Homework Assignment No. 2 - Solutions

Problem 1 - (10 points) (Problem 5.20 of Gray, Hurst, Lewis and Meyer)

(a). Device Currents

	Hand Calculations	SPICE Simulations
I_{C1}	420μA	473μA
I_{C2}	340μA	390μA
I_{C3}	420μA	415μA
I_{C5}	100μA	100μA
I_{C6}	500μA	504μA
I_{D1}	80μA	83.9μA
I_{D2}	80μA	83.9μA
I_{D3}	30μA	28.4μA

(b). $V_{o(max)}$ = 3.95V $V_{o(mim)}$ = -4.00V

P_{max} = 8.3 mW

(c). SPICE Results

- A plot of the DC transfer characteristic and mark on your results where clipping begins to occur.
- Plots of i_{C1}, i_{C2}, and i_{D2} for v_o = 2V peak and v_o = 4V peak. Assume a signal frequency of 10 KHz. (Note: you will have to apply the appropriate dc bias and signal amplitude to achieve the desired output conditions.)
- Also use SPICE to compute the THD of the output voltage for both the 2V and 4 V conditions.

	Hand Calculations	SPICE Simulations
Clipping		
max	3.95V	4.0V
min	-3.975V	-4.0V
THD of v_o		
2V peak	NA	0.4875%
4V peak	NA	0.3678%

(For the remainder of the problems, see plots on following pages.)

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$K_p := 26 \cdot 10^{-6}$ $V_T := 0.7$ $V_t := .026$ $\beta_P := 20$ $\beta_N := 80$

$I_o := 100 \cdot 10^{-6}$ $S_1 := 500$ $S_2 := 500$ $S_3 := 1$

$R_1 := 10 \cdot 10^3$ $R_2 := 10 \cdot 10^3$ $R_3 := 500$ $R_4 := 100$ $R_L := 200$

$V_{BE} := .8$ $V_{CEsat} := .2$ $V_{CC} := 5$ $V_{EE} := -5$

$I_{C5} := I_o$ $I_{C6} := 5 \cdot I_o$

$I_{D3} := \dfrac{I_{C5} + I_{C6}}{\beta_P}$ $I_{D1} := \dfrac{V_{BE}}{R_1}$ $I_{D2} := \dfrac{V_{BE}}{R_2}$

$I_{C3} := I_{C6} - I_{D1}$

Note: VGS1+VBE3=VGS2+VBE. Therefore, VGS1=VGS2, so that.

$I_{C1} := I_{C3}$ $I_{C2} := I_{C1} - I_{D2}$

Summary

$I_{C1} = 4.2 \cdot 10^{-4}$ $I_{D1} = 8 \cdot 10^{-5}$

$I_{C2} = 3.4 \cdot 10^{-4}$ $I_{D2} = 8 \cdot 10^{-5}$

$I_{C3} = 4.2 \cdot 10^{-4}$ $I_{D3} = 3 \cdot 10^{-5}$

$I_{C5} = 1 \cdot 10^{-4}$

$I_{C6} = 5 \cdot 10^{-4}$

(b)

$V_{omax} := V_{CC} - V_{BE} - V_{CEsat} - I_{C6} \cdot R_4$

$V_{omax} = 3.95$

For Vomin, the limit will be M2 entering the active region where VDS = VGS -VT. M2 will carry the extra base drive current to permit Q2 to sink about 20 mA of load current. The following are seed values to set up the four simultaneous equations for solution.

$V_{omin} := -4$ $I_{C2} := 20 \cdot 10^{-3}$ $I_{B2} := 250 \cdot 10^{-6}$ $V_{GS1} := 1$

Given

$$\frac{-V_{omin}}{R_L} + I_{C1} = I_{D2} + I_{C2}$$

$$I_{C2} = \beta_N \cdot I_{B2}$$

$$I_{D2} = \frac{V_{BE}}{R_2} + I_{B2}$$

$$V_{omin} - (V_{EE} + V_{BE}) = \sqrt{\frac{2 \cdot I_{D2}}{K_p \cdot S_2}}$$

$$\begin{bmatrix} V_{omin} \\ I_{C2} \\ I_{B2} \\ I_{D2} \end{bmatrix} := \mathrm{Find}(V_{omin}, I_{C2}, I_{B2}, I_{D2})$$

$V_{omin} = -3.975$

$I_{C2} = 0.01996$

$I_{B2} = 2.496 \cdot 10^{-4}$

$I_{D2} = 3.296 \cdot 10^{-4}$

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Prob. 5.20 from G&M - BiCMOS Output Stage
Vcc 1 0 5V
Vee 2 0 -5V
R3  1 3 500
Q5 6 4 3 PNP
M3 0 6 4 4  PMOS W=2U L=2U
Ibias 6 0 100UA
R4 1 5 100
Q6 7 4 5 PNP 5
M1 8 8 7 7 PMOS W=500U L=1U
R1 8 9 10K
Q3 7 8 9 NPN 25
Q1 1 7 10 NPN 25
M2 13 9 10 10 PMOS W=500U L=1U
R2 13 2 10K
Q2 10 13 2 NPN 25
RL 10 0 200
Vi 9 0 SIN -0.8093 2.1V  10K 0 0
.TRAN 4US 100US
.FOUR 10K V(10)
.MODEL NPN NPN RB=200 BF=80 IS=1E-18 VAF=130
.MODEL PNP PNP RB=300 BF=20 IS=1E-18 VAF=50
.MODEL PMOS PMOS KP=26U Lambda=0.0125 Vto=-0.7V LD=0
.OP
.Probe
.END
```

NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
(1)	5.0000	(2)	-5.0000	(3)	4.9476	(4)	4.1135
(5)	4.9472	(6)	1.9712	(7)	.7909	(8)	-.0221
(9)	-.8093	(10)	981.8E-06	(13)	-4.2151		

VOLTAGE SOURCE CURRENTS

NAME	CURRENT
Vcc	-1.105E-03
Vee	4.735E-04
Vi	4.985E-04

TOTAL POWER DISSIPATION   8.30E-03 WATTS

NAME	Q5	Q6	Q3	Q1	Q2
MODEL	PNP	PNP	NPN	NPN	NPN
IB	-4.79E-06	-2.36E-05	5.15E-06	5.72E-06	4.73E-06
IC	-1.00E-04	-5.04E-04	4.15E-04	4.73E-04	3.90E-04
VBE	-8.34E-01	-8.34E-01	7.87E-01	7.90E-01	7.85E-01
VBC	2.14E+00	3.32E+00	-8.13E-01	-4.21E+00	-4.22E+00
VCE	-2.98E+00	-4.16E+00	1.60E+00	5.00E+00	5.00E+00
BETADC	2.09E+01	2.13E+01	8.05E+01	8.26E+01	8.26E+01
GM	3.86E-03	1.95E-02	1.60E-02	1.83E-02	1.51E-02
RPI	5.39E+03	1.09E+03	5.02E+03	4.52E+03	5.47E+03
RX	3.00E+02	6.00E+01	8.00E+00	8.00E+00	8.00E+00
RO	5.21E+05	1.06E+05	3.16E+05	2.84E+05	3.44E+05
BETAAC	2.08E+01	2.13E+01	8.05E+01	8.26E+01	8.26E+01
FT	6.15E+16	3.10E+17	2.55E+17	2.91E+17	2.40E+17

**** MOSFETS

NAME	M3	M1	M2
MODEL	PMOS	PMOS	PMOS
ID	-2.84E-05	-8.39E-05	-8.32E-05
VGS	-2.14E+00	-8.13E-01	-8.10E-01
VDS	-4.11E+00	-8.13E-01	-4.22E+00
VBS	0.00E+00	0.00E+00	0.00E+00
VTH	-7.00E-01	-7.00E-01	-7.00E-01
VDSAT	-1.44E+00	-1.13E-01	-1.10E-01
GM	3.94E-05	1.48E-03	1.51E-03
GDS	3.38E-07	1.04E-06	9.88E-07

Prob. 5.20 from G&M - BiCMOS Output Stage

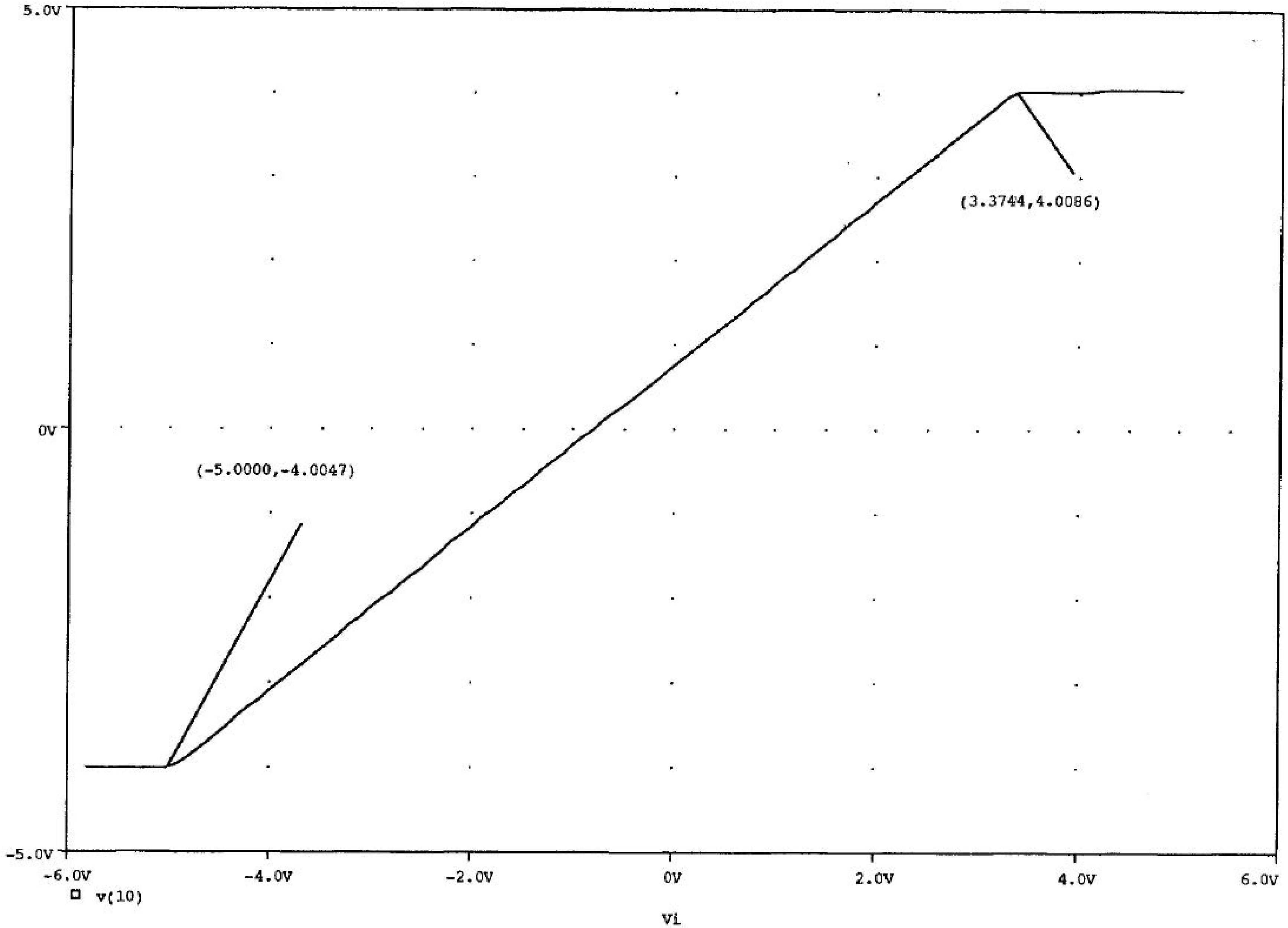
**** FOURIER ANALYSIS TEMPERATURE = 27.000 DEG C

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(10)

DC COMPONENT = -1.100053E-02

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+04	2.012E+00	1.000E+00	2.390E-03	0.000E+00
2	2.000E+04	1.323E-03	6.576E-04	8.992E+01	8.991E+01
3	3.000E+04	8.405E-03	4.178E-03	1.794E+02	1.794E+02
4	4.000E+04	2.628E-03	1.306E-03	8.991E+01	8.991E+01
5	5.000E+04	3.001E-03	1.492E-03	1.783E+02	1.783E+02
6	6.000E+04	1.954E-03	9.714E-04	8.979E+01	8.979E+01
7	7.000E+04	1.316E-03	6.541E-04	1.761E+02	1.761E+02
8	8.000E+04	1.393E-03	6.926E-04	8.955E+01	8.955E+01
9	9.000E+04	6.102E-04	3.033E-04	1.715E+02	1.715E+02

TOTAL HARMONIC DISTORTION = 4.874744E-01 PERCENT



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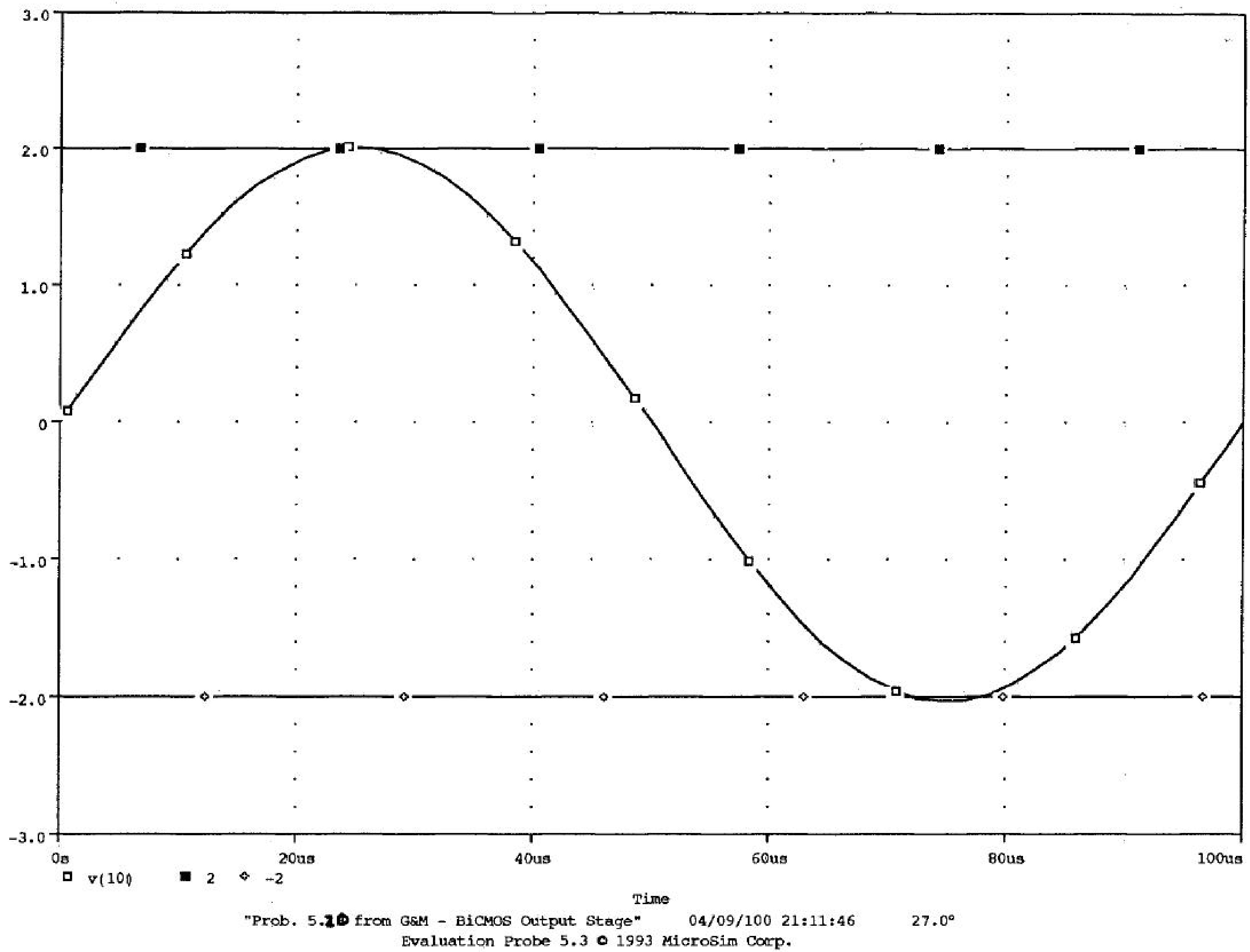
Evaluation Probe 5.3 © 1993 MicroSim Corp.


3.0
2.0
1.0
0
-1.0
-2.0
-3.0
0s
20us
40us
60us
80us
100us
v(10)
2
-2
Time
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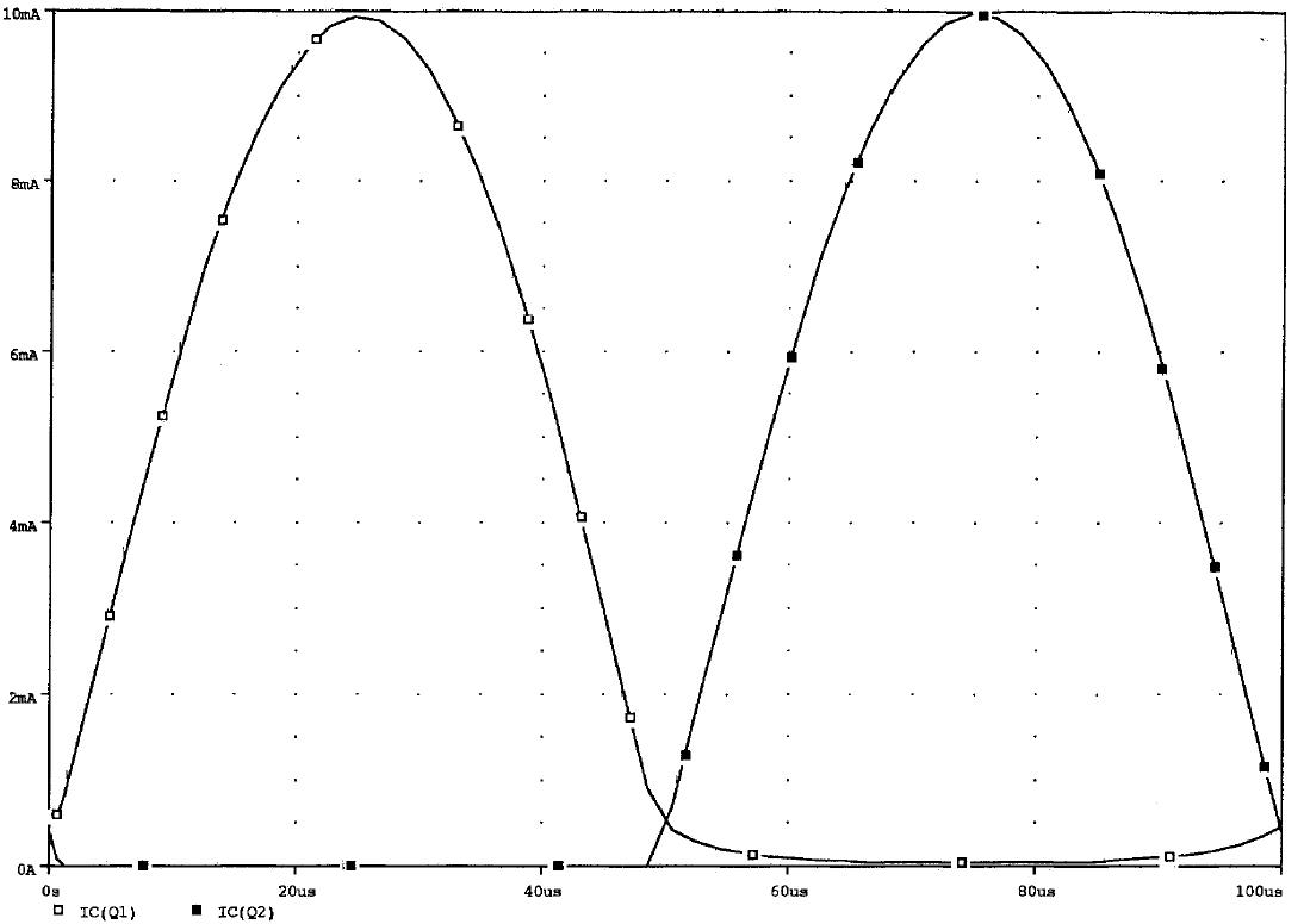
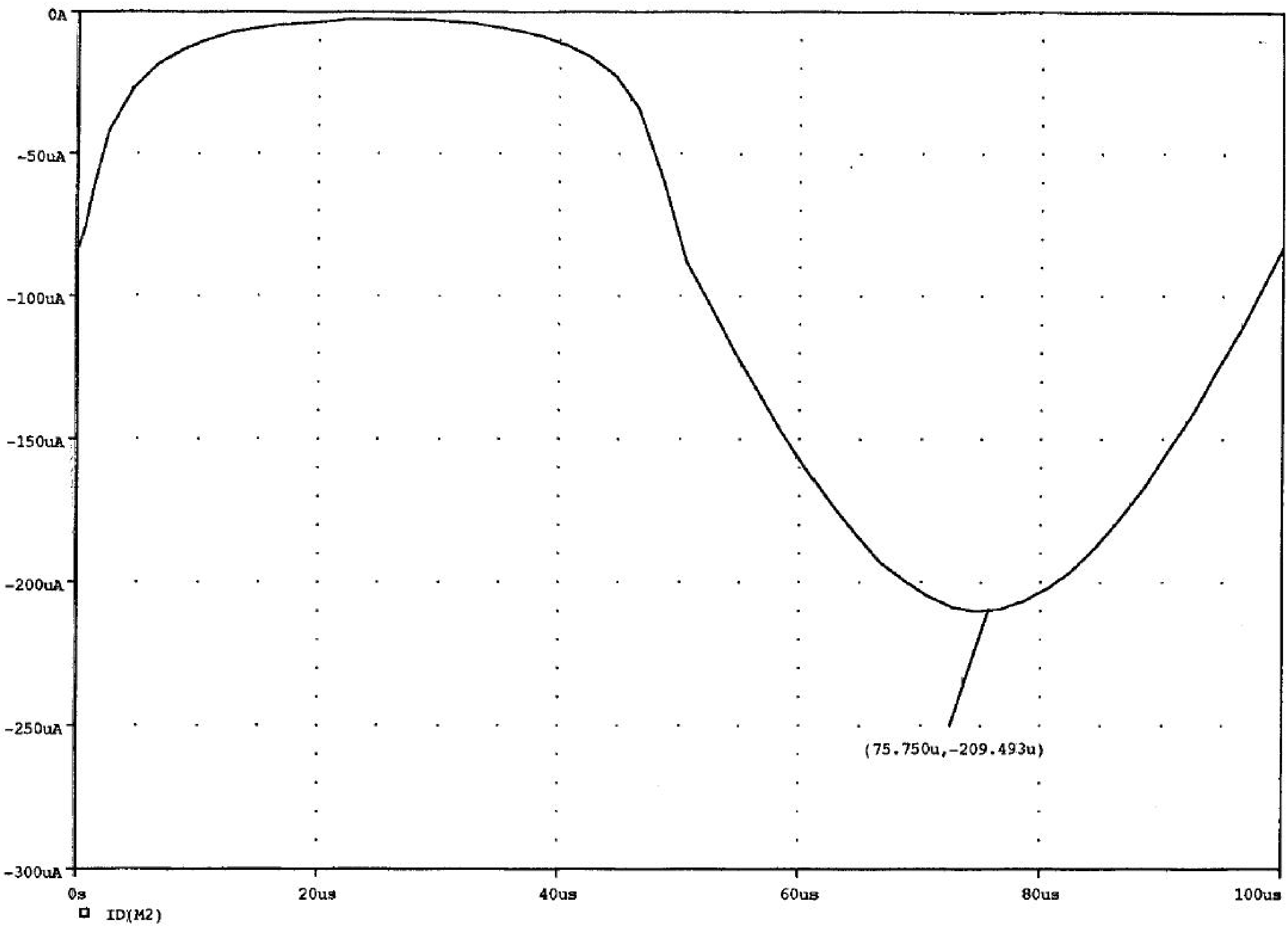
Evaluation Probe 5.3 © 1993 MicroSim Corp.



"Prob. 5.20 from G&M - BiCMOS Output Stage"   04/09/100 21:11:46   27.0°
Evaluation Probe 5.3 © 1993 MicroSim Corp.



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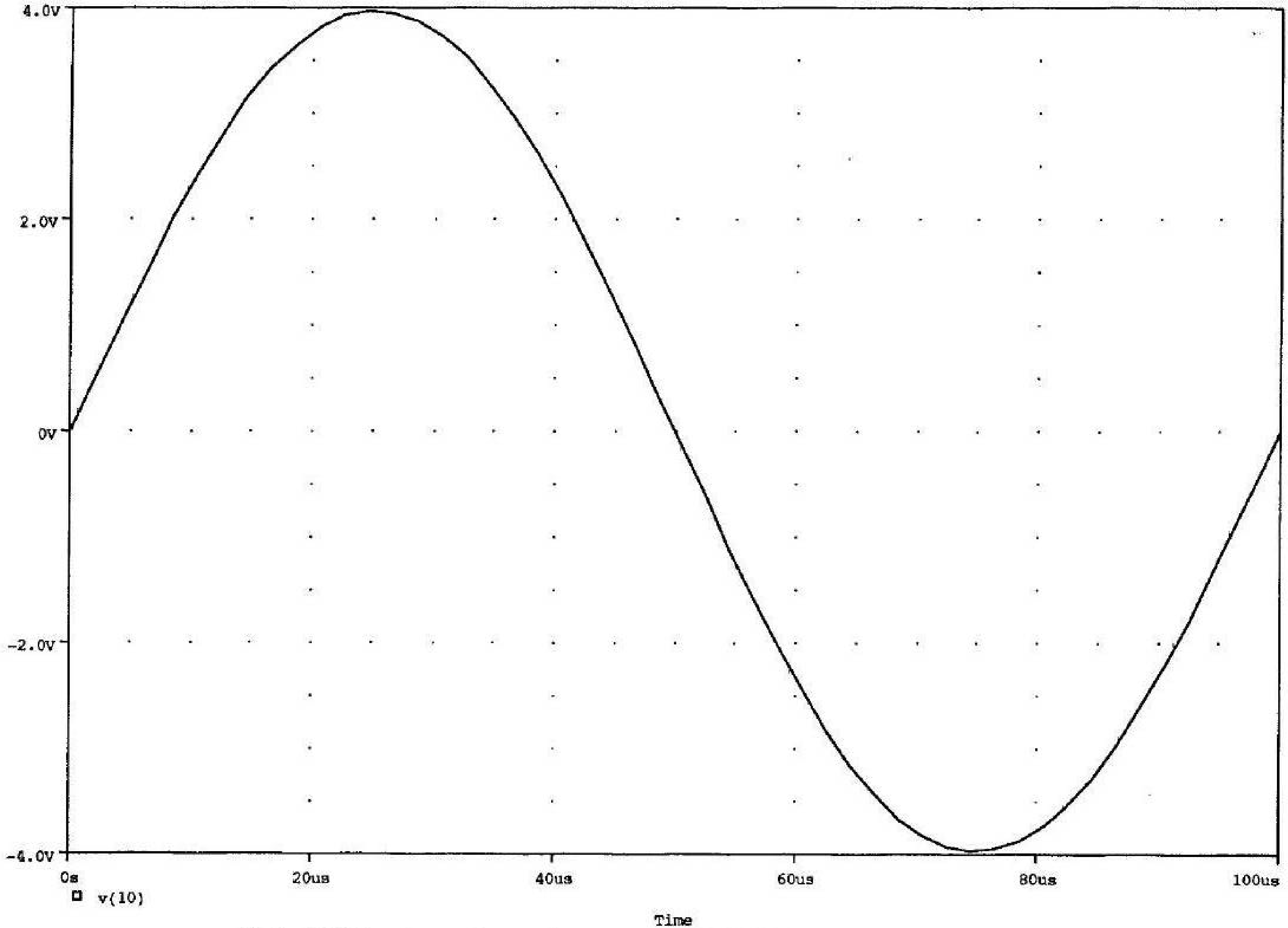

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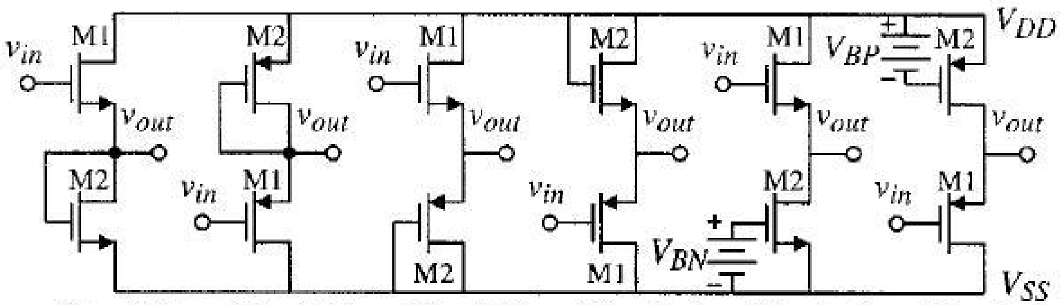
Problem 2 - (10 points)

Six versions of a source follower are shown below. Assume that $K'_N = 2K'_P$, $\lambda_P = \lambda_N$, all W/L ratios of all devices are equal, and that all bias currents in each device are equal. Neglect bulk effects in this problem and assume no external load resistor. Identify which circuit or circuits have the following characteristics: (a.) highest small-signal voltage gain, (b.) lowest small-signal voltage gain, (c.) the highest output resistance, (d.) the lowest output resistance, (e.) the highest v_{out}(max) and (f.) the lowest v_{out}(max).



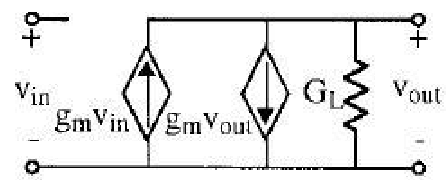
Circuit 1 Circuit 2 Circuit 3 Circuit 4 Circuit 5 Circuit 6
FS02E1P1

Solution

(a.) and (b.) - Voltage gain. Small signal model:

The voltage gain is found as: $\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + G_L}$

where G_L is the load conductance. Therefore we get:



Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{g_{mN}}{g_{mN}+g_{mN}}$	$\frac{g_{mP}}{g_{mP}+g_{mP}}$	$\frac{g_{mN}}{g_{mN}+g_{mP}}$	$\frac{g_{mP}}{g_{mP}+g_{mN}}$	$\frac{g_{mN}}{g_{mN}+g_{dsN}+g_{dsP}}$	$\frac{g_{mP}}{g_{mP}+g_{dsN}+g_{dsP}}$

But $g_{mN} = \sqrt{2}\, g_{mP}$ and $g_{dsN} = g_{dsP}$, therefore

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{1}{2}$	$\frac{1}{2}$	0.5858	0.4142	$\frac{g_{mP}}{g_{mP}+(g_{dsP}+g_{dsN})/\sqrt{2}}$	$\frac{g_{mP}}{g_{mP}+g_{dsP}+g_{dsN}}$

Thus, circuit 5 has the highest gain and circuit 4 the lowest gain

(c.) and (d.) - Output resistance.

The denominators of the first table show the following:

Ckt.6 has the highest output resistance and Ckt. 1 the lowest output resistance.

(e.) Assuming no current has to be provided by the output, circuits 2, 4, and 6 can pull the output to V_{DD}. ∴ Circuits 2, 4 and 6 have the highest output swing.

(f.) Assuming no current has to be provided by the output, circuits 1, 3, and 5 can pull the output to ground. ∴ Circuits 1, 3 and 5 have lowest output swing.

Summary

(a.) Ckt. 5 has the highest voltage gain
(b.) Ckt. 4 has the lowest voltage gain
(c.) Ckt. 6 has the highest output resistance
(d.) Ckt. 1 has the lowest output resistance
(e.) Ckts. 2,4 and 6 have the highest output
(f.) Ckts. 1,3 and 5 have the lowest output

Problem 3 - (10 points)

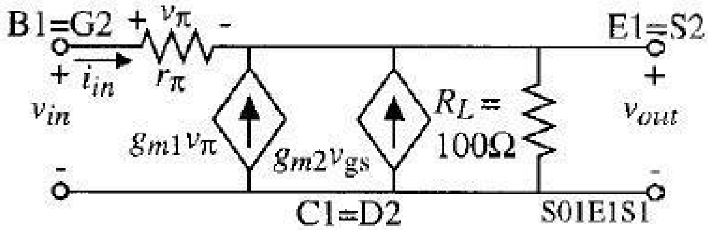
A push-pull follower is shown which uses an NPN BJT and a p-channel MOSFET. In this problem, ignore the bulk effect, the channel length modulation, and the Early voltage. The parameters for the NPN BJT are $\beta_F = 100$, $I_s = 10\text{fA}$ and $V_t = 25.9\text{mV}$. The model parameters for the PMOS are $K_P' = 50\mu\text{A/V}^2$ and $V_{TP} = -0.7\text{V}$. (a.) Find the value of the dc batteries, V_1 and V_2, which will cause 120µA to flow in Q1 and M2 when the dc value of $v_{IN} = 0\text{VDC}$. (b.) Find the small-signal input resistance, output resistance (not including R_L) and voltage gain when the dc value of $v_{IN} = 0\text{VDC}$.

Solution

(a.) $V_1 = V_{BE1} = V_t \ln\left(\frac{i_C}{I_s}\right) = 0.0259\ln\left(\frac{120\mu\text{A}}{10\text{fA}}\right) = 0.5964\text{V}$ $\boxed{V_1 = 0.6\text{ V}}$

$V_2 = V_{SG2} = \sqrt{\frac{2I_D}{K_P'(\text{W/L})}} + |V_{TP}| = \sqrt{\frac{2\cdot 120}{50\cdot 100}} + 0.7 = 0.92\text{V}$ $\boxed{V_2 = 0.92\text{V}}$

(b.) Small-signal model (simplified):



$g_{m1} = \frac{I_{C1}}{V_t} = \frac{120\mu\text{A}}{25.9\text{mV}} = 4.63\text{ mS}$

$r_{\pi 1} = \frac{1+\beta_F}{g_{m1}} = 21.80\text{ k}\Omega$

$g_{m2} = \sqrt{\frac{2K_P' W_2 I_{D2}}{L_2}} = \sqrt{2\cdot 50\cdot 100\cdot 120} = 1.09\text{mS}$

R_{in}: $v_{in} = r_{\pi 1} i_{in} + (i_{in} + g_{m1} v_\pi + g_{m2} v_{gs2})R_L = r_{\pi 1} i_{in} + (i_{in} + g_{m1} r_{\pi 1} i_{in} + g_{m2} r_{\pi 1} i_{in})R_L$

$$R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi 1} + R_L + g_{m1} r_{\pi 1} R_L + g_{m2} r_{\pi 1} R_L = r_{\pi 1} + R_L(1+\beta_F) + g_{m2} r_{\pi 1} R_L$$

$\therefore R_{in} = 21.80 + 101\text{x}.100 + 1.09\text{x } 21.80\text{x } 0.1 = 34.28\text{ k}\Omega$ $\boxed{R_{in} = 34.28\text{ k}\Omega}$

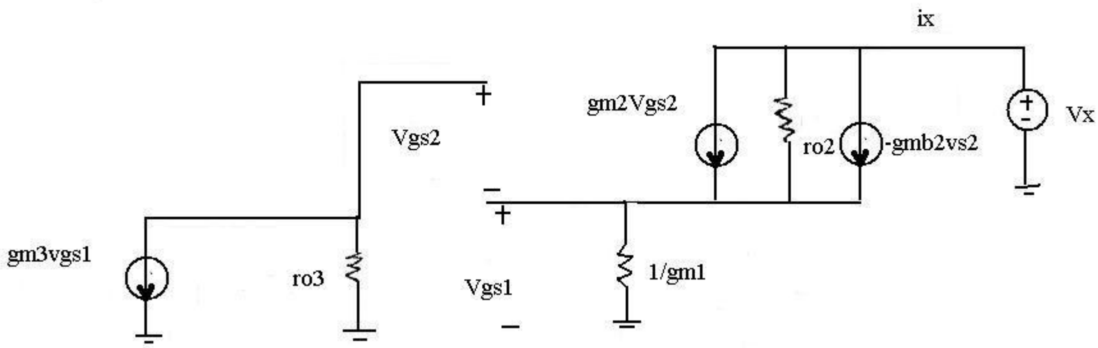
R_{out}: $R_{out} = \frac{1}{g_{m1}} \| \frac{1}{g_{m2}} = \frac{1}{4.63+1.09\text{mS}} = 174.8\text{k}$ $\boxed{R_{out} = 174.8\text{ k}\Omega}$

$\frac{v_{out}}{v_{in}}$: $\frac{v_{out}}{v_{in}} = \frac{v_{out}}{i_{in}}\frac{i_{in}}{v_{in}} = \frac{R_L(1+\beta_F) + g_{m2} r_{\pi 1} R_L}{r_{\pi 1} + R_L(1+\beta_F) + g_{m2} r_{\pi 1} R_L} = \frac{12.47}{34.28}$

$\boxed{\frac{v_{out}}{v_{in}} = 0.364\quad \text{V/V}}$

Problem 4 - (10 points)

Derive an expression for the output resistance of the MOS Wilson current mirror shown in Figure 4.15 of GHLM .



$$v_{gs_2} = v_{g_2} - v_{s_2} = -g_{m3} v_{gs_1} r_{o3} - \frac{i_x}{g_{m1}} = -(g_{m3} r_{o3} + 1)\frac{i_x}{g_{m1}}$$

$$V_x = (i_x - g_{m2} v_{gs2} + g_{mb2} v_{s_2}) r_{o2} + v_{gs_1}$$

$$V_x = \left(i_x + i_x \frac{g_{m2}(1 + g_{m3} r_{o3}) + g_{mb2}}{g_{m1}}\right) r_{o2} + \frac{i_x}{g_{m1}}$$

$$\rightarrow \frac{V_x}{i_x} = r_{o2} + \frac{1}{g_{m1}} + \frac{g_{m2} r_{o2}(1 + g_{m3} r_{o3}) + g_{mb2} r_{o2}}{g_{m1}}$$

$$\frac{V_x}{i_x} \simeq r_{o2} + g_{m2} r_{o2} r_{o3} \frac{g_{m3}}{g_{m1}} + \frac{g_{m2} r_{o2}}{g_{m1}} + \frac{g_{mb2} r_{o2}}{g_{m1}}$$

$$R_{out} \simeq r_{o2}\left(2 + g_{m2} r_{o3} + \frac{g_{mb2}}{g_{m1}}\right)$$

$$\boxed{R_{out} \simeq r_{o2}(2 + g_{m2} r_{o3})}$$