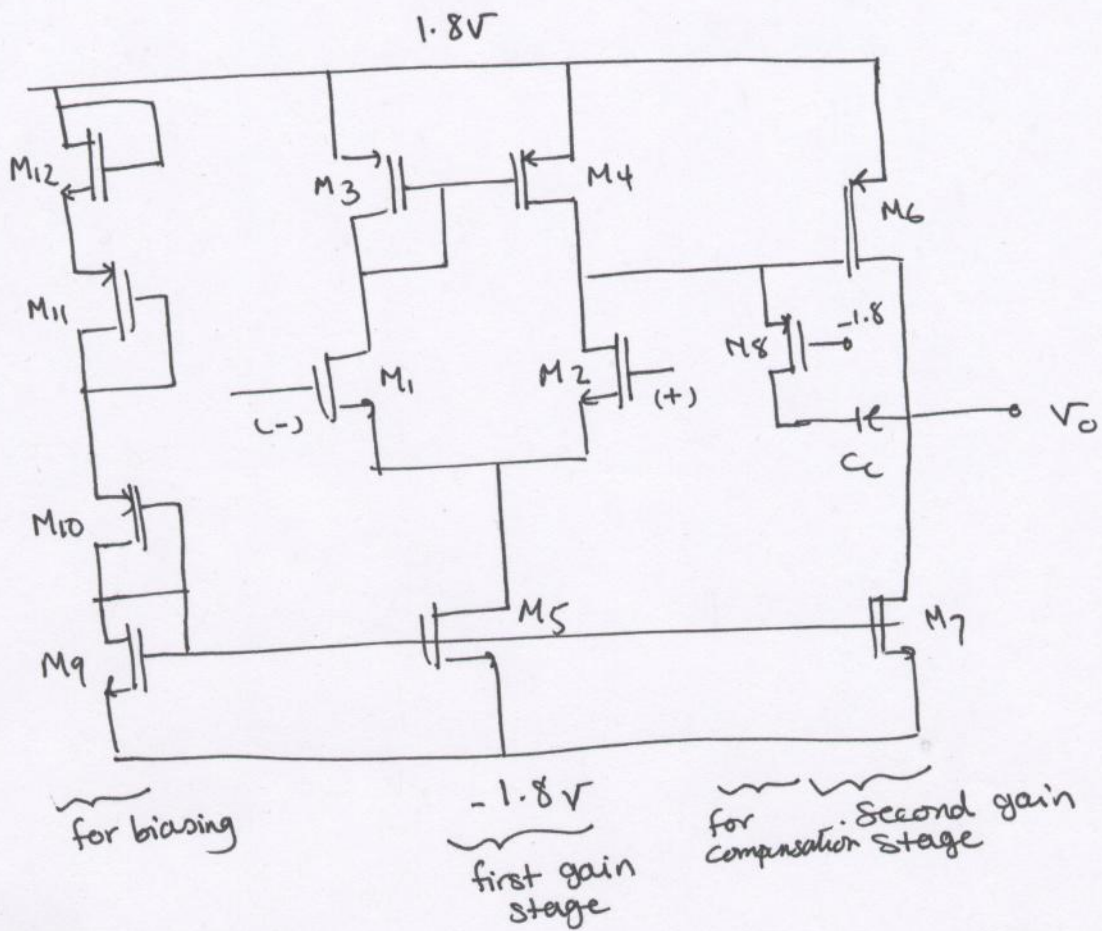
λ_n from spice = 0.05 (for $L = 1 \mu\text{m}$)

if considering the given nominal parameters :

$$\left\{ \begin{array}{l} \mu_n = 119.8 \\ \lambda_n \text{ (from simulation)} = 0.05 \mid L = 1 \mu\text{m} \\ V_{TN} = 0.45 \text{ V} \end{array} \right.$$

$$\left\{ \begin{array}{l} \mu_p = -24.6 \\ \lambda_p = 0.054 \\ V_{TP} = -0.56 \text{ V} \end{array} \right.$$

We use these values for the hand calculations.



1) For power dissipation

$$[1.8V - (-1.8V)] I_{total} = 720 \mu W \rightarrow I_{total} \leq 200 \mu A$$

2) phase margin $\geq 60^\circ \Rightarrow C_c > 0.22 C_L$

$$C_L = 5 \text{ pF} \rightarrow C_c > 1.1 \text{ pF}$$

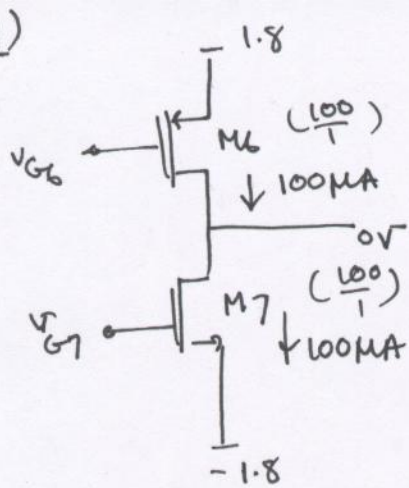
$$\rightarrow \text{use } \underline{\underline{C_c = 2 \text{ pF}}}$$

3) $SR = \frac{I_S}{C_c} = \frac{I_S}{2 \text{ pF}} \geq \frac{10V}{\mu\text{sec}} \rightarrow I_S \geq 20 \mu A$

$$\rightarrow \text{take } \underline{\underline{I_S = 20 \mu A}}$$

for the output stage use $I_6 = I_7 = 5 \times 20 \mu A = \underline{\underline{100 \mu A}}$

4)



$$V_{O(max)} = 1.8 - |V_{SD_{sat6}}|$$

$$100\mu A = \frac{24.6\mu}{2} \times \frac{100}{1} \times V_{OD6}^2$$

$$\rightarrow |V_{OD6}| = 0.285V$$

$$V_{Omax} = \underline{\underline{1.51V}}$$

$$V_{out\ min} = -1.8 + V_{OD7}$$

$$V_{OD7} = \sqrt{\frac{2 \times 100\mu A}{120\mu \times 100}} = 0.129V$$

$$\rightarrow V_{out\ min} = \underline{\underline{-1.67V}}$$

$$\rightarrow OVSR = 1.51 + 1.67 = \underline{\underline{3.18V}} > 3V \text{ (OK)}$$

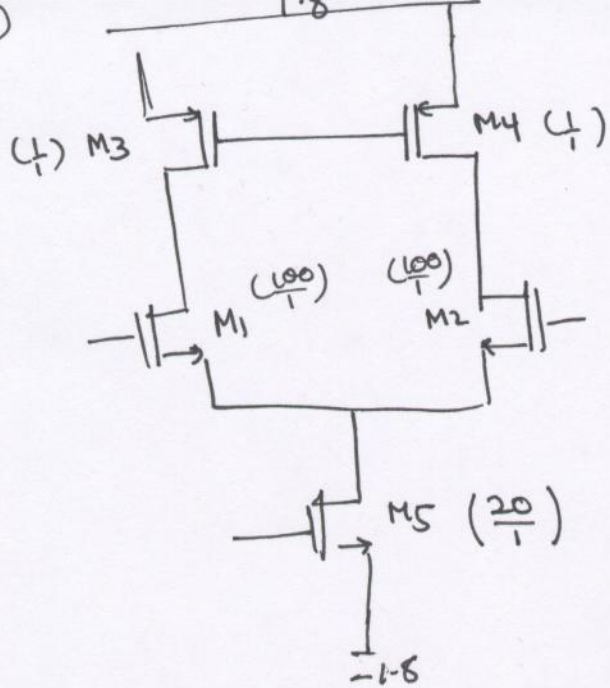
For $V_O = 0 \rightarrow$ for biasing condition

$$|V_{OD6}| = 0.285V \rightarrow |V_{GS6}| = \boxed{0.845V}$$

$$\rightarrow \underline{\underline{V_{G6} = 0.955V}}$$

$$V_{OD7} = 0.129V \rightarrow V_{GS7} = \boxed{0.609V}$$

$$\rightarrow \underline{\underline{V_{G7} = -1.191V}}$$



$$V_{ic}(\min) = -1.8 + V_{DS5}(\text{sat}) + V_{GS1}$$

$$V_{DS5}(\text{sat}) = \sqrt{\frac{2 \times 20 \mu}{120 \mu \times 20}} = 0.129$$

$$V_{GS1} = \sqrt{\frac{2 \times 10 \mu}{120 \times 100}} + 0.45 = 0.49$$

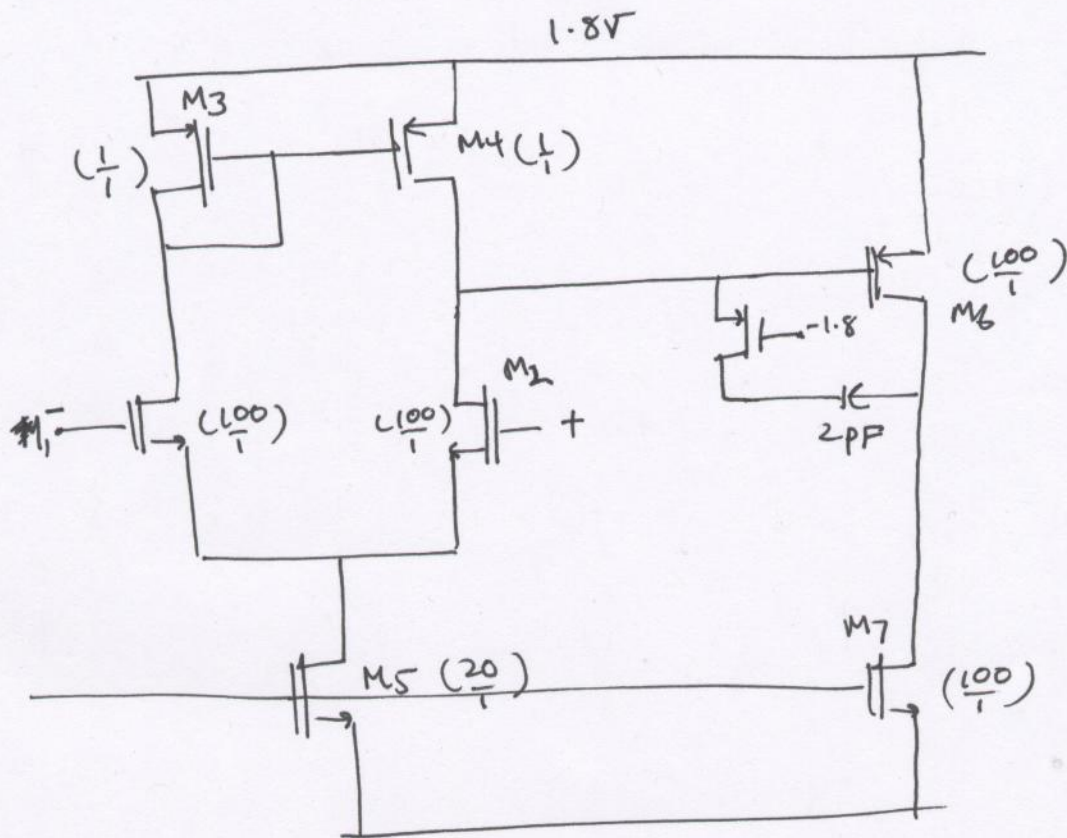
$$\rightarrow V_{ic \min} = -1.8 + 0.129 + 0.49 = -1.18 \text{ V}$$

$$V_{ic}(\max) = V_{DD} - V_{DS3}(\text{sat}) + V_{TN}$$

$$V_{DS3}(\text{sat}) = \sqrt{\frac{2 \times 10}{1 \times 25}} = 0.9 \text{ V}$$

$$V_{ic}(\max) = 1.8 - 0.9 + 0.45 = 1.34 \text{ V}$$

$$\rightarrow I_{CMR} = 1.34 + 1.18 \text{ V} = \underline{\underline{2.53 \text{ V}}} > 2 \text{ V (OK)}$$



$$6) \quad GB = \frac{g_{m1}}{C_c}$$

$$g_{m1} = \sqrt{2 \times 120 \mu \times 100 \times 10 \mu} = 0.49 \text{ mS}$$

$$GB = \frac{0.49 \text{ mS}}{2 \text{ pF}} = 2.45 \times 10^8 \text{ rad/sec} = \underline{\underline{38.98 \text{ MHz} > 25 \text{ MHz}}}$$

OK

7) $A_{V(0)}$ checking:

$$g_{m1} = g_{m2} = 0.49 \text{ mS}$$

$$R_{\Sigma} = r_{ds2} \parallel r_{ds4} \rightarrow R_{\Sigma} = 0.962 \text{ M}\Omega$$

$$r_{ds2} = \frac{1}{0.05 \times 10 \mu} = 2 \text{ M}\Omega$$

$$\rightarrow A_{V1} = 0.962 \text{ M} \times 0.49 \text{ m} = 471.4$$

$$r_{ds4} = \frac{1}{0.054 \times 10 \mu} = 1.85 \text{ M}\Omega$$

$$\rightarrow A_V = 31,808 > 10,000$$

OK

$$g_{m6} = \sqrt{2 \times 24.6 \times 100 \times 100} = 0.701 \text{ mS}$$

$$r_{ds6} = \frac{1}{0.054 \times 100 \mu} = 0.185 \text{ M}\Omega$$

$$\rightarrow R_{\Sigma} = 96.2 \text{ k}\Omega$$

$$r_{d7} = \frac{1}{0.05 \times 100 \mu} = 200 \text{ k}\Omega$$

$$A_{V2} = 96.2 \text{ k} \times 0.701 \text{ m} = 67.5$$

(8) poles and zeros

$$\text{pole 1} = - \frac{g_{m1}}{A_V C_C} = \frac{0.49 \text{ mS}}{3.18 \times 10^4 \times 2 \times 10^{-12}} = 7704.4 \text{ rad/sec}$$

$$\rightarrow P_1 = 1.226 \text{ kHz}$$

$$P_2 = - \frac{g_{m6}}{C_C} = - \frac{0.701 \text{ m}}{5 \times 10^{-12}} = 140.2 \text{ Mrad/sec} = 22.31 \text{ MHz}$$

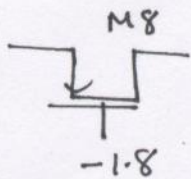
$$P_2 = 22.31 \text{ MHz} < 38 \text{ MHz}$$

↑
phase margin will be ruined \rightarrow need zero nulling

9) Zero nulling

$$Z = \frac{-g_{m6}}{C_C}$$

$$R_Z = \frac{C_C + C_C}{g_{m6} C_C} = \frac{7}{0.701 \times 2} = 4.992 \text{ k}\Omega$$



$$R_Z = \frac{1}{\mu'_p \left(\frac{W}{L}\right)_8 (|V_{GS}| - |V_t|)}$$

↓

$$+1.8 + 0.955 - 0.56$$

$\rightarrow \left(\frac{W}{L}\right)_8 = 1.78$

$$\rightarrow \underline{\underline{\left(\frac{W}{L}\right)_8 = 2}}$$

10) check phase margin

$$\phi = 180^\circ - \tan^{-1} \left[\frac{GB}{|P_1|} \right] - \tan^{-1} \left[\frac{GB}{|P_2|} \right] - \tan^{-1} \left[\frac{GB}{|Z_1|} \right]$$

$$= 180^\circ - \tan^{-1} \left[\frac{38.98 \text{ MHz}}{1.226 \text{ kHz}} \right] - \tan^{-1} \left[\frac{GB}{|P_3|} \right]$$

is gone with ~~zero~~ zero nulling

$$= 180 - \underline{89.998} - \tan^{-1} \left[\frac{GB}{|P_3|} \right]$$

close to 90°
as expected

$$P_3 = \frac{1}{R_2 C_Z} = \frac{1}{4.992 \text{ k} \times 0.5 \text{ p}} = 33.78 \text{ MHz}$$

$$C_Z = C_{db4} + C_{gd4} + C_{bd2} + \underline{C_{gs6}} + \underline{C_{gs8}} \approx 0.5 \text{ pF}$$

$$\rightarrow \phi = 90 - \tan^{-1} \left[\frac{38.98}{\underline{33.78}} \right] = \underline{58.56^\circ} \text{ close to } \underline{60^\circ}$$

can increase gmb to get a better PM

$$\underline{5} = \gamma \left(\frac{W}{J} \right)$$