

**Ph.D. Preliminary Examination  
Fall 2009 Solutions**

**Code Number** \_\_\_\_\_

**Instructions:**

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Note that **Problem A occupies B pages, Problem C occupies D pages, and Problem E occupies F pages.** Including the cover sheet, you should have **31 pages.** There should be no blank pages in the booklet.
2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.
3. All wireless devices must be turned off for the entire duration of the exam.
4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.
5. Your examination code number **MUST APPEAR ON EVERY SHEET.** This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. **DO NOT** write your name on any of these sheets. Use the preprinted numbers whenever possible, or **WRITE LEGIBLY!!!**
6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. **DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!**
7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM.
8. When you hand in the exam:
  - (a) Separate the 8 problems to be graded as explained above.
  - (b) Check to see that your Code Number is in **EVERY** sheet you are turning in.
  - (c) On the section at the bottom of this page, **CIRCLE** the problem numbers that you are turning in for grading.
  - (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
  - (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25		

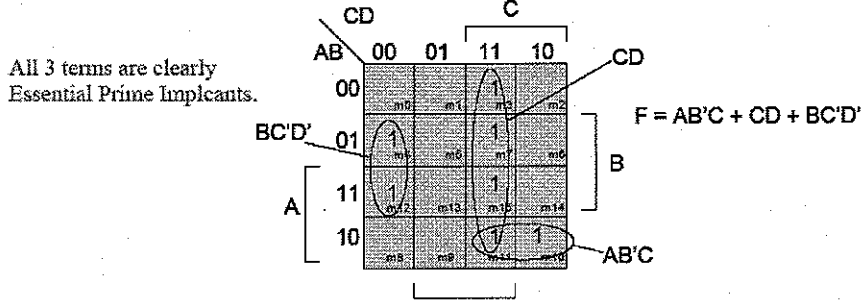
# Problem 1 (Core: CompE-ECE2030)

Code Number: \_\_\_\_\_

Use CMOS technology (nMOS and pMOS FETs) to implement the function below as a minimized complex gate (with a minimum number of transistors). Assume that the variables and their complements are available as inputs to your circuit.

$$F(A,B,C,D) = AB'C + CD + BC'D'$$

(a) Use a four-variable Karnaugh-Map (K-Map) to show that this expression is in the **minimum-sum-of-products** form. (Hint: What are the essential prime implicants for this function?)



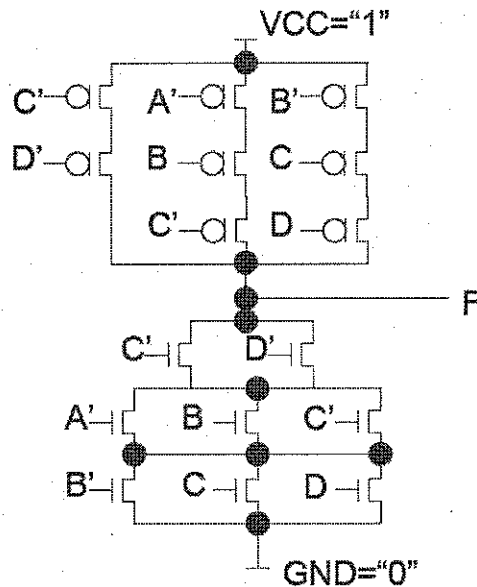
(b) Write the **complement** of the function (i.e.  $F'$ ) as a **product-of-sums** form (Hint: deMorgan).

$$F'(A,B,C,D) = (AB'C + CD + BC'D')' = (A'+B+C')(C'+D')(B'+C+D) \quad (\text{by DeMorgan's Theorem})$$

Note: This is useful for constructing the nMOS section in part c, below.

(c) Draw a minimized transistor-level schematic for the function, using standard symbols to represent the nMOS and pMOS transistors.

Note:  $F(A',B',C',D') = A'BC' + C'D' + B'CD$  which is useful in constructing the pMOS section



**Problem 2 (Core: CompE-ECE2030)**

**Code Number:** \_\_\_\_\_

- a. (3pts) Express the following function as a Sum-of-Minterms. Write your answer in the box below.

$$F = AB + \bar{B}(\bar{A} + \bar{C})$$

$$F = AB + \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$F = ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$\Sigma m(0, 1, 4, 6, 7)$

- b. (3pts) Express the following function as a Product-of-Maxterms. Write your answer in the box below.

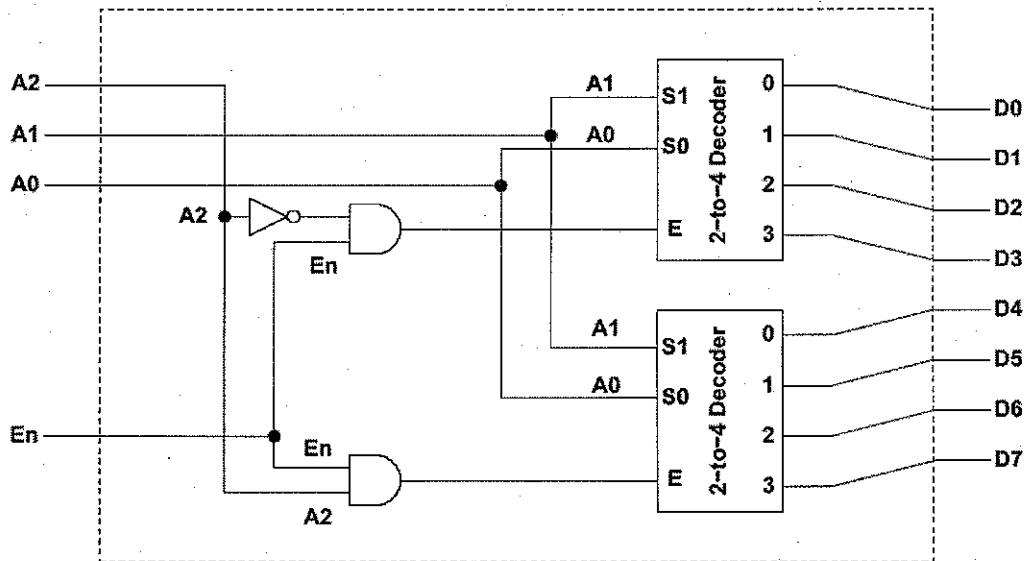
$$F = AB + \bar{B}(\bar{A} + \bar{C})$$

from the table above

$$F = (A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

$\Pi M(2, 3, 5)$

- c. (4pts) Make the appropriate additions to the two decoders shown below to make the entire system act like a 3-to-8 bit decoder. All necessary lines must be shown (except power and ground) and the lines should be clear and neat—you may also use labels in place of drawing wires. You may add inverters and AND or OR gates as needed. Note, when the enable line is low, all outputs are low.



**Problem 3 (Core: CompE-ECE3055)****Code Number:** \_\_\_\_\_**[solution]**

a)

$$1\text{GB}/8\text{KB} = 2^{17} = 131,072 \text{ entries}$$

b)

8KB indicates a 13-bit page offset

(1) =	0000	0000	0001	0010	1010	1111	0000	0000
(2) =	0000	0000	0001	0010	1010	1111	0010	0000
(3) =	0000	0000	0001	0010	1011	1111	0000	0000
(4) =	0000	0000	0001	0010	1010	1111	0001	0000
(5) =	0000	0000	0001	0010	1110	1111	0000	0000

There are two pages allocated, thus 2 entries.

The 1<sup>st</sup> page's entry content: VPN value = **0x95**The 2<sup>nd</sup> page's entry content: VPN value = **0x97**

c)

16KB/32 = 512-set direct mapped cache. Thus, index = 9 bits, line offset = 5 bits

(1) =	0000	0000	0001	0010	1010	1111	0000	0000
(2) =	0000	0000	0001	0010	1010	1111	0010	0000
(3) =	0000	0000	0001	0010	1011	1111	0000	0000
(4) =	0000	0000	0001	0010	1010	1111	0001	0000
(5) =	0000	0000	0001	0010	1110	1111	0000	0000

There are 3 lines at the end. Line (5) evicts Line (1) and (4), the latter 2 shares the same line.

The 1<sup>st</sup> line for access (2): set#=0x179 Tag=0x4AThe 2<sup>nd</sup> line for access (3): set#=0x1F8 Tag=0x4AThe 3<sup>rd</sup> line for access (5): set#=0x178 Tag=0x4B

**Problem 4 (Core: CompE-ECE3060) Code Number: \_\_\_\_\_**

Consider the function  $f = \overline{(a+b).(b+c).d}$

Implement the above function using a static-CMOS logic using NO MORE THAN 8 transistors.

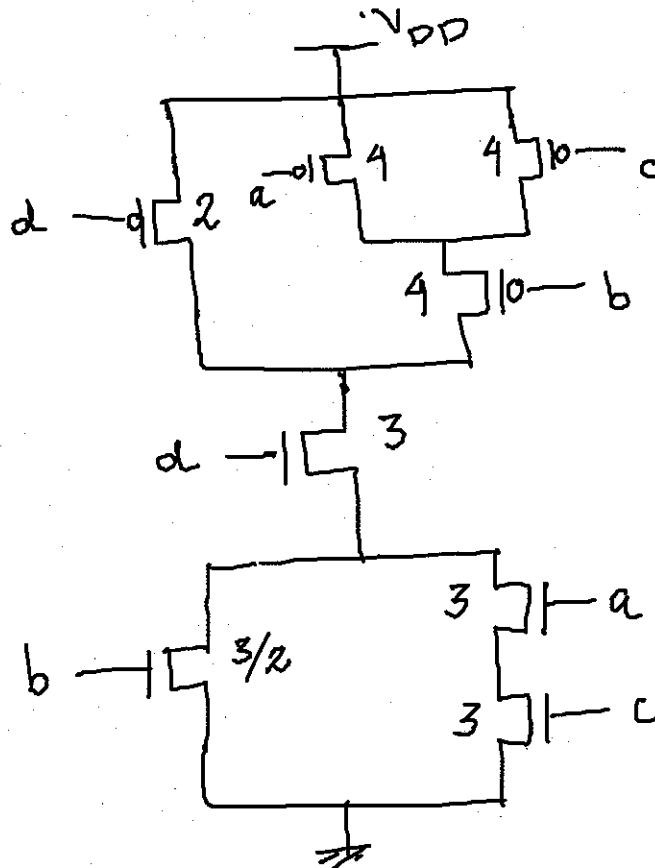
(a) Show the reduced functional form that you will implement.

$$f = \overline{(a+b).(b+c).d} = \overline{(ab+bb+bc+ac).d} = \overline{(ab+b+bc+ac).d}$$

$$= \overline{(ab+b(1+c)+ac).d} = \overline{((a+1)b+ac).d} = \overline{(b+ac).d}$$

(b) Draw the transistor level circuit diagram for the reduced functional obtained from part-a. You need to ensure that the junction capacitance connected to the output node is as small as possible.

Size the NMOS and PMOS devices so that the worst-case pull-up and pull-down resistances are same as that of an inverter with width of NMOS= $W$  and PMOS= $2W$  (the channel length of NMOS and PMOS are assumed to be same).



(c) Consider a step input and only **one input is allowed to make a transition** at one time. What are the input patterns that give the worst case high-to-low delay (i.e. output switches from '1' to '0'). State clearly what is the initial input pattern and what is the final pattern in the table below. For full-credit you need to consider the capacitances at the intermediate node.

	A	B	C	D
INITIAL				
FINAL				

Answer:

Answer that will fetch "1" Point

Input making transition – 'c'

	A	B	C	D
INITIAL	1	0	0	1
FINAL	1	0	1	1

Answer that will fetch "0.5" Point

Input making transition – 'a'

	A	B	C	D
INITIAL	0	0	1	1
FINAL	1	0	1	1

Input making transition – 'd'

	A	B	C	D
INITIAL	1	0	1	0
FINAL	1	0	1	1

Problem 5 (Core: E&M ECE3025) Code Number: \_\_\_\_\_

A coaxial transmission line is constructed from two conducting cylindrical shells, one of radius 2 mm and the other of radius 5 mm, centered along the  $z$ -axis and separated by a material with  $\mu_r = 1.0$  and  $\epsilon_r = 2.2$ . When charged from a DC source, the electric field between the cylinders is

$$\bar{E} = \frac{1.637}{\rho} \hat{\rho} \quad (\text{V/m})$$

where  $\rho$  is in units of meters. You may use  $\epsilon_0 = 8.854 \times 10^{-12}$  F/m. Where possible, provide numerical answers for the following:

- (a) What is the voltage difference between the inner and outer conductors?

Solution:  $V = \int_{0.002}^{0.005} \bar{E} \cdot d\bar{\ell} = 1.637 \ln\left(\frac{0.005}{0.002}\right) = 1.5 \text{ V}$

- (b) What is the displacement field (electric flux density)  $\bar{D}$  between the cylinders?

Solution:  $\bar{D} = \epsilon_0 \epsilon_r \bar{E} = \frac{(8.854 \times 10^{-12})(2.2)(1.637)}{\rho} \hat{\rho} = \frac{3.189 \times 10^{-11}}{\rho} \hat{\rho} \text{ C/m}^2$

- (c) Find the magnitude of the surface charge density  $\rho_s$  on either cylinder.

Solution:  $\rho_s|_{\rho=0.002} = \hat{\rho} \cdot \bar{D}|_{\rho=0.002} = \frac{3.189 \times 10^{-11}}{0.002} = 1.594 \times 10^{-8} \text{ C/m}^2$

$$\rho_s|_{\rho=0.005} = -\hat{\rho} \cdot \bar{D}|_{\rho=0.005} = -\frac{3.189 \times 10^{-11}}{0.005} = -6.378 \times 10^{-9} \text{ C/m}^2$$

- (d) What is the capacitance per unit length of the coaxial line?

Solution:  $C/\ell = \frac{2\pi\rho\rho_s}{V}\bigg|_{\rho=0.002} = \frac{(2\pi)(0.002)(1.594 \times 10^{-8})}{1.5} = 1.336 \times 10^{-10} \text{ F/m}$

- (e) What is the total energy per unit length stored in this field?

Solution #1:  $W_e/\ell = \frac{1}{2}CV^2 = \frac{(1.336 \times 10^{-10})(1.5)^2}{2} = 1.503 \times 10^{-10} \text{ J/m}$

Solution #2:  $W_e/\ell = \iint \frac{1}{2} \epsilon_0 \epsilon_r \bar{E} \cdot \bar{E} dv = \frac{(8.854 \times 10^{-12})(2.2)(1.637)^2}{2} 2\pi \int_{0.002}^{0.005} \frac{1}{\rho^2} \rho d\rho$   
 $= 1.503 \times 10^{-10} \text{ J/m}$

**Problem 6 (Core: E&M-ECE3065)**

**Code Number:** \_\_\_\_\_

1. (15p)

A lossless transmission line of electrical length  $l=0.35 \lambda$  and characteristic impedance  $Z_0=100\Omega$  is terminated in a load impedance  $Z_L=60\Omega$ .

(a) Find the reflection coefficient  $\Gamma$  and the standing wave ratio  $S$  at the load.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 - 100}{60 + 100} = -0.25$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.67$$

(b) Assuming that the incident power density to  $Z_L$  is  $0.15 \text{ W/m}^2$ , what is the reflected power density?

$$S_{\text{refl}} = S_{\text{inc}} |\Gamma|^2 = 0.15 \times 0.25^2 = 0.009375 \text{ W/m}^2$$

(c) Find the input impedance  $Z_{in}$  at the input of the transmission line [Formula:  $Z_{in} = Z_0 \times (Z_L + jZ_0 \tan(\beta l)) / (Z_0 + jZ_L \tan(\beta l))$ ]

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} = 100 \cdot \frac{60 + j100 \tan(0.7\pi)}{100 + j60 \tan(0.7\pi)} = 115.78 \angle -26.9^\circ$$

$$= 103.25 - j52.4 \Omega$$

(d) Design a quarter-wavelength transformer matching the  $Z_L$  to  $Z_0$ . Give the electrical length, the physical length and the characteristic impedance  $Z_1$  of the transformer. ( $\mu_r = 1.5 \times 10^8$ ,  $f = 150 \text{ MHz}$ )

Electrical length:  $\lambda/4$

Physical length:  $l = \lambda/4 = \frac{u_p/f}{4} = \frac{1.5 \times 10^8}{150 \times 10^6 \times 4} = 0.25 \text{ m}$

Characteristic impedance:  $Z_1 = \sqrt{Z_0 \cdot Z_L} = \sqrt{6000} = 77.46 \Omega$

(e) If  $Z_L$  gets replaced by  $Z_L = j60 \Omega$  can you still match it to  $Z_0$  with a quarter-wavelength transformer and why?

No, because  $Z_1 = \sqrt{Z_0 \cdot Z_L}$  would be a complex number.

**Problem 7 (Core: EDA-ECE2040)****Code Number:** \_\_\_\_\_

As far as the natural response of the circuit is concerned, the two resistors are in parallel; hence,

$$\tau = R_{eq}C = (5\Omega)(2\mu\text{F}) = 10\mu\text{s}$$

By continuity,  $v_C(0^+) = v_C(0^-) = 0$ . Furthermore, as  $t \rightarrow \infty$ , the capacitor becomes an open circuit, leaving  $20\Omega$  in series with the  $50\text{V}$ . That is,

$$i(\infty) = \frac{50}{20} = 2.5\text{A} \quad v_C(\infty) = (2.5\text{A})(10\Omega) = 25\text{V}$$

Knowing the end conditions on  $v_C$ , we can write

$$v_C = [v_C(0^+) - v_C(\infty)]e^{-t/\tau} + v_C(\infty) = 25(1 - e^{-t/10}) \quad (\text{V})$$

wherein  $t$  is measured in  $\mu\text{s}$ .

The current in the capacitor is given by

$$i_C = C \frac{dv_C}{dt} = 5e^{-t/10} \quad (\text{A})$$

and the current in the parallel  $10\Omega$  resistor is

$$i_{10\Omega} = \frac{v_C}{10\Omega} = 2.5(1 - e^{-t/10}) \quad (\text{A})$$

Hence,

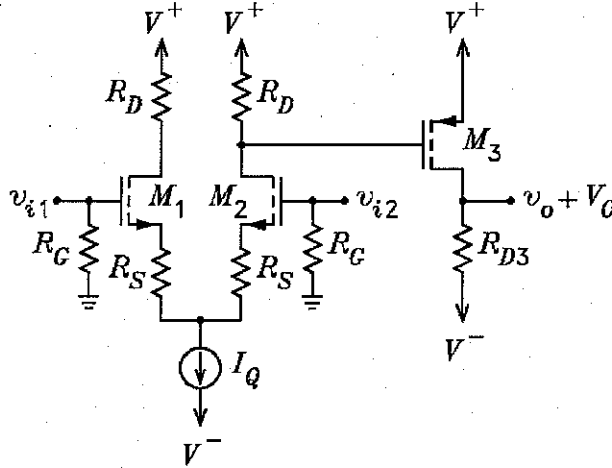
$$i = i_C + i_{10\Omega} = 2.5(1 + e^{-t/10}) \quad (\text{A})$$

The problem might also have been solved by assigning mesh currents and solving simultaneous differential equations.

**Problem 8 (Core: EDA-ECE3050)**

**Code Number:** \_\_\_\_\_

Given  $V^+ = +15\text{V}$ ,  $V^- = -15\text{V}$ ,  $I_Q = 2.5\text{mA}$ ,  $R_G = 10\text{M}\Omega$ ,  $R_D = 1.6\text{k}\Omega$ ,  $R_S = 100\Omega$ ,  $R_{D3} = 12\text{k}\Omega$ . The current flowing into the drains of  $M_1$  and  $M_2$  is given by  $i_D = 0.004(v_{GS} - 1.5)^2$ . The current flowing out of the drain of  $M_3$  is given by  $i_{D3} = 0.004(v_{SG} - 1.5)^2$ .



- (a) With  $v_{i1} = v_{i2} = 0$ , solve for the dc output voltage  $V_O$ .

$$V_{SG3} = \frac{I_Q R_D}{2} = 2\text{V} \quad I_{D3} = 0.004(2 - 1.5)^2 = 1\text{mA} \quad V_O = V^- + I_{D3} R_{D3} = -3\text{V}$$

- (b) Solve for the small-signal ac output voltage  $v_o$  as a function of the small-signal input voltages  $v_{i1}$  and  $v_{i2}$ .

$$g_{m1} = g_{m2} = 2\sqrt{0.004 \times I_Q / 2} = 4.471\text{mA/V} \quad g_{m3} = 2\sqrt{0.004 \times I_{D3}} = 4\text{mA/V}$$

$$v_o = \frac{(v_{i1} - v_{i2})}{g_{m1}^{-1} + 2R_S + g_{m2}^{-1}} \times R_D \times (-g_{m3} R_{D3}) = -118.7(v_{i1} - v_{i2})$$

**Problem 9 (Core: Power-ECE3070)****Code Number: \_\_\_\_\_**

(\* Check with Fall 2005 prelim problem?)

**1. SOLUTION**

a)

$$\text{Output power} = 1.732 \times 13,800 \times I \times 0.8$$

$$\text{Line current} = I = 784.46 \text{ Amperes}$$

$$\text{Apparent power} = 1.732 \times 13,800 \times 784.46 = 18.750 \text{ MVA}$$

$$\text{Reactive power} = (\text{Apparent power}^2 - \text{Load power}^2)^{0.5} = 11.250 \text{ MVAR}$$

b)

At full current, the Apparent Power can be calculated to be

$$\text{Apparent Power} = 1.732 \times 13,800 \times 1000 = 23.901 \text{ MVA}$$

$$\text{Net Real Power} = 22.5 + 0.7*S$$

$$\text{Net Reactive Power} = 11.25 + 0.7141*S$$

$$23.9^2 = (15 + 0.7*S)^2 + (11.25 + 0.7141*S)^2$$

$$\text{Solving for } S = 5 \text{ MVA, } P = 3.5 \text{ MW, } Q = 3.6 \text{ MVAR}$$

c)

Capacitance should be 14.85 MVAR to give unity power factor

This is 4.95 MVAR/capacitor

In delta – 13,800 volts across it –  $X_c = 38.47$  ohms or 68.9 microfarads.

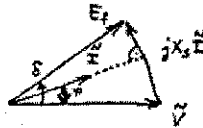
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### Problem 10 (Core: Power-ECE3070)

Code Number: \_\_\_\_\_

1. Synchronous machine :  $S_n = 7,500$  kVA  
 3-phase, Y-connected  
 $V_n = 2,300$  V  
 $f_n = 60$  Hz  
 $X_s = 1.95$   $\Omega$ /phase  
 $r_s = 0$   $\Omega$ /phase

- a) Synchronized to 3-phase network.  $I_f = 100$  A at synchronization. Turbine valve opened until active power reaches  $P_g = 300$  kW. Find the power angle  $\delta$ , stator current  $I_s$ , and power factor in this operating condition.

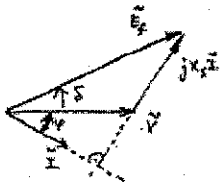


$$P_g = \frac{200 \cdot 10^3}{3} = \frac{E_f \cdot V}{X_s} \cdot \sin \delta \Rightarrow \delta = \sin^{-1} \left\{ \frac{P_g \cdot X_s}{E_f \cdot V} \right\} = 6.35^\circ$$

$$\varphi = \frac{\delta}{2} = \frac{6.35}{2} = 3.18^\circ \Rightarrow \text{pf} = \cos 3.18^\circ = 0.9985 \text{ leading}$$

$$\vec{I}_s = \frac{\vec{E}_f - \vec{V}}{jX_s} = \frac{2,300}{\sqrt{3}} \cdot \frac{1/\angle 6.35^\circ - 1/\angle 0^\circ}{j1.95} = 75.4 \angle 3.18^\circ \text{ A}$$

- b) Rotor current has then been increased to  $I_f = 180$  A without other changes. Calculate the power angle  $\delta$ , power factor, and stator current in this operating condition, as well as  $Q_g$ .



$$\vec{E}_f = \frac{I_f^{\text{new}}}{I_f^{\text{old}}} \cdot \vec{E}_f^{\text{old}} = \frac{180}{100} \cdot \frac{2,300}{\sqrt{3}} = 2,390.2 \text{ V} \quad \left( \frac{2,300}{\sqrt{3}} = 1,328 \right)$$

$$\delta = \sin^{-1} \left\{ \frac{P_g \cdot X_s}{E_f \cdot V} \right\} = \sin^{-1} \left\{ \frac{300 \cdot 10^3 \cdot 1.95}{2,390.2 \cdot 1,328} \right\} = 3.52^\circ$$

$$\vec{I}_s = \frac{\vec{E}_f - \vec{V}_a}{jX_s} = \frac{2,390.2 \angle 3.52^\circ - 1,328 \angle 0^\circ}{j1.95} = 547.6 \angle -82.1^\circ \text{ A}$$

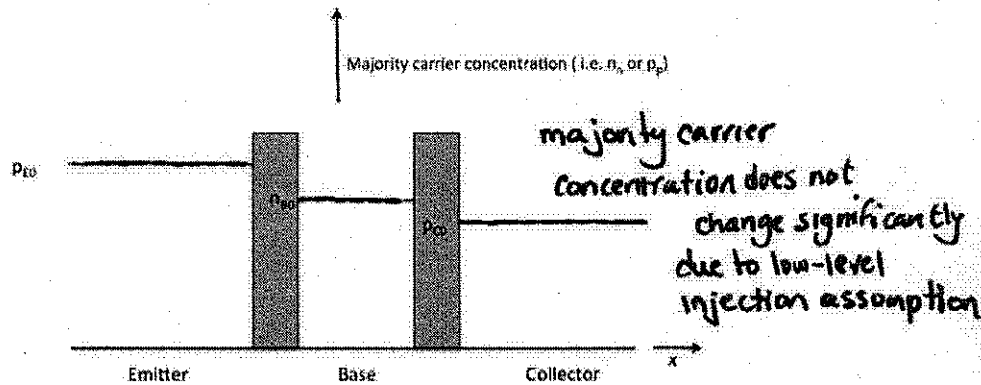
$$\text{pf} = \cos(-82.1^\circ) = 0.1374 \text{ lagging}$$

$$Q_g = 3VI \sin \varphi = 3 \cdot 1,328 \cdot 547.6 \cdot \sin(82.1^\circ) = 2,160.9 \text{ kVAR}$$

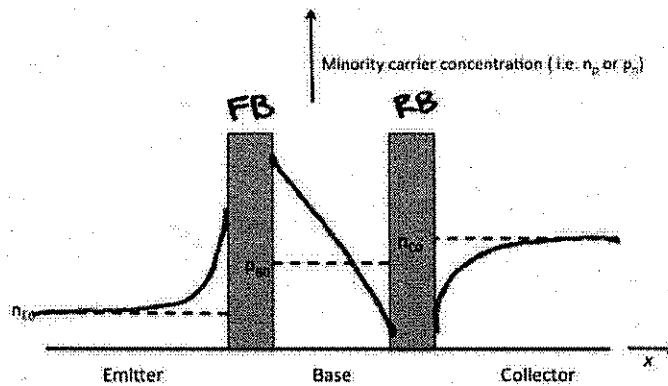
**Problem 11 (Core: Microsystems-ECE3040) Code Number: \_\_\_\_\_**

In this problem, you will be asked to show qualitatively the steady state majority or minority carrier concentration in the quasi-neutral region of a bipolar junction transistor (BJT) on a given set of plots. Please note that the x and y axis on these plots have a linear scale. The y-axis is carrier concentration and the x-axis represents the position in the BJT. The regions in grey represent the depletion regions around each junction. The carrier concentrations are not drawn to proper scale, but only reflect the fact that doping concentration in the emitter ( $N_E$ ), base ( $N_B$ ), and the collector ( $N_C$ ) have the following relationship:  $N_E \gg N_B > N_C$ . You may assume that the biasing allows for a low-level injection approximation to carrier dynamics, and that the equilibrium majority ( $p_{E0}$ ,  $n_{B0}$ ,  $p_{C0}$ ) and the minority ( $n_{E0}$ ,  $p_{B0}$ , and  $n_{C0}$ ) concentrations are represented by dashed lines in the below plots.

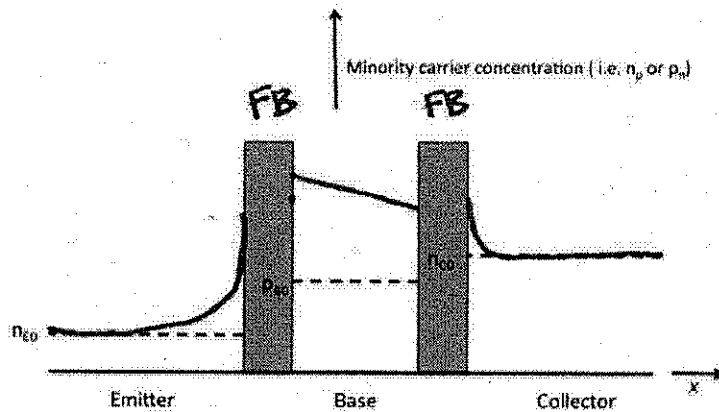
- a) Please assume that the BJT is in forward active mode of biasing. On the following plot, draw the approximate steady state **majority** carrier concentrations in the quasi-neutral regions.



- b) Please assume that the BJT is in forward active mode of biasing. On the following plot, draw the approximate steady state **minority** carrier concentrations in the quasi-neutral regions.



- c) Please assume that the BJT is in saturation mode of biasing. On the following plot, draw the approximate steady state minority carrier concentrations in the quasi-neutral regions.



- d) From the given description so far in this problem, please indicate whether this is a *pn*p or *n*p*n* BJT.

*pn*p

- e) In a typical BJT, please comment on why the emitter doping is much higher than the base doping.

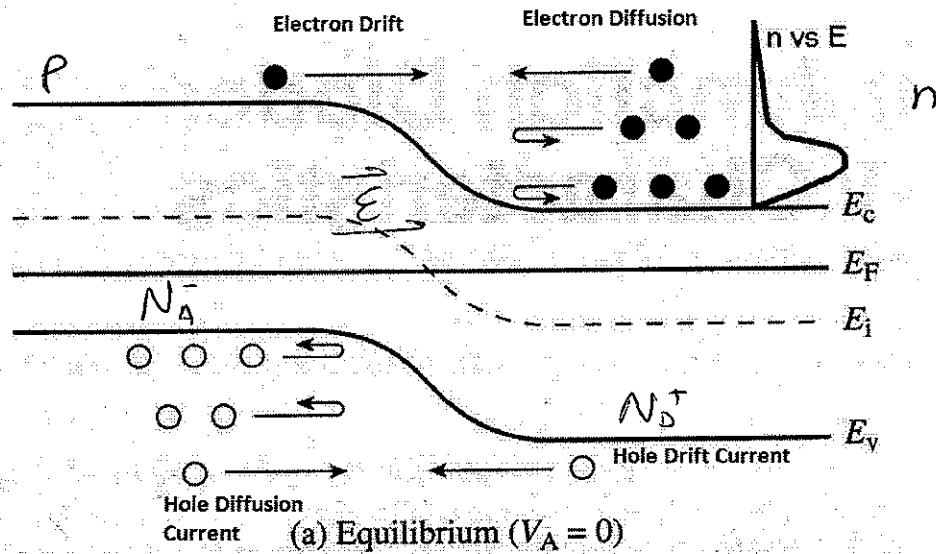
This is so that the hole current for this *pn*p BJT dominates the total current across the *EB* junction. This causes the emitter efficiency to approach one and results in high current gain. The source of this high current gain is in part due to the fact that small changes in the electron base current result in large changes in the hole current across the *EB* junction.

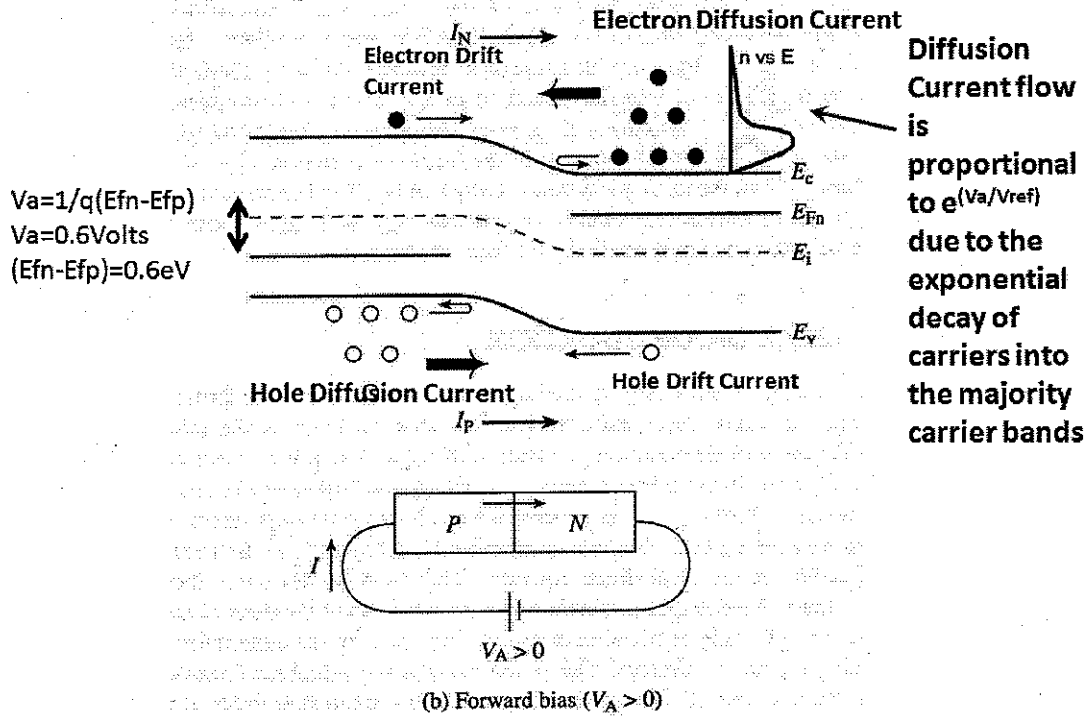
**Problem 12 (Core: Microsystems-ECE3080) Code Number: \_\_\_\_\_**

Problem statement:

- Draw the energy band diagram of a p-n homojunction diode at zero bias AND biased into forward bias at 0.6 volts. Be sure to indicate and label the position of the quasi-fermi levels (qualitatively with respect to the bands), the conduction band, valence band, and position of the dopants.
- Indicate the direction of motion due to electron drift, electron diffusion, hole drift, and hole diffusion as well as the direction of all four currents related to these motions.
- What is the difference (numeric answer) in the quasi-fermi levels?

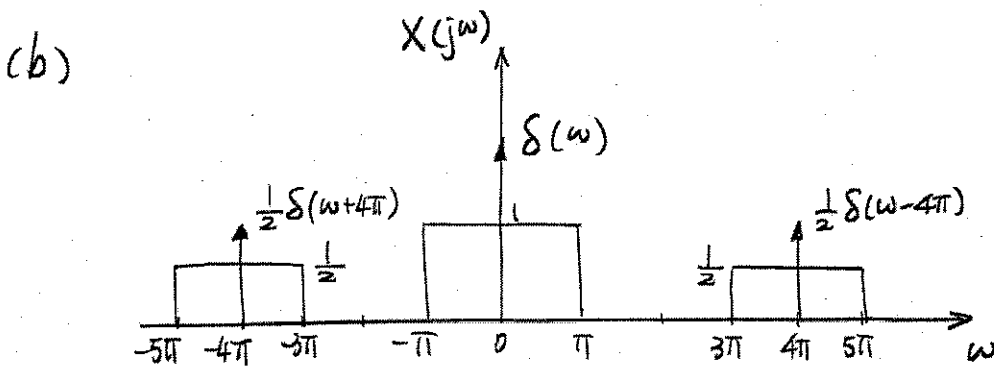
In Equilibrium, the Total current balances due to the sum of the individual components



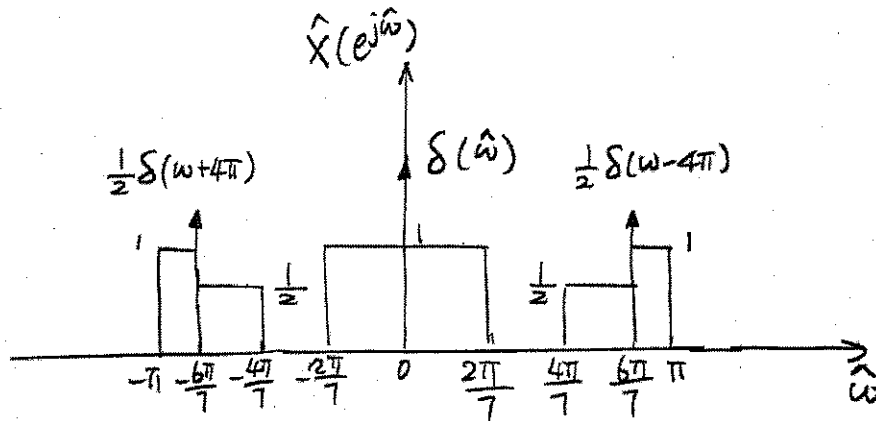


Note: Hole motion and hole current are in the same directions whereas electron motion and electron current are in opposite directions.  $E_{fn} - E_{fp} = 0.6 \text{ eV}$

$$\begin{aligned}
 (a) \quad m(t) &= \frac{1}{2\pi} \int_{-3\pi}^{-\pi} e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{\pi}^{3\pi} e^{j\omega t} d\omega \\
 &\quad + \frac{1}{2\pi} \int_{-\infty}^{+\infty} \delta(\omega+2\pi) e^{j\omega t} d\omega \\
 &\quad + \frac{1}{2\pi} \int_{-\infty}^{+\infty} \delta(\omega-2\pi) e^{j\omega t} d\omega \\
 &= \frac{1}{2\pi} \left( \frac{1}{jt} (e^{-j\pi t} - e^{-j3\pi t}) + e^{j2\pi t} - e^{j\pi t} \right) \\
 &\quad + e^{-j2\pi t} + e^{j2\pi t} \\
 &= \frac{1}{2\pi} \left( \frac{1}{jt} (2j \sin(2\pi t) - 2j \sin(\pi t)) + 2 \cos(2\pi t) \right) \\
 &= \frac{1}{\pi t} \sin(\pi t) \cdot \cos(2\pi t) + \frac{1}{\pi} \cos(2\pi t)
 \end{aligned}$$



(c)

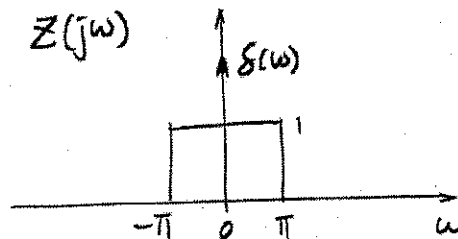


$$\hat{\omega} = \frac{\omega}{f_s} \quad \pm \frac{\pi}{3.5} = \pm \frac{2\pi}{7} \quad \pm \frac{4\pi}{3.5} = \pm \frac{8\pi}{7} \xrightarrow{\text{folded}} \pm \frac{6\pi}{7}$$

$$\pm \frac{3\pi}{3.5} = \pm \frac{6\pi}{7} \quad \pm \frac{5\pi}{3.5} = \pm \frac{10\pi}{7} \xrightarrow{\text{folded}} \pm \frac{4\pi}{7}$$

$$\pm \frac{3.5\pi}{3.5} = \pm \pi$$

(d)  $\frac{2\pi}{7} < \hat{\omega}_p < \frac{4\pi}{7}$



Note  $Y(j\omega) = M(j\omega) = Z(j\omega) * (\delta(\omega + 2\pi) + \delta(\omega - 2\pi))$

Thus,  $s(t) = 2 \cos(2\pi t)$

**Problem 14 (Core: DSP-ECE3075)**

**Code Number:** \_\_\_\_\_

Let  $X$  and  $Y$  be *i.i.d.*  $\mathcal{N}(0, 1)$  random variables, and let  $Z = \frac{X}{Y}$  be the ratio.

- (a)  $Y$  and  $Z$  are [positively correlated] [negatively correlated] [uncorrelated] (circle one)  
**Explain.**

They are uncorrelated because  $E(YZ) = E(Y)E(Z)$ : the right-hand side is zero because  $E(Y) = 0$  by definition, and the left-hand side is zero because  $E(YZ) = E(X) = 0$ .

- (b) Find  $P(X > 2Y)$ .

The difference  $X - 2Y$  is  $\sim \mathcal{N}(0, 5)$ , so the probability is  $1/2$ .

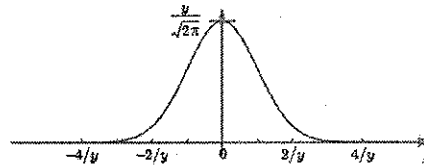
- (c) Find  $E(\cos(X))$ .

Using Euler  $\cos(X) = 0.5e^{jX} + 0.5e^{-jX}$  and the characteristic function  $E(e^{j\omega X}) = e^{-\omega^2/2}$ , yields  $E(\cos(X)) = e^{-1}$ .

- (d) Find and sketch the conditional pdf  $f(z|y)$ , carefully labeling both axes.

When  $Y = y$ ,  $Z$  reduces to  $Z = X/y \sim \mathcal{N}(0, y^{-2})$ , so that the conditional pdf is:

$$f(z|y) = \frac{y}{\sqrt{2\pi}} e^{-(yz)^2/2}$$



- (e) Find  $P(Z > 1)$ .

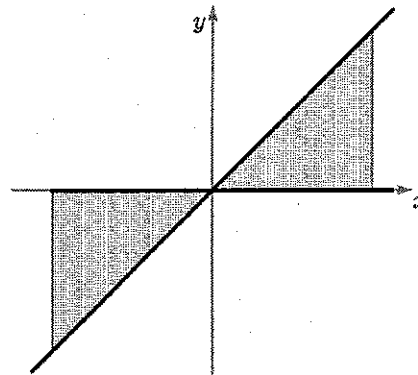
$$\begin{aligned} P(Z > 1) &= P(Z > 1, Y > 0) + P(Z > 1, Y < 0) \\ &= P(X/Y > 1, Y > 0) + P(X/Y > 1, Y < 0) \\ &= P(X > Y, Y > 0) + P(X < Y, Y < 0) \end{aligned}$$

$$= \iint_{\text{SHADED REGIONS}} f(x, y) dx dy$$

SHADED  
REGIONS

$$= 1/8 + 1/8$$

$$= 1/4.$$



We don't need to perform the integral, we can simply exploit the circular symmetry of the joint pdf  $f(x, y)$  to see that the shaded region accounts for  $1/4$  of the total volume under the joint pdf.

**Problem 15 (Core: S&C-ECE3085)**

Code Number: \_\_\_\_\_

Solution:

(a)

$$G_o(s) = \frac{2K}{s(15(10s+1)(3s+1) + 2K_T)}$$

$$G_c(s) = \frac{2K}{s(15(10s+1)(3s+1) + 2K_T) + 2K}$$

For the denominator:  $450s^3 + 195s^2 + (15 + 2K_T)s + (2K)$ 

Routh Array is:

$s^3$ :	450	15 + 2K <sub>T</sub>
$s^2$ :	195	2K
$s^1$ :	$\frac{195 + 26K_T - 60K}{13}$	0
$s^0$ :	2K	0

For stability:

$$195 + 26K_T - 60K > 0$$

$$2K > 0$$

Therefore, required condition is:

$$0 < K < \frac{13(15 + 2K_T)}{60}$$

**Problem 16 (Core: S&C-ECE3085)**

**Code Number:** \_\_\_\_\_

a) (i) PID, (ii) PD, (iii) P, (iv) PI

PD and PID speed up the response, PI and PID improve accuracy

b)  $H(s) = F(s)/G(s)$

**Problem 17 (Specialized: Comp Science-CS3210) Code Number: \_\_\_\_\_**

<i>Virtual Address</i>	<i>Page Fault? (Yes/No)</i>
2A000	<u>Y</u>
2A004	<u>N</u>
C5328	<u>Y</u>
2A008	<u>N</u>
E73AC	<u>Y</u>
2A00C	<u>N</u>
2A010	<u>N</u>
C7118	<u>Y</u>
2D200	<u>Y</u>
2D204	<u>N</u>
9A388	<u>Y</u>
2D208	<u>N</u>
9A38C	<u>N</u>
2D20C	<u>N</u>
2D210	<u>N</u>
B329C	<u>Y</u>
27358	<u>Y</u>
2735C	<u>N</u>
9B400	<u>Y</u>
27360	<u>N</u>
B89AC	<u>Y</u>

**Final contents of physical memory:**

9B	B8	E7	C7	2D	9A	B3	27
----	----	----	----	----	----	----	----

**Problem 18 (Specialized: Software Sys- ECE3035) Code Number: \_\_\_\_\_**

Below is a snapshot of heap storage. Values that are pointers are denoted with a "\$". The heap pointer is \$6160. The heap has been allocated contiguously beginning at \$6000, with no gaps between objects.

addr	value	addr	value	addr	value	addr	value	addr	value	addr	value
6000	12	6032	\$6080	6064	\$6148	6096	12	6128	12	6160	0
6004	48	6036	8	6068	4	6100	\$6080	6132	8	6164	0
6008	\$6080	6040	24	6072	12	6104	\$6136	6136	4	6168	0
6012	16	6044	4	6076	8	6108	\$6148	6140	\$6100	6172	0
6016	8	6048	8	6080	\$6032	6112	16	6144	12	6176	0
6020	8	6052	\$6136	6084	0	6116	8	6148	4	6180	0
6024	24	6056	\$6020	6088	4	6120	16	6152	8	6184	0
6028	4	6060	12	6092	\$6004	6124	8	6156	12	6188	0

**Part A** Suppose the stack holds a local variable whose value is the memory address \$6052 and a local variable whose value is \$6004. No other registers or static variables currently hold heap memory addresses. List the addresses of all objects in the heap that are *not* garbage.

**Addresses of Non-Garbage Objects:** 6052, 6136, 6020, 6100, 6080, 6148, 6032, 6004

**Part B** Create a *sorted* (by size) free list by scanning the memory for garbage, starting at address \$6000 and inserting each garbage object into the free list in increasing size order. List the *base address* of each object (not the address of its header) on the free list (in order) at the end of the scan.

**Free List:** 6092 (4), 6040 (8), 6064 (12), 6116 (16)

**Part C** If the local variable whose value is the address \$6052 is popped from the stack, which addresses will be reclaimed by each of the following strategies? If none, write "none." (You do not have to list addresses already on the free list from part B.)

<b>Reference Counting:</b>	6052, 6020
<b>Mark and Sweep:</b>	6052, 6136, 6020, 6100, 6148
<b>Old-New Space (copying):</b>	6052, 6136, 6020, 6100, 6148

**Problem 19 (Specialized: Telecom-ECE3076) Code Number: \_\_\_\_\_**

How does a reliable transport protocol (e.g. TCP) overcome the following problems:

1. Lost packet?                    **Retransmits if not Sequence Number not Acknowledged (ACKed)**    **\_\_\_\_\_**
2. Bit error in packet?            **Checksum compared to that in TCP header**    **\_\_\_\_\_**
3. Packets arrive in wrong order?    **Sequence number determines placement in received buffer**    **\_\_\_\_\_**
4. Packets from receiver lost?        **Segment retransmitted if no ACK within RTO timeout**    **\_\_\_\_\_**
5. Flow control?                    **Each TCP header carries a "window" (which limits bytes that can be sent).**    **\_\_\_\_\_**

6. What network device forwards IP datagrams? **\_\_\_\_\_ a router \_\_\_\_\_**

7. What network device protects a network from harmful IP datagrams? **\_\_\_\_\_ a firewall \_\_\_\_\_**

Name two Internet or Web parameters that must be assigned by a central organization (or coordinated organizations) to prevent duplication?

8. **\_\_\_\_\_ IP address blocks assigned to organizations \_\_\_\_\_**

9. **\_\_\_\_\_ Domain names assigned to organizations (e.g., gatech.edu)**

10. If signals travel at  $2E8$  m/s through 100 Mb/s fiber and wire connections, what is the maximum data transfer rate for a single TCP connection between a host in Atlanta and a host in Oregon (3600 km apart, 65,000 byte TCP window size).

$$RTT = 2 * 3.6E6 / 2E8 = 0.036 \text{ s}$$

$$\text{Maximum Transfer Rate} = 8 \text{ (bits/byte)} * 65,000 \text{ bytes} / 0.036 \text{ s} = \mathbf{14.4 \text{ Mb/s}}$$

**Problem 20 (Specialized: Optics-ECE4500) Code Number: \_\_\_\_\_**

**Part 1**

A maximum transmitting thin film is an anti-reflecting thin film.

$$d(\text{maximum transmission}) = \frac{\lambda}{4n_2}$$
$$n_2(\text{maximum transmission}) = \sqrt{n_3}$$

For  $\lambda = 1.000 \mu\text{m}$  and  $n_3 = 2.000$

$$d(\text{maximum transmission}) = 0.1768 \mu\text{m}$$

$$n_2(\text{maximum transmission}) = 1.414$$

**Part 2**

For  $\lambda = 1.000 \mu\text{m}$  and  $n_3 = 2.000$

$$d(\text{maximum reflection}) = \frac{\lambda}{4n_2} = 0.0625 \mu\text{m}$$

$$n_2(\text{maximum reflection}) = 4.000 \quad (\text{largest available value})$$

**Problem 21 (Specialized: Optics -ECE 4501) – Code Number: \_\_\_\_\_**

A point-to-point optical fiber communication link is to be constructed to join transmitter and receiver stations that are 100 km apart. The link consists of three segments that are end-to-end joined. The first segment is 25 km of old fiber whose quadratic dispersion is  $D_1 = 16.0$  ps/nm-km, and whose decibel loss coefficient is  $\alpha_1 = 0.30$  dB/km. The second segment is newer fiber, having dispersion  $D_2 = 6.0$  ps/nm-km, loss coefficient  $\alpha_2 = 0.15$  dB/km, and length  $L_2$ . The final segment (completing the 100 km overall distance) is dispersion-compensating fiber, having  $D_3 = -100$  ps/nm-km, loss coefficient  $\alpha_3 = 1.50$  dB/km, and length  $L_3$ . Assume all splice losses are negligible.

- a. Determine the required lengths  $L_2$  and  $L_3$  such that the 100-km link has exactly zero net quadratic dispersion.

$$D_1L_1 + D_2L_2 + D_3L_3 = 0 = 16(25) + 6L_2 - 100L_3$$

where  $L_2 = 75 - L_3$ .

Obtain  $L_2 = 67.0$  km,  $L_3 = 8.0$  km.

- b. Suppose the receiver sensitivity is -30 dBm. What is the minimum required power input to the link from the transmitter. Express your result in *both* dBm and mW.

Link loss will be

$$\text{Loss (dB)} = \alpha_1L_1 + \alpha_2L_2 + \alpha_3L_3 = 0.3(25) + 0.15(67.0) + 1.5(8.0) = 29.6 \text{ dB}$$

Transmitted power in dBm is therefore:

$$P_t(\text{dBm}) = 29.6 \text{ dB} - 30.0 \text{ dBm} = -0.4 \text{ dBm}$$

in mW this is  $P_t(\text{mW}) = 10^{-0.04} = 0.9 \text{ mW}$ .

- c. Suppose that a path-average dispersion magnitude of  $|D_{avg}| = 3$  ps/nm-km could in fact be tolerated for this link. Keeping the 25-km old fiber segment as is, what new choices could be made for  $L_2$  and  $L_3$  to reduce the overall loss, and what would the new minimum transmitter power be? (in dBm only)

$$D_{avg} = \frac{D_1L_1 + D_2L_2 + D_3L_3}{L_1 + L_2 + L_3} = \pm 3$$

$$\Rightarrow 16(25) + 6(75 - L_3) - 100L_3 = 300 \text{ (choose positive case)}$$

Obtain  $L_3 = 5.2$  km and  $L_2 = 69.8$  km.

Loss is now  $0.3(25) + 0.15(69.8) + 1.5(5.2) = 25.8$  dB, or  $P_t = 25.8 - 30.0 = -4.2$  dBm.

Problem 22 (Specialized: Microsystems-ECE4752) Code Number: \_\_\_\_\_

Solution:

Dry Oxidation

$$x^2 + Ax = B(t + \tau)$$

$$x^2 + 0.165x = 0.0117(1.0 + 0.37)$$

$$\underline{x = 0.0686 \mu\text{m}}$$

Wet Oxidation

$$\tau = \frac{d_0^2 + Ad_0}{B} = \frac{0.0686^2 + (0.11)(0.0686)}{0.51}$$

$$\Rightarrow \tau = 0.024$$

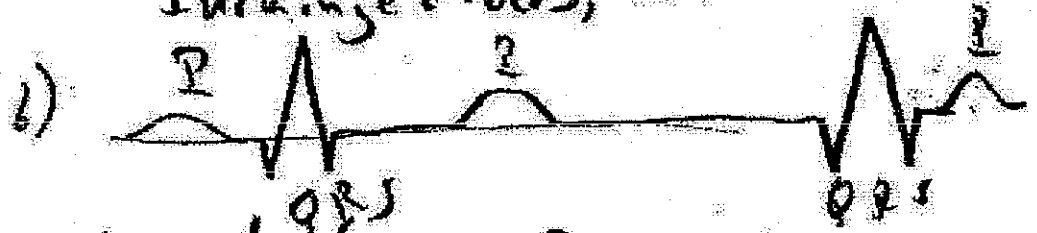
$$x^2 + Ax = B(t + \tau)$$

$$x^2 + 0.11x = 0.51(5.0 + 0.024)$$

$$\underline{x = 1.54 \mu\text{m}}$$

Problem 23 (Specialized: Bio Eng-ECE4784) Code Number: \_\_\_\_\_

a) S/A node, atrial muscles, AV node  
Purkinje Fibers, ventricle muscle



c) time between P & QRS is v  
atrial contraction will tra  
the normal so the potential  
points in the opposite direct

**Problem 24 (Specialized: Bio Eng-ECE4781) Code Number: \_\_\_\_\_**

$h_1 = [1/\text{Power of Stimulus}] [\text{Cross-correlation between the stimulus and response}]$

Best feature--

the kernels are easy to calculate quickly as opposed to Volterra Kernels ....

Worst feature--

the kernels are difficult to interpret ....

A white-noise light stimulus could be created by pulse modulating LED's or an arc lamp

**Problem 25 (Specialized: Bio Eng-ECE4782) Code Number: \_\_\_\_\_**

The simplest design would be to limit the total amplification to a small level that will allow the bias drifting to occur without causing any clipping, and to then use a high-bit A-to-D converter, e.g. 32 bit. The low-frequency bias could be then removed via digital filtering without producing any phase shifts at the frequencies of interest. Other answers will be acceptable.