**Ph.D. Preliminary Examination** **Code Number** ___________
**Spring 2011 SOLUTIONS**

## Instructions:

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Including the cover sheet, you should have **39 pages**. There should be no blank pages in the booklet.
2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.
3. All wireless devices must be turned off for the entire duration of the exam.
4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.
5. Your examination code number **MUST APPEAR ON EVERY SHEET**. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. **DO NOT** write your name on any of these sheets. Use the preprinted numbers whenever possible, or **WRITE LEGIBLY!!!**
6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. **DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!**
7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM.
8. When you hand in the exam:
   - (a) Separate the 8 problems to be graded as explained above.
   - (b) Check to see that your Code Number is in **EVERY** sheet you are turning in.
   - (c) On the section at the bottom of this page, **CIRCLE** the problem numbers that you are turning in for grading.
   - (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   - (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

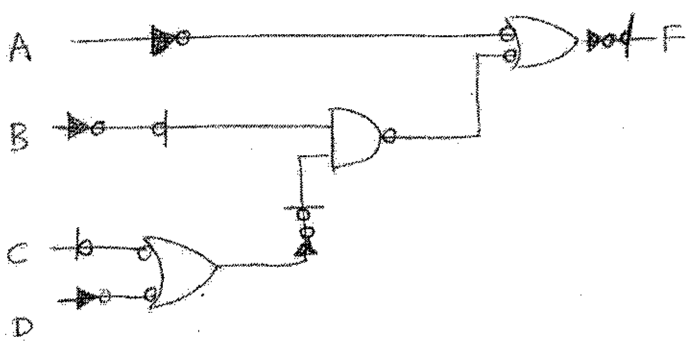
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |
| **19** | **20** | **21** | **22** | **23** | **24** | **25** | | |

## Problem 1 (Core: VSDD-ECE2030)      Code Number:________

**Problem X : This problem has 2 parts.**

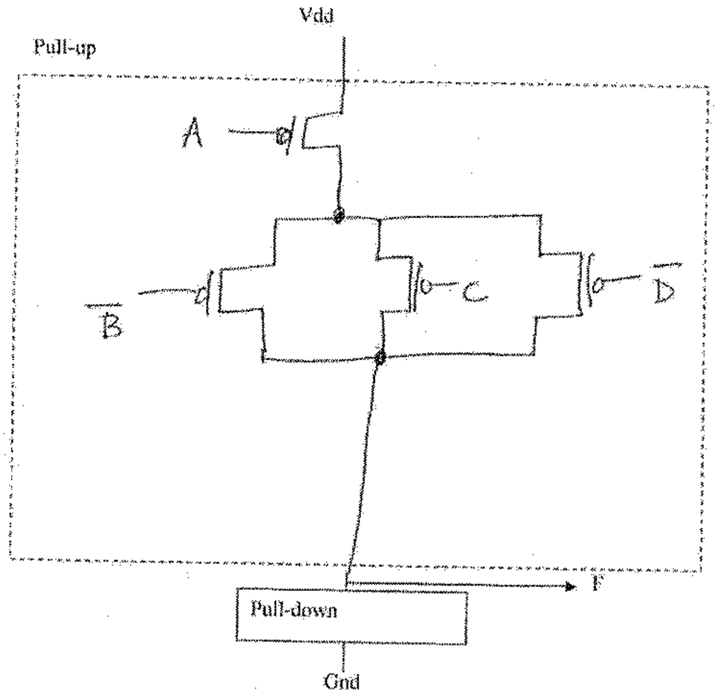
You are to implement the function $F = \overline{A + (\overline{B} \bullet \overline{(\overline{C} + D)})}$ using (a) inverters and NAND gates and (b) using a complex gate (pull-up/pull-dn chain combination):

(a) Draw the circuit below that realizes the function F using mixed-logic notation (i.e. use the slash "/" notation to represent inversion in the original function F above and cancel bubbles, etc. ).



## Problem 1 (Core: VSDD-ECE2030) Code Number:________

(b) For the complex gate implementation of the function F, draw the pull-up chain below. We assume that the pull-dn chain has already been designed correctly.



$$F = \overline{A} \cdot \overline{\overline{B} \cdot (\overline{C+D})} = \overline{A + (\overline{B} \cdot (\overline{\overline{C}+D}))}$$

$$= \overline{A} \cdot ( B + \overline{C} + D )$$

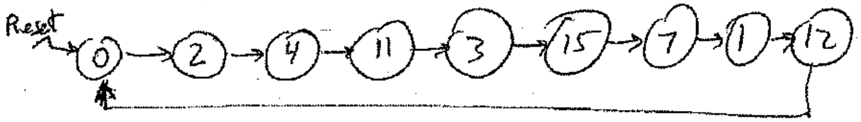
# Problem 2 (Core: VSDD-ECE2030) Code Number:__________

Solution
updated 3/25/11

Design a synchronous finite state machine that counts in the sequence 0,2,4,11,3,15,7,1,12 (and repeats). Assume that the 4-bit state register will be constructed using positive-edge-triggered D-type flip flops, with resets. Denote the four bits as "A,B,C,D" where "A" is the most-significant-bit (i.e. count 4 is encoded as ABCD=0100). Treat unused states as "don't care" conditions where appropriate (to minimize the logic).

(a) Draw a state diagram for the counter.

(2)



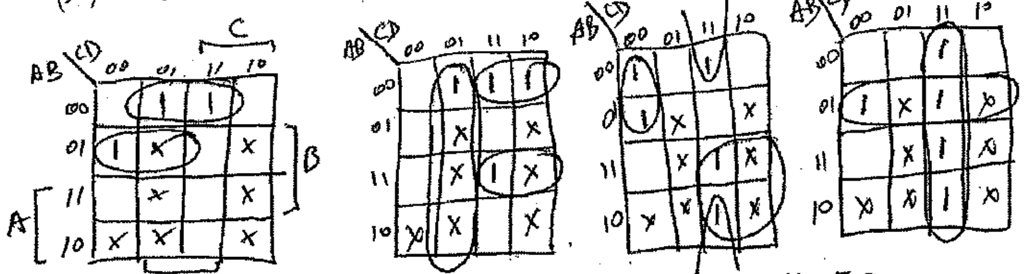
(b) Draw the complete encoded state transition table that defines the operation of the counter. Be sure to show how unused states are handled in the table.

(3)

| A B C D | (A B C D)+ |
|---|---|
| 0 0 0 0 | 0 0 1 0 |
| 0 0 0 1 | 1 1 0 0 |
| 0 0 1 0 | 0 1 0 0 |
| 0 0 1 1 | 1 1 1 1 |
| 0 1 0 0 | 1 0 1 1 |
| 0 1 0 1 | X X X X |
| 0 1 1 0 | X X X X |
| 0 1 1 1 | 0 0 0 1 |
| 1 0 0 0 | X X X X |
| 1 0 0 1 | X X X X |
| 1 0 1 0 | X X X X |
| 1 0 1 1 | 0 0 1 1 |
| 1 1 0 0 | 0 0 0 0 |
| 1 1 0 1 | X X X X |
| 1 1 1 0 | X X X X |
| 1 1 1 1 | 0 1 1 1 |

(5)

(c) Minimize the flip flop input logic functions and specify these as sum-of-products (SOP) Boolean expressions



$A^+ = D_A = \bar{A}\bar{B}D + \bar{A}B\bar{C}$

or $\bar{A}B\bar{D}$

$B^+ = D_B = \bar{C}D + \bar{A}\bar{B}C + ABC$

or $\bar{C}D + \bar{A}\bar{B}C + ABD$

or $C\bar{D} + \bar{A}\bar{B}D + ABD$

or $C\bar{D} + \bar{A}\bar{B}D + ABC$

$C^+ = D_C = AC + \bar{B}CD + \bar{A}\bar{C}\bar{D}$

or $AD$

$D^+ = D_D = \bar{A}B + CD$

## Problem 3 (Core: CSS-ECE3055) Code Number:_________

The memory system of a 32-bit processor is comprised of a 4Gbyte virtual address space with 4Kbyte pages, 4-entry fully associative data Translation Lookaside Buffer (TLB) with LRU replacement and a 2 Gbyte main memory. Memory is byte addressed. The L1 data cache is a 32Kbyte direct mapped cache with 64 byte lines that can be accessed in 1 cycle including TLB access. The L2 is a 1Mbyte 16-way set associative cache with 64 byte lines with a hit time of 8 cycles (this includes the line transfer to L1).

1. **(2 pts)** Show the breakdowns of the physical addresses used to access the L1 and the L2 caches, i.e., identify the fields used to access each cache. Clearly label each field.

|  | Tag | Line | Byte |
|---|---|---|---|
| 0 | 16 | 9 | 6 |

L1

|  | Tag | Set | Byte |
|---|---|---|---|
| 0 | 15 | 10 | 6 |

L2

---

2. **(2 pts)** Assuming a page table entry has i) a disk or page frame address (24 bits), ii) valid bit, iii) dirty bit, and iv) 6 bits for miscellaneous protection and page management, how large would a flat page table be in bytes?

4 Mbytes = #virtual_pages ($2^{20}$) * bytes/entry ($2^2$)

3. **(3 pts)** The main memory has 16 interleaved banks accessed over a bus with a bank access time of 200 cycles for a 32-bit word. The memory bus can transmit a single address from the L2 to memory in 1 cycle. The data width of the bus is 8 32-bit words, i.e., 8 words can be transmitted to the L2 in 1 cycle. Write an expression for the average memory access time in terms of the L1 and L2 miss rates ($m_{L1}$ and $m_{L2}$) and the <u>actual</u> miss penalties at each level of the memory hierarchy. State any assumptions.

$t_{avg} = 1 + m_{L1}\,(8 + m_{L2}\,(1 + 200 + 2))$

Transfer time between L1 and L2 = 8 cycles = L2 hit time

Transfer time between memory and L2 = Address_time (1 cycle) + Memory_access_time (200 cycles) + Transfer_time (16 words@ 8 words/cycle = 2 cycles).

# Problem 3 (Core: CSS-ECE3055) Code Number:_________

4. **(3 pts)** Consider a 32x32 matrix stored in memory in row major form starting at address 0x10010000, i.e., all elements in the first row followed by all elements in the second row, and so on. Each element is an integer number that takes 4 bytes. In what L2 set would you find element (10, 12) of the matrix (the first element of the first row is (0, 0)).

Each row of the matrix is stored in 2 cache lines (each cache line is 16 words = 16*4 = 64 bytes). The first 10 rows of the matrix are stored in 20 consecutive lines in memory. The element (10,12) is in the first half of the row and therefore is stored in the 21st line. There are 1024 sets in the L2 cache. Lines in main memory mapped to set 0 start on a 64 byte boundary whose lower order 6 bits are 0B000000. The first line of the matrix starts in set 0 and element (10,12) can be found in the 21st set or set number 0x14.

## Problem 4 (Core: VSDD-ECE3060) Code Number:_________

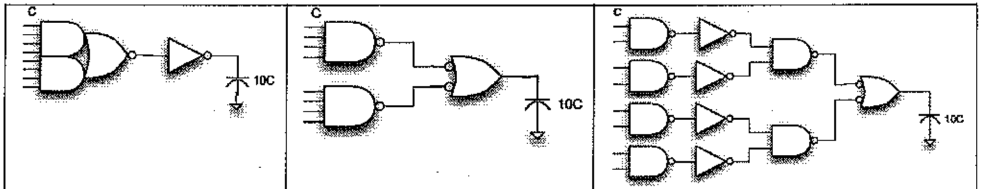
Consider implementing a logic function in static complementary CMOS style gates. Suppose we wish to minimize the delay for a given function, assuming the rise time and fall time for a given gate is equal in the worst case. Assume that the mobility of electrons is twice the mobility of holes, and thus a minimum size inverter is constructed from an nfet with width $W_{\min}$ and a pfet having width $2W_{\min}$. For both devices, the gate length $L = L_{\min}$. Using the method of Logical Effort, or other techniques, answer the following questions:

1. (6 pts) Given the boolean function $U = QRST + WXYZ$ explore three alternative implementations for this function, assuming that the load capacitance is 10 times the input capacitance for the first stage of the implementation. One implementation must use an AOI44 gate ( $V = \overline{QRST + WXYZ}$ ) and some additional logic, one must use NAND4 and NAND2 gates, and one must use only NAND2 and INV gates. Implementations that do not follow these specifications will receive no credit. For each implementation, give the minimum achievable delay $D_{\min}$ including parasitic and effort delay, in units of $\tau = R_n C_{inv}$, where $R_n$ is the resistance of a minimum size nfet, and $C_{inv}$ is the gate capacitance of a minimum size inverter.

2. (4 pts) For the implementation above with the lowest delay, size each transistor in units of $W_{\min}$ and $L_{\min}$ assuming that $C = C_{inv}$.

**Solution**

Figures (a), (b) and (c) show the schematics for the three cases



(a) AOI44 - INV

(b) NAND4–NAND2

(c) NAND2–INV–NAND2–NAND2

To calculate the minimum delay $D_{\min}$ for each, we first compute the path effort $F = GBH$, where the path logical effort $G = \prod g_i$ is the product of the gate logical efforts along the path, and the path branching effort $B = \prod b_i$ is the product of branching efforts along the path, and $H = C_{out} / C_{in} = 10$ is the given path electrical effort. Then in each case we compute the ideal stage effort $\hat{f} = F^{1/N}$ where $N$ is the given number of stages in the path. Finally we calculate $D_{\min} = N\hat{f} + P$, where $P = \sum p_i$ is the sum of parasitic delays along the path. In the following solution we assume that the parasitic delay of an $i$ input device is $i\tau$, which can easily be derived. We also use

## Problem 4 (Core: VSDD-ECE3060) Code Number:_________

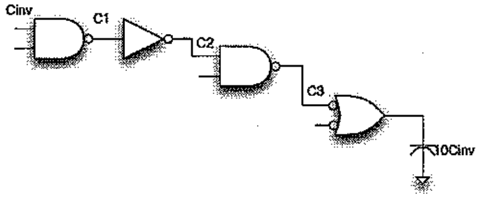
the fact that the logical effort of a NANDi is $(i+2)/3$, and the logical effort of an AOI44 is 8/3 which is also easily derived. Then the delays for each case are computed as:

(a) $G = 8/3$, $B = 1$, $H = 10$, and thus $F = 26.67$ and $\hat{f} = 5.16\tau$. Since $P = 8 + 1 = 9\tau$ for this circuit, we have $D_{\min} = 19.3\tau$.

(b) $G = 2 \cdot 4/3 = 8/3$, $B = 1$, $H = 10$, and thus $F = 26.67$ and $\hat{f} = 5.16\tau$. Since $P = 4 + 2 = 6\tau$ for this circuit, we have $D_{\min} = 16.3\tau$.

(c) $G = (4/3)^3$, $B = 1$, $H = 10$, and thus $F = 23.7$ and $\hat{f} = 2.2\tau$. Since $P = 2 + 1 + 2 + 2 = 7\tau$ for this circuit, we have $D_{\min} = 15.8\tau$.

For part 2, we work with implementation (c) since it has lowest minimum delay. To compute the actual gate sizes we identify the size of each gate by its input capacitance as shown below:



Then since the delay in each stage is given by $\hat{f} = gh = g(c_{out}/c_{in})$, we can solve $c_{in} = gc_{out}/\hat{f}$ working from right to left. Thus $C3 = 4/3 \cdot 10/2.2 = 6C_{inv}$, $C2 = 4/3 \cdot 6/2.2 = 3.6C_{inv}$, and $C1 = 1 \cdot 3.6/2.2 = 1.6C_{inv}$.

To convert size in units of capacitance to size in units of $W_{\min}$ and $L_{\min}$, we need the ratio of pfet to nfet width for the gate which is 1:1 for the NAND2 and 2:1 for the inverter. We also note that $C_{inv}$ is the gate capacitance of a FET with width $3W_{\min}$ and $L = L_{\min}$ for all transistors. Thus we have (from left to right on the path):

| Gate | Input Cap | Nfet Width | Pfet Width |
|---|---|---|---|
| NAND2 | 1 | 1.5 | 1.5 |
| INV | 1.6 | 1.6 | 3.2 |
| NAND2 | 3.6 | 5.4 | 5.4 |
| NAND2 | 6 | 9 | 9 |

## Problem 5 (Core: E&M-ECE3025) Code Number:_________

**Prelim Problem - 3025 - Solution**

A coaxial transmission line has inner and outer conductor radii $a$ and $b$. Between conductors ($a < \rho < b$) lies a "leaky" dielectric, having conductivity $\sigma(\rho) = \sigma_0/\rho$, where $\sigma_0$ is a constant. The inner conductor is charged to potential $V_0$, and the outer conductor is grounded.

a) Assuming dc radial current $I$ per unit length in $z$, determine the radial current density field $\mathbf{J}$ in A/m$^2$: This will be the current divided by the cross-sectional area that is normal to the current direction:

$$\mathbf{J} = \frac{I}{2\pi\rho(1)}\mathbf{a}_\rho \text{ A/m}^2$$

b) Determine the electric field intensity $\mathbf{E}$ in terms of $I$ and other parameters, given or known:

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} = \frac{I\rho}{2\pi\sigma_0\rho}\mathbf{a}_\rho = \frac{I}{2\pi\sigma_0}\mathbf{a}_\rho \text{ V/m}$$

c) by taking an appropriate line integral of $\mathbf{E}$, find an expression that relates $V_0$ to $I$:

$$V_0 = -\int_b^a \mathbf{E}\cdot d\mathbf{L} = -\int_b^a \frac{I}{2\pi\sigma_0}\mathbf{a}_\rho \cdot \mathbf{a}_\rho \, d\rho = \frac{I(b-a)}{2\pi\sigma_0} \text{ V}$$

d) find an expression for the conductance of the line per unit length, $G$:

$$G = \frac{I}{V_0} = \frac{2\pi\sigma_0}{(b-a)} \text{ S/m}$$

e. If the capacitance of the line per unit length is known to be $C$, express the ratio of charge to current in terms of $C$ and $G$. Use $Q = CV_0$ to obtain

$$\frac{Q}{I} = \frac{C}{G}$$

# Problem 6 (Core: E&M-ECE3065) Code Number:_________

4.

A 50-MHz plane wave with electric field amplitude of 30 V/m is normally incident in air onto a semi-infinite, perfect (lossless and nonmagnetic) dielectric medium with $\epsilon_r$=36.

[Use $\mu_o = 4 \times \pi \times 10^{-7}$ [H/m] and $\epsilon_o = 10^{-9} / (36\ \pi)$ [F/m] ]

(a) Determine the reflection coefficient Γ.

Air | Dielectric

$\eta_1$ | $\eta_2$

$$\eta_1 = \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi\ (\Omega)$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}} = 120\pi/6 = 20\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.71$$

(b) Determine the average power densities of the incident and reflected waves.

$$S_{av}^{i} = \frac{|E_o^i|^2}{2\eta_1} = \frac{30^2}{2 \times 120\pi} = 1.19\ (W/m^2)$$

$$S_{av}^{r} = |\Gamma|^2 S_{av}^{i} = 0.71^2 \times 1.19 = 0.60\ (W/m^2)$$

(c) Can you design a material transformer (calculate dielectric constant $\epsilon_r$, electrical length and physical length) to place in-between the two media in order to produce no reflection at the incidence point of the air medium?

Air | Trans. | Dielectric

$\eta_1$ | $\eta_3$ | $\eta_2$

$\xleftrightarrow{}$ d

Quarter-wa-elength tranformer $\Rightarrow d = \lambda/4$

$$\eta_3 = \sqrt{\eta_1 \eta_2} \Rightarrow \sqrt{\frac{\mu_o}{\varepsilon_{3r}}} = \left(\sqrt{\frac{\mu_o}{\varepsilon_o}} \sqrt{\frac{\mu_o}{\varepsilon_r}}\right)^{1/2} \Rightarrow \varepsilon_{3r} = \sqrt{\varepsilon_o \varepsilon_r}$$

$$\Rightarrow \varepsilon_{3r} = \sqrt{36} = 6$$

Physical length: $d = \frac{\lambda}{4} = \frac{(c/\sqrt{\varepsilon_{3r}})/f}{4} = \frac{\left(\frac{3 \times 10^8}{50 \times 10^6 \sqrt{6}}\right)}{4} = 0.61m$

(c) Repeat (a) after replacing the dielectric medium with a conductor with $\epsilon_r$=1, $\mu_r$=1 and $\sigma$=2.78 × $10^{-3}$ S/m.

Air | Conductor

$\eta_1$ | $\eta_{2c}$

$$\eta_{2c} = \sqrt{\frac{\mu_2}{\varepsilon_{2c}}} = \sqrt{\frac{\mu_o}{\varepsilon_2' - j\varepsilon_2''}} = \sqrt{\frac{\mu_o}{\varepsilon_o - j\frac{\sigma}{\omega}}} =$$

$$= \sqrt{\frac{\mu_o}{\varepsilon_o}} \left(1 - j\frac{\sigma}{\omega\varepsilon_o}\right)^{-1/2} = 120\pi \left(1 - j\frac{2.78 \times 10^{-3}}{2\pi 50 \times 10^6 (10^{-9}/36\pi)}\right)^{-1/2}$$

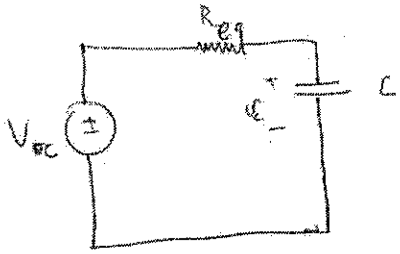
$$= 292.88 + j\,121.31\ (\Omega)$$

$$\Gamma = \frac{\eta_{2c} - \eta_1}{\eta_{2c} + \eta_1} = -0.09 + j0.12 = 0.22 \angle 114.5^\circ$$

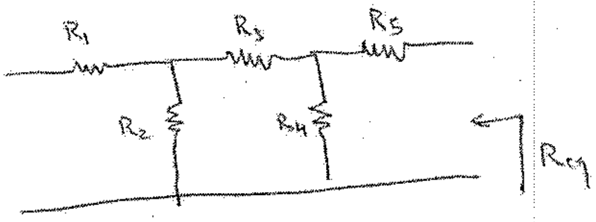
**Problem 7 (Core: EDA-ECE2040)** **Code Number:_________**

Solutions

Derive first the Thévenin equivalent circuit,



It can be seen that Req is



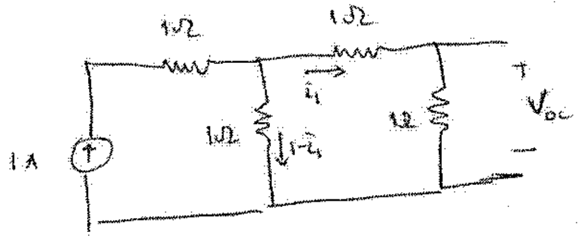
$$R_{eq} = R_5 + R_4 \parallel (R_2 + R_3) = \frac{1}{3} + (1 \parallel 2) =$$

$$= \frac{1}{3} + \frac{2}{3} = 1\ \Omega .$$

**Problem 7 (Core: EDA-ECE2040)** **Code Number:**__________

2

$V_{oc}$ is given by the following circuit,

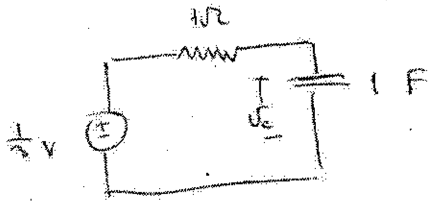


$1 - i_1 = 2 i_1$

$i_1 = \frac{1}{3}$ A

$V_{oc} = \frac{1}{3}$ V

The Thevenin circuit :



**Problem 7 (Core: EDA-ECE2040)** **Code Number:__________**

3

$$\frac{dv_c}{dt} + v_c = \frac{1}{3}$$

$v_c(t) = A + Be^{-t}$ for some A and B

By the above equation

$$-Be^{-t} + A + Be^{-t} = \frac{1}{3}$$

$$A = \frac{1}{3}$$

Initial conditions:

$v_c(0) = 0$

$A + B = 0$

$B = -A = -\frac{1}{3}$

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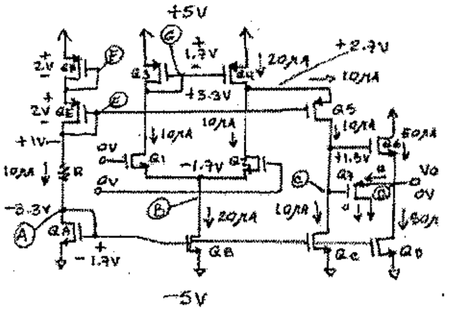
$$v_c(t) = \frac{1}{3}(1 - e^{-t}).$$

## Problem 8 (Core: EDA-ECE3050) Code Number:_________



DC analysis.

(a) $I_{REF} = 10\mu A = \frac{1}{2} \times 40 \times \frac{5}{5} (V_{GS_A} - V_t)^2$

$\Longrightarrow V_{GS_A} = 1.71V \simeq \underline{1.7V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5} (V_{GS_{E,F}} - 1)^2$

$\Longrightarrow V_{GS_{E,F}} = 2V$

$R = \frac{1-(-3.3)}{10\mu A} = \underline{\underline{430 K\Omega}}$

(b) See figure above

$V_{GS1} = V_{GS2} = V_{GSA} \simeq 1.7V$

$V_{GS3} = \sqrt{\frac{2 \times 10}{20 \times \frac{10}{5}}} + 1 = 1.71V \simeq 1.7V$

$V_{GS5} = V_{GS3} \simeq 1.7V$

For $Q_6$: $50 = \frac{1}{2} \times 40 \times \frac{50}{5} (V_{GS_6} - V_t)^2$

$\Longrightarrow V_{GS6} = 1.50V$

$V_A = \underline{\underline{-3.3V}}$ $V_B = \underline{\underline{-1.7V}}$

$V_C = \underline{\underline{+1.5V}}$ $V_D = \underline{\underline{0V}}$

$V_E = \underline{\underline{+1V}}$ $V_F = \underline{\underline{+3V}}$

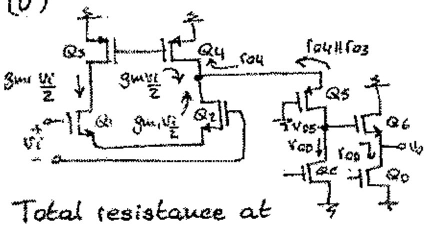
$V_G = \underline{\underline{+3.3V}}$ $V_H = \underline{\underline{+2.7V}}$

(c)

| Transistor | $I_D$ (μA) | $V_{GS}$ (V) | $g_m$ (μA/V) | $r_o$ (MΩ) |
|---|---|---|---|---|
| $Q_1$ | 10 | 1.7 | 28.3 | 5 |
| $Q_2$ | 10 | 1.7 | 28.3 | 5 |
| $Q_3$ | 10 | 1.7 | 28.3 | 5 |
| $Q_4$ | 20 | 1.7 | 56.6 | 2.5 |
| $Q_5$ | 10 | 1.7 | 28.3 | 5 |
| $Q_6$ | 50 | 1.5 | 200 | 1 |
| $Q_7$ | 0 | -1.5⊛ | 0 | $\infty$ |
| $Q_A$ | 10 | 1.7 | 28.3 | 5 |
| $Q_B$ | 20 | 1.7 | 56.6 | 2.5 |
| $Q_C$ | 10 | 1.7 | 28.3 | 5 |
| $Q_D$ | 50 | 1.7 | 141.4 | 1 |
| $Q_E$ | 10 | 2 | 20 | 5 |
| $Q_F$ | 10 | 2 | 20 | 5 |

⊛ cut-off.

(d)



Total resistance at the drain of $Q_5$, $R$, is:

$R = (g_{m5} r_{o5})(r_{o4} \| r_{o2}) \| r_{oC}$

$= [(28.3 \times 5)(2.5 \| 2)] \| 5$

$= 4.9 M\Omega$

Thus, $\frac{v_{d5}}{v_i} = g_{m1} R$

$= 28.3 \times 4.9 = 138.7$ V/V

and $\frac{v_o}{v_{d5}} = \frac{(r_{oD} \| r_{o6})}{(r_{oD} \| r_{o6}) + \frac{1}{g_{m6}}}$

$= \frac{(1 \| 1)}{(1 \| 1) + \frac{1}{200}} \simeq 1$

$\frac{v_o}{v_i} = \underline{\underline{138.7}}$ V/V

$R_{in} = \infty$

$R_{out} = r_{oD} \| r_{o6} \| 1/g_{m6}$

$= 1 \| 1 \| 1/200$ MΩ

$\simeq \underline{\underline{5 K\Omega}}$

**Problem 8 (Core: EDA-ECE3050)** **Code Number:__________**

(e) $v_{ICM}\big|_{max} = V_G + V_t$

$= +\underline{\underline{4.3V}}$

$v_{ICM}\big|_{min} = V_{GS1} + V_{Bmin}$

$= V_{GS1} + V_A - V_t$

$= 1.7 - 3.3 - 1 = -\underline{\underline{2.6}}V$

(f) $v_{O\,max} = v_{C\,max} - V_{GS6}$

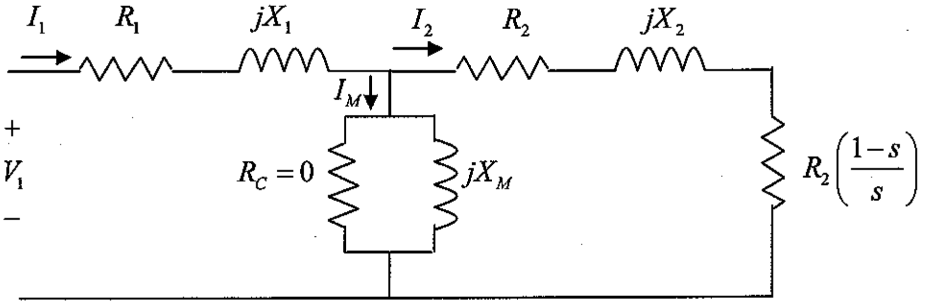
$= V_E + |V_t| - V_{GS6}$

$= +1 + 1 - 1.5 = +\underline{\underline{0.5V}}$

$v_{O\,min} = V_A - V_t = -3.3 - 1 = \underline{\underline{-4.3}}V$

## Problem 9 (Core: Power-ECE3070) Code Number:__________

**Solution:** a) Per-phase equivalent circuit:



b) The synchronous speed is:

$$n_{synch} = \frac{120 f_e}{p} = \frac{120\times 60}{4 poles} = 1800\ [rpm];\quad \omega_{synch} = 1800\left[\frac{r}{\min}\right]\times 2\pi\left[\frac{rad}{r}\right]\times\frac{1}{60}\left[\frac{\min}{\sec}\right] = 188.5[rad/s]$$

The mechanical shaft speed is:

$$n_m = (1-s)n_{synch} = (1-0.022)\times 1800\left[\frac{r}{\min}\right] = 1760 rpm;\quad \omega_m = 184.4[rad/s]$$

To find the stator current, we get the equivalent impedance. The referred rotor impedance is:

$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464 = 15.10\angle 1.76^\circ\Omega$$

$$Z_{tot} = Z_{stat} + jX_m \,\|\, Z_2 = 14.07\angle 33.6^\circ\Omega$$

$$I_1 = \frac{V_1}{Z_{tot}} = \frac{266\angle 0^\circ}{14.07\angle 33.6^\circ} = 18.88\angle -33.6^\circ$$

The power factor is: $PF = \cos 33.6^\circ = 0.833$ lagging

The input power to the motor is:

$$P_{in} = 3V_1 I_1 \cos\theta = 3\times 266\times 18.88\times 0.833 = 12{,}530W$$

The air gap power is: $P_{AG} = P_{in} - 3I_1^2 R_1 = 11{,}845W$

The converted power is: $P_{conv} = P_{out} = (1-s)P_{AG} = (1-0.022)\times 11{,}845 = 11{,}585W$

The motor efficiency is: $\eta = \frac{P_{out}}{P_{in}} = \frac{11{,}585}{12{,}530} = 92.46\%$

## Problem 10 (Core: Power-ECE3070) Code Number:_________

**SOLUTION**

Reluctance of the core:
$R_c = 2(0.09)+0.09+(0.09-0.003)/[(4\pi 10^{-7})(250)(0.01)(0.01)] = 11.36 \times 10^6$

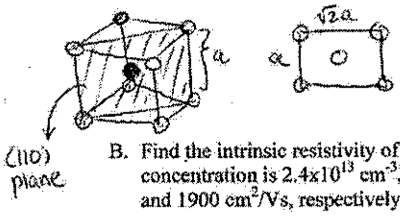
Then reluctance of the gap: $R_{gap} = 0.003/[(4\pi 10^{-7})(0.01)(0.01)] = 23.87 \times 10^6$
Total amp turns F required to produce the 1 mWb flux is
$(1 \text{ mWb})(11.36+23.87)10^6 = 35230 = F$

And this has to be equal to $N_3 i_3 + N_2 i_2 + N_1 i_1 = N_3 i_3 + 1600 - 300$
So $N_3 i_3 = 35230 - 1600 + 300 = 33930$
Thus $i_3 = 113.1$ amp in the direction shown.

# Problem 11 (Core: Microsystems-ECE3040) Code Number:________

**Problem 11 (Core: Microsystems-ECE3040)** **Code Number:** ____________

A. Show the atoms on the (110) plane of a body center cubic (bcc) lattice structure with a lattice constant of 5 Angstrom. Calculate the atoms per cm$^2$ in this plane.



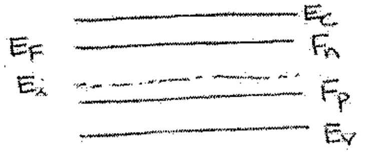
FOR 2-D ATOMIC DENSITY WE HAVE

$$\frac{2\text{ ATOMS}}{a^2\sqrt{2}} = \frac{2}{(5\times10^{-8}\text{cm})^2\sqrt{2}} = \boxed{5.656\times10^{14}\ \frac{\text{ATOMS}}{\text{cm}^2}}$$

B. Find the intrinsic resistivity of Ge at room temperature. The intrinsic concentration is 2.4x10$^{13}$ cm$^{-3}$, and the electron and hole mobility is 3900 cm$^2$/Vs and 1900 cm$^2$/Vs, respectively.

$$\rho = \frac{1}{q n_i(\mu_n+\mu_p)} = \frac{1}{(1.6\times10^{-19})(2.4\times10^{13})(3900+1900)} = \boxed{44.89\ \Omega\text{-cm}}$$

C. Assume that electron-hole pairs are created in an n-type silicon bar that is uniformly illuminated with light for a long period of time. Qualitatively show the quasi-Fermi levels for electrons and holes on an energy band diagram under this non-equilibrium condition. Assume *low-level injection* conditions prevail. For reference on your band diagram show where the equilibrium Fermi level, E$_F$, and the intrinsic Fermi level, E$_i$, might be to match the above conditions.



D. For part C, assume that the intrinsic carrier concentration of silicon at room temperature is 1.45e10 cm$^{-3}$ and that the quasi-Fermi level for the minority carriers lie 0.1 eV below the intrinsic Fermi level. Calculate the minority carrier concentration.

$$p = n_i e^{(E_i-F_p)/kT} = 1.45\times10^{10}\, e^{\frac{0.1\text{eV}}{0.02591\text{eV}}} = \boxed{6.889\times10^{11}\ \text{cm}^{-3}}$$

E. If the silicon is doped with phosphorus atoms at a density of 5 x 10$^{16}$ cm$^{-3}$, is the low-level injection assumption valid for the n-type semiconductor from part C and D. Why or why not?

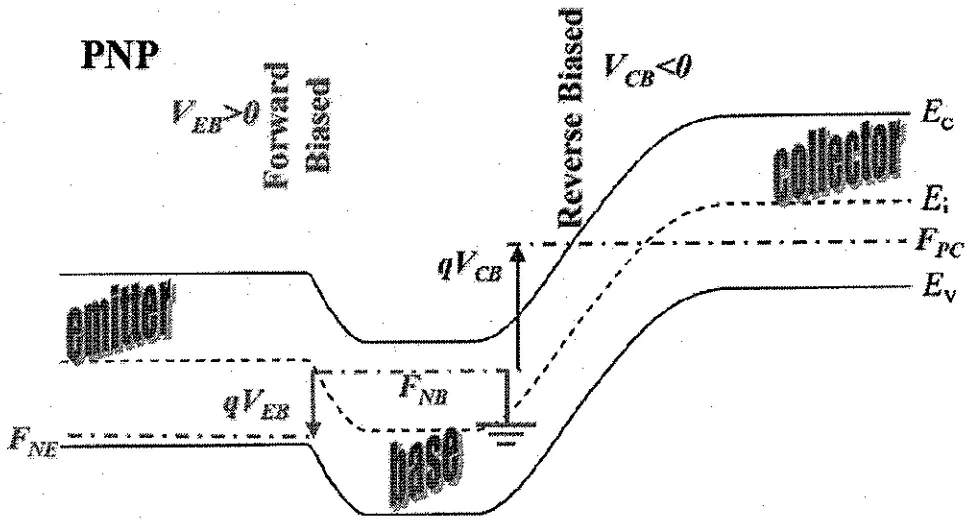
YES, BECAUSE $P_{\text{part D}} \approx \Delta p = \Delta n \ll 5\times10^{16}\text{cm}^{-3}$.

THEREFORE, THE MAJORITY CARRIER CONCENTRATION DOES NOT CHANGE SIGNIFICANTLY WITH THE GIVEN LEVEL OF ILLUMINATION.

## Problem 12 (Core: Microsystems-ECE3080) Code Number:_________

Part 1: In the diagram below, **using solid lines**, show the $E$ vs. $x$ diagram for this **PNP** bipolar homojunction transistor in the ***NORMAL ACTIVE REGION*** of operation. Assume that the **BASE** is grounded and ***show and label the arrows*** corresponding to the energies related to the BASE-EMITTER and BASE-COLLECTOR bias voltages, $V_{EB}$ and $V_{CB}$ and indicate the polarities of these voltages. Draw and label $E_C$, $E_V$, $E_i$, and the "***majority-carrier Quasi-Fermi Energies***" in the each of the layers and label them appropriately. I have given you the starting point for the diagram in the Emitter layer.

Solution:



Part 2: Why is the EMITTER region of a homojunction BJT generally more heavily doped than the BASE region?

Solution: To improve the emitter injection efficiency, $\gamma_E = I_{Ep}/(I_{Ep}+I_{En})$ for pnp transistor.

## Problem 13 (Core: DSP-ECE2025) Code Number:__________

**PROBLEM 2025 SOLUTION:**

Nyquist Rate for sampling the signal $x(t) = \sin(400\pi(t + 0.001))\cos(900\pi t - \pi/4)$.

The Nyquist Rate is twice the highest frequency in the signal, which is $1300\pi$ rad/s, or 650 Hz. So the Nyquist Rate is 1300 Hz.

(b) The MATLAB function `soundsc(xn, samplingRate)` will play a sound. If `xn=cos(1.7*pi*(0:99999))` and `samplingRate=10000`, determine the frequency (in hertz) of the sound you will hear.

The MATLAB code effectively defines a discrete-time signal $x[n] = \cos(1.7\pi n)$ which aliases to $x[n] = \cos(0.3\pi n)$. Then the output frequency will be $(0.3\pi) f_s/(2\pi) = 1500$ Hz.

(c) Draw a stem plot of the signal $y[n] = u[n-2] * (\delta[n-3] - \delta[n-5])$, for $0 \leq n \leq 9$. (Note: $*$ denotes convolution.)

The convolution gives: $y[n] = u[n-2] * \delta[n-3] - u[n-2] * \delta[n-5] = u[n-5] - u[n-7] = \delta[n-5] + \delta[n-6]$. Thus we get two nonzero stems: one at $n = 5$ and one at $n = 6$.

(d) Determine the Continuous-time Fourier transform of $x(t) = \left(\frac{4\pi}{t}\right)\sin(9t)$

This signal is a *sinc function:* $x(t) = (4\pi^2)\frac{\sin(9t)}{\pi t}$, so $X(j\omega) = 4\pi^2\{u(\omega+9) - u(\omega-9)\}$, which is a rectangle.

(e) A signal $x[n]$ has a $z$-Transform $X(z) = \frac{7z^{-2}}{3+2z^{-1}}$. Determine the signal $x[n]$ from $X(z)$.

$$X(z) = \frac{7}{3}\left(\frac{z^{-2}}{1+(2/3)z^{-1}}\right) = \frac{7}{3}z^{-2}\left(\frac{1}{1+(2/3)z^{-1}}\right)$$

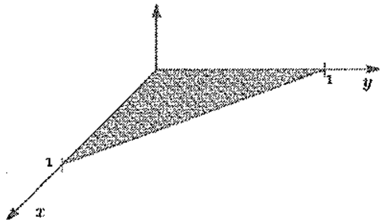
Then recall $\frac{1}{1-az^{-1}} \leftrightarrow a^n u[n]$. Take the inverse $z$-transform and use the shifting property:

$\boxed{x[n] = \frac{7}{3}(-2/3)^{n-2}u[n-2]}$

## Problem 14 (Core: DSP-ECE3075) Code Number:_________

3075 SOLUTION

**Consider the joint probability density function for a pair of continuous random variables $X$ and $Y$:**



$$f(x, y) = \begin{cases} 24xy, & \text{for } x > 0,\, y > 0,\, x + y < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

**The joint pdf is nonzero only for $(x, y)$ falling within the shaded triangle shown above.**

(a) YES ☐ NO ☒ Are $X$ and $Y$ are independent? Explain briefly.

The footprint is not rectangular, so they cannot be independent.
For example, when $x = y = 0.9$ we find that the left-hand side of the independence condition $f(x, y) \neq f(x)f(y)$ is zero, while the right-hand side is not.

(b) **$X$ and $Y$ are [ positively ][✓ negatively][ un- ]correlated. (Circle one.) Explain briefly.**

Without quantifying correlation, we can see from the joint pdf that $X$ and $Y$ cannot both be large; when one is large, the other must be small.

(c) **The expected value of $\frac{1}{XY}$ is $E\left(\frac{1}{XY}\right) = \boxed{\quad}$.**

From the fundamental theorem of expectation:

$$E\left(\frac{1}{XY}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{xy} f(x, y)\,dx\,dy$$

$$= \iint_{\text{triangle}} \frac{24xy}{xy}\,dx\,dy = 24 \iint_{\text{triangle}} dx\,dy = 24(\text{triangle area}) = \boxed{12}.$$

(d) The pdf for $X$ is $f(x) = \boxed{\qquad}$ for $0 < x < 1$, and $f(x) = 0$ elsewhere.

Integrating the joint pdf yields the marginal pdf:

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\,dy = 24x \int_{0}^{1-x} y\,dy = \boxed{12x(1-x)^2}, \quad \text{for } 0 < x < 1 \text{ only.}$$

(e) **The conditional pdf for $Y$ given $X$ is $f(y|x) = \boxed{\qquad}$, for $0 < y < \boxed{\quad}$, and $f(y|x) = 0$ elsewhere.**

The conditional pdf can be found using:

$$f(y|x) = \frac{f(x, y)}{f(x)} = \frac{24xy}{12x(1-x)^2} = \boxed{\frac{2y}{(1-x)^2}}, \quad \text{for } 0 < y < \boxed{1-x} \text{ only.}$$

## Problem 15 (Core: S&C-ECE3085) Code Number:_________

Part A) Suppose that frequency response data is recorded from a physical system and is shown on a Bode plot below.

System used to generate plot is $G(s) = \dfrac{1}{(s^2 + 1.6s + 4)(s + 0.1)}$;

The system is 3rd order with break frequencies at 0.1rad/sec and 2 rad/sec. The one at 2 rad/sec is a double pole (you can tell by the slope change) but it has a little resonant peak, indicating a complex pole. In the solution, the accuracy of the damping ratio is not critical, but students should know that the pole is complex.

The gain margin is 16dB, so the range for stability is $K < 10^{16/20} = 6.3$

Part B) The magnitude plot shows a zero at 1 and a pole at 10. However, the phase does not follow, which indicates that the zero is nonminimum phase. Moreover, the phase at DC is -180 indicating the following form for the system: $G(s) = A\dfrac{s-1}{s+10}$. The DC gain is -20 dB or 0.1, which means that A = 1.

The range of K for stability is found from the Routh-Hurwitz criteria or from the Nyquist Stability Test. $K < 10$

## Problem 16 (Core: S&C-ECE3085) Code Number:_________

Consider four systems with the following transfer functions:

$$\begin{aligned} G_A(s) &= \frac{500s + 10000}{s^2 + 20s + 10000}, \\ G_B(s) &= \frac{50s + 100}{s^2 + 2s + 100}, \\ G_C(s) &= \frac{5000s - 10000}{s^2 + 40s + 10000}, \\ G_D(s) &= \frac{-50s + 100}{s^2 + 4s + 100}. \end{aligned}$$

1. You are required to match these with their matching pole/zero plots, step responses, Bode plots, and Nyquist plots shown in the next four figures, Figures 1-4. (E.g., $G_A(s)$ corresponds to the first pole-zero plot. Then we should mark *zp*1 in the corresponding position as shown, etc.) **Each entry in the table should be filled with a distinct plot name** (you are not allowed to fill a column with the same plot name in order to guarantee one correct match). You'll notice that the Nyquist plots *nq*2 and *nq*4 are identical which means that you may choose from two different correct combinations for the last column.

| Transfer fn | Pole-zero plot | Step response | Bode plot | Nyquist plot |
|---|---|---|---|---|
| A | $\boxed{zp1}$ | sr4 | bd3 | nq2 |
| B | zp4 | sr1 | bd4 | nq4 |
| C | zp2 | sr2 | bd2 | nq3 |
| D | zp3 | sr3 | bd1 | nq1 |

2. Explain why the two Nyquist plots *nq*2 and *nq*4 are identical.

---

SOLUTIONS:

1. To match the graphs you can proceed as follows:

For the root-location shown in the zero-pole plot it is straight forward. For the step response, the step response of $G_A$ has a limit of 1 at infinity, also it is a minimum phase system, so the response will rise, oscillate and settle to 1. The period should roughly be $\frac{2\pi}{100} \sim 0.06$. Hence it is sr4. Note that sr3 has similar limit value, however it starts by going negative first, therefore it must correspond to the nonminimum phase $G_D(s)$ which has a right-half-plane zero. (Of course it also has different period, which may also help distinguish it.) Similarly for the rest.

The Bode plots have frequency and range of phase changes as distinguishing features. Hence, bd1 which shows phase change by $-270^o$ must have a right-half-plane zero. Since the phase at $\omega = 0$ is $0^o$ it must be $G_D(s)$. Etc.

2. The correspond transfer functions relate through a frequency scaling, i.e., $G_A(s) = G_B(\frac{s}{10})$.

## Problem 16 (Core: S&C-ECE3085) Code Number:_________

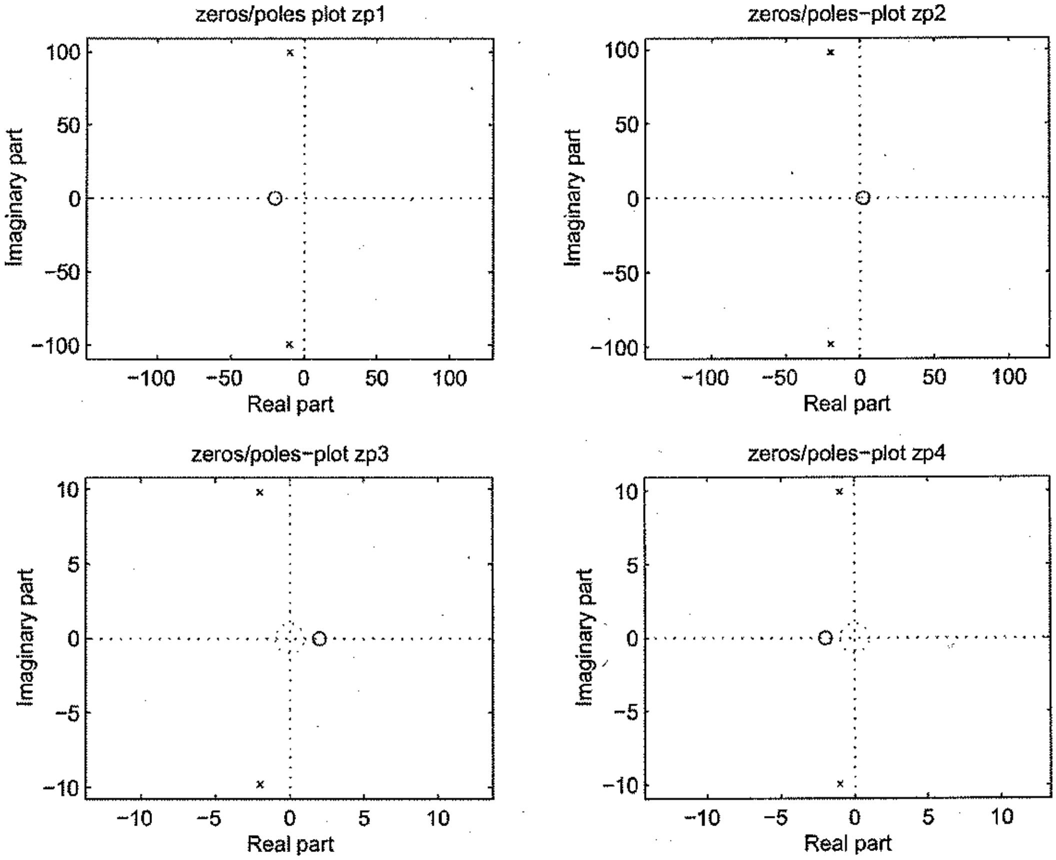


Figure 1: Pole-zero plots

## Problem 16 (Core: S&C-ECE3085) Code Number:________

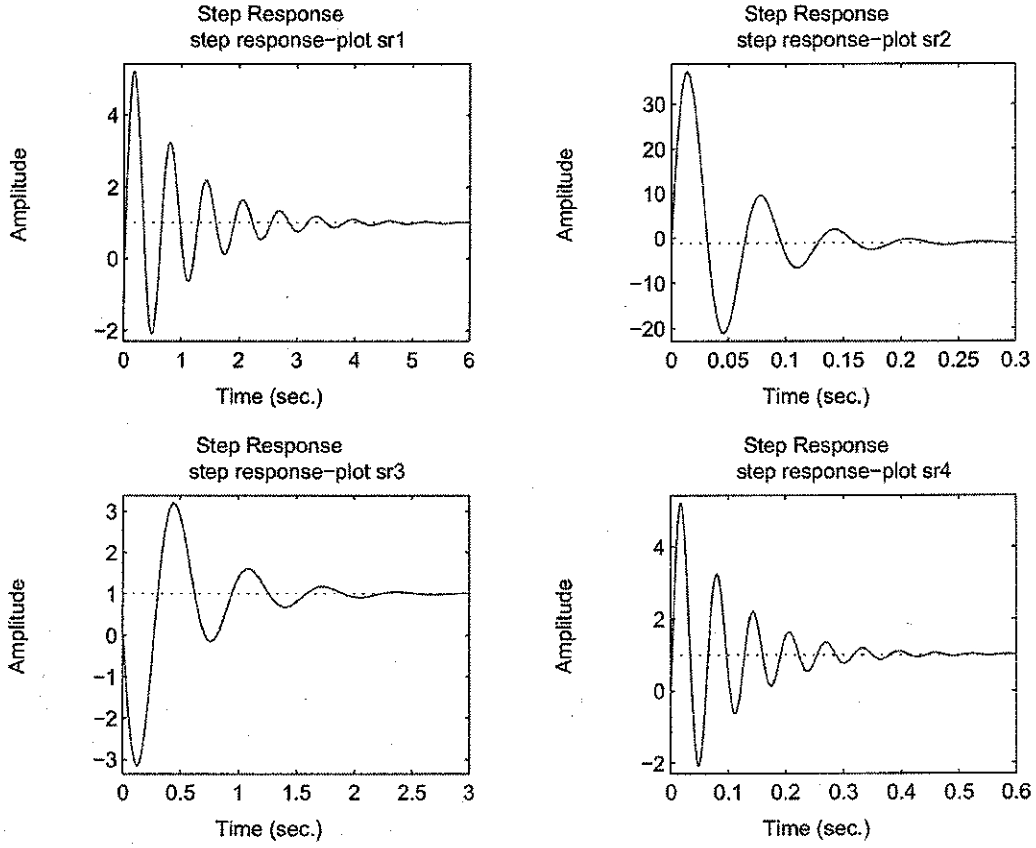


Figure 2: Step responses

## Problem 16 (Core: S&C-ECE3085) Code Number:________

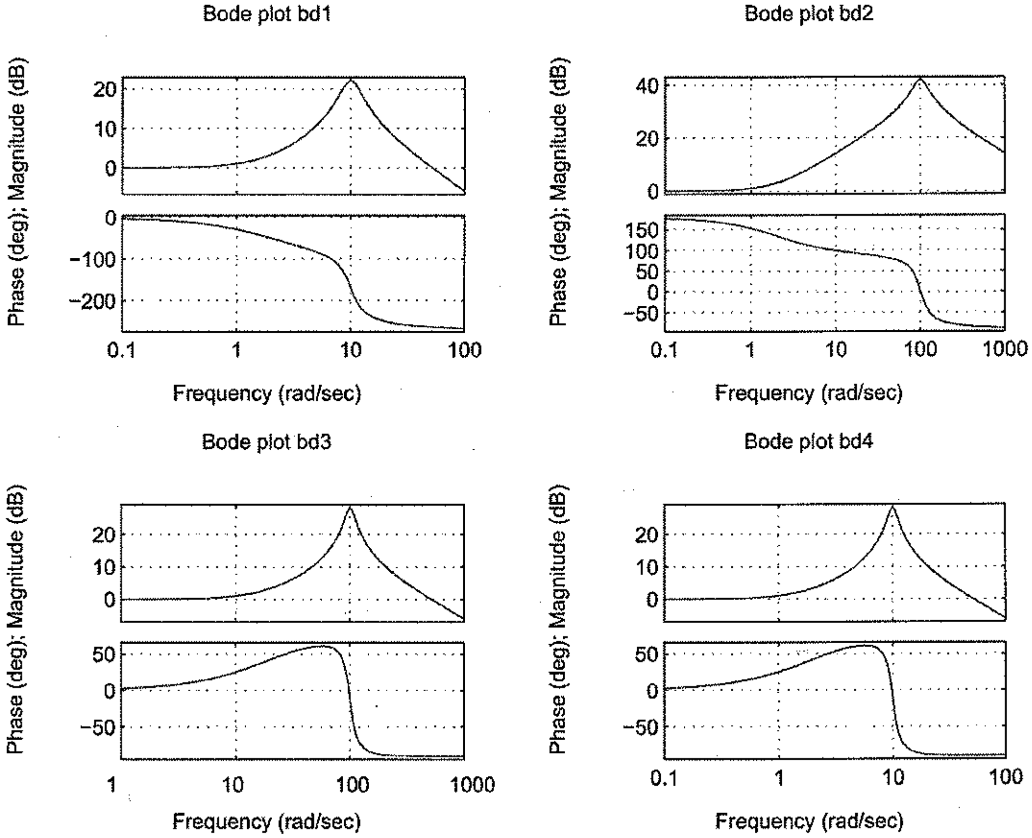


Figure 3: Bode plots

## Problem 16 (Core: S&C-ECE3085) Code Number:__________

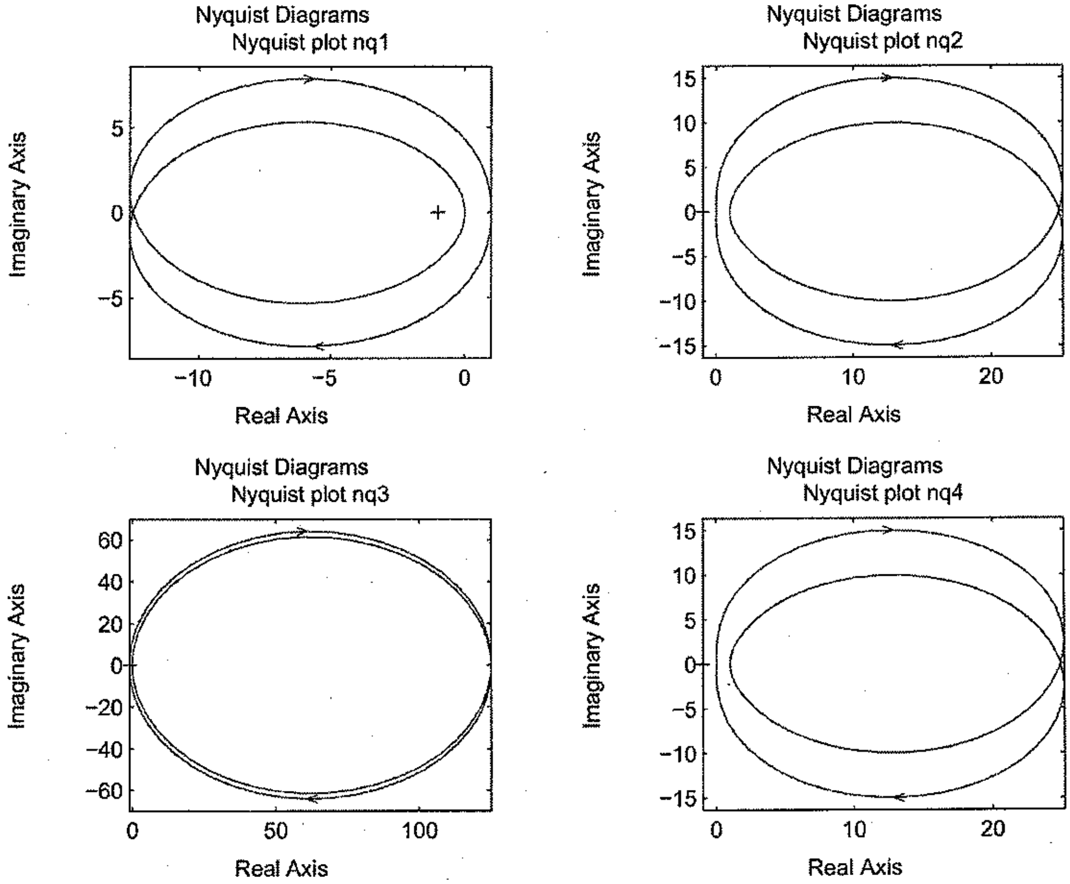


Figure 4: Nyquist plots

## Problem 17 (Specialized: CSS-CS3210) Code Number:______

Consider the problem of finding the best order in which to multiply n matrices of different sizes together:

$$M_1M_2M_3\cdots M_n$$

The overall cost of computing this multiplication sequence depends on the order chosen to do the individual matrix multiplications. Assume that $M_i$ has $r_i$ rows. (Note that the number of columns of $M_i$ must be $r_{i+1}$ in order for this to be a valid matrix multiplication sequence. For simplicity, let the number of columns of $M_n$ be denoted by $r_{n+1}$ also.) Assume cost is measured by the number of scalar multiplications performed, which for a single matrix multiplication $M_iM_{i+1}$ is equal to $r_i \cdot r_{i+1} \cdot r_{i+2}$.

Write a polynomial-time algorithm, using C++ or Java-like pseudo-code, to calculate the exact minimum number of scalar multiplications necessary to compute the entire multiplication sequence.

*Use dynamic programming:*

```
for i = 1 to n {
        cost[i, i] = 0;
}
for h = 2 to n {
        for i = 1 to n-h+1 {
                j = i+h-1;
                cost[i, j] = MAXINT;
                for k = i to j-1 {
                        q = cost[i, k] + cost[k+1, j] + $r_i \cdot r_{k+1} \cdot r_{j+1}$;
                        if q < cost[i, j] then cost[i, j] = q;
                }
        }
}
return cost[1, n];
```

## Problem 18 (Specialized: CSS - ECE3035) Code Number: ____

Part A: What is the content of the stack right before 'foo' returns? The input parameter and the return address has already been filled for you.

| address | content |
|---|---|
| … | … |
| 1000 | 6 (input parameter for 'foo') |
| 996 | Return address |
| 992 | 6 (j=6 when the loop ends) |
| 988 | 5 ( a[5] ) |
| 984 | 4 ( a[4] ) |
| 980 | 3 ( a[3] ) |
| 976 | 2 ( a[2] ) |
| 972 | 1 ( a[1] ) |
| 968 | 0 ( a[0] ) |
| 964 | undefined |

## Problem 18 (Specialized: CSS - ECE3035) Code Number: _____

Part B: What is the content of the stack right before the first and second 'bar' returns?

Stack right before the first 'bar' returns

| address | content |
|---|---|
| ... | ... |
| 1000 | 4 (second input parameter for the first 'bar') |
| 996 | 2 (first input parameter for the first 'bar') |
| 992 | Return address |
| 988<br>984<br>980<br>976<br>972<br>968<br>964 | 4 (k=4 when the loop ends)<br>5 ( a[1] )<br>4 ( a[0] )<br>1 ( b[3] )<br>0 ( b[2] )<br>-1 ( b[1] )<br>undefined (b[0]) |

Stack right before the second 'bar' returns

| address | content |
|---|---|
| ... | ... |
| 1000 | 2 (second input parameter for the first 'bar') |
| 996 | 4 (first input parameter for the first 'bar') |
| 992 | Return address |
| 988<br>984<br>980<br>976<br>972<br>968<br>964 | 2 (k=2 when the loop ends)<br>6 ( a[3] )<br>5 ( a[2] )<br>2 ( a[1] )<br>1 ( a[0] )<br>-2 ( b[1] )<br>undefined ( b[0] ) |

## Problem 19 (Specialized: Telecom-ECE3076) Code Number:________

1. What devices forward IP datagrams across the Internet?

*Routers*

2. How do they build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular IP datagram?

*They communicate with each other, using a routing protocol, so that they find out the minimum cost routes to other areas of the Internet (address space).*

3. How do Ethernet switches build a Forwarding Table that is used to decide what outbound network link is best for forwarding a particular Ethernet frame?

*When Ethernet frames are received, the switch remembers the source address and which link the frame came from. Frames to that address are then sent out over that link.*

4. What is the primary advantage of a packet store-and-forward network?

*Statistical multiplexing allows many users to share the available bandwidth.*

5. What is the primary advantage of a circuit-switched network?

*Users can be guaranteed resources (minimum bandwidth, maximum delay, ...)*

6. Which type of network (packet or circuit-switched) is the Internet?

*Packet store and forward.*

7. How does your PC determine the 32-bit IP address for "www.nationalcarbuy.com"?

*It sends a UDP request to the local DNS server for a recursion-requested lookup.*

8. How does your browser authenticate a Web site that is using https?

*It receives a Certificate from the Web server that is digitally signed by a Certificate Authority whose own certificate is stored in the browser.*

9. What is the advantage of the TCP transport-layer protocol?

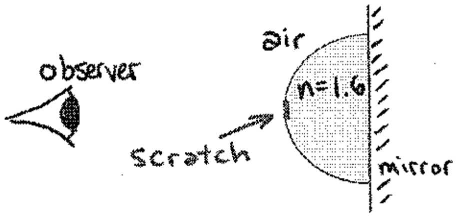
*It provides an error-free transport of data.*

10. What is the advantage of the UDP transport-layer protocol?

*It is simple and fast, particularly for sending short messages or streaming media.*

# Problem 20 (Specialized: Optics-ECE4500) Code Number:__________

(a) The flat face of a glass hemisphere with refractive index n = 1.6 and radius R = 5mm is cemented a perfect planar mirror. There is a small scratch on the surface of the hemisphere at the point farthest from the mirror. This is diagrammed below. What will be the apparent depth and magnification of the image of the scratch as seen by the observer? (Please give depth relative to the mirror surface.)



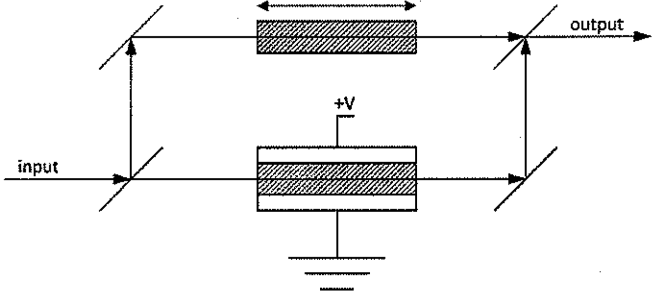
Before the light exits the hemisphere, the object will appear @ $s' = -5$ mm (in the mirror). After refraction:

$$\frac{1.6}{10} + \frac{1}{s'} = \frac{1-1.6}{-5} \Rightarrow s' = -25\text{mm} \Rightarrow \text{image appears}$$

20mm below surface

$$m = \frac{1.6(25)}{10} = 4$$

(b) A Kerr electro-optic modulator is diagrammed below.



The input light is split equally into two paths. Each path has identical length, and each path passes through a Kerr cell (represented by a crosshatched rectangle) of length L = 3 cm. A voltage V is applied to one of the Kerr cells, which changes the refractive index of that cell by an amount

$$\Delta n = KV^2\lambda$$

where λ is the free-space wavelength of the light passing through the cell, and K = 2.4e-8 $V^2$ $m^{-1}$. Neglect all reflections from the Kerr cell. Find the voltage V necessary to minimize the light intensity at the output.

$$\frac{2\pi}{\lambda_0}(n_1 L - n_2 L) = \pi \quad \text{for destructive interference}$$

$$\Delta n = \frac{\lambda_0}{2L} = KV^2\lambda_0$$

$$\Rightarrow V = \sqrt{\frac{1}{2KL}} = 26.4\text{kV}$$

## Problem 21 (Specialized: Optics-ECE4501) Code Number:_________

**Prelim Problem Solution, ECE 4501**

Consider a simple direct-detection optical receiver, based on a PIN photodiode that is known to have (for practical purposes) zero dark current. When using binary on-off keying, the bit error rate (BER) can be related to the respective current levels of the "one" and "zero" bits, $I_1$ and $I_0$, through the approximate formula:

$$\text{BER} \approx \frac{1}{Q\sqrt{2\pi}} \exp\left[-Q^2/2\right]$$

where $Q = (I_1 - I_0)/(\sigma_1 + \sigma_0)$, and where $\sigma_1$ and $\sigma_0$ are the rms noise currents associated with $I_1$ and $I_0$. The noise powers are known to be composed of shot ($\sigma_s$) and thermal ($\sigma_T$) contributions according to $\sigma_1^2 = \sigma_{s1}^2 + \sigma_T^2$ and $\sigma_0^2 = \sigma_{s0}^2 + \sigma_T^2$. In general, current is related to the optical power, $P$, through $I_i = RP_i$ ($i = 0, 1$), where $R$ is the detector responsivity. $P_1$ and $P_0$ are the optical powers in the "one" and "zero" bits.

a. The signal-to-noise ratio (SNR) is defined as the average *electrical* power divided by the noise power. Determine an expression for the SNR in terms of $I_1$, $I_0$, $\sigma_{s1}$, $\sigma_{s0}$, and $\sigma_T$ (as necessary) in which there is complete extinction (optical power in the zero bits is zero).

*Answer:* with full extinction, the average optical power is $P_1/2$, so that the average electrical power in the signal is proportional to $[R(1/2)P_1]^2 = I_1^2/4$, and $I_0 = 0$. The noise power (with equal time contributions from the "one" and "zero" bits) is (in the same proportion) $(1/2)[\sigma_1^2 + \sigma_0^2] = (1/2)[\sigma_{s1}^2 + \sigma_{s0}^2] + \sigma_T^2$. The SNR becomes:

$$\text{SNR} = \frac{(1/4)I_1^2}{(1/2)\sigma_{s1}^2 + \sigma_T^2}$$

b. Construct the SNR expression for again, complete extinction, but in which shot noise is negligible compared to thermal noise.

*Answer:* Set $\sigma_{s1} \approx 0$ to get

$$\text{SNR} = \frac{I_1^2}{4\sigma_T^2}$$

c. Determine an expression for $Q$ in terms of the SNR under the conditions of part *b*.

*Answer:* Under the part *b* conditions, we have $I_0 = 0$ and $\sigma_1 = \sigma_0 = \sigma_T$. $Q$ becomes

$$Q = \frac{I_1}{2\sigma_T} = \sqrt{\text{SNR}}$$

d. Again, under conditions as specified in part *b*, suppose the BER for the given setup is known to be $10^{-9}$ ($Q = 6$). What BER would be obtained if the average *optical* power is reduced by 3dB? What is the effect on the SNR?

*Answer:* With optical power 3dB down, $I_1$ is cut in half and thermal noise is unaffected, which means that $Q = 3$. Substitute $Q = 3$ into the given formula to find BER = $\underline{1.5 \times 10^{-3}}$.

With $I_1$ down by one-half, $I_1^2$ is down by 1/4, and so is the SNR (6dB down).

## Problem 22 (Specialized: Microsystems-ECE4752) Code Number:____

From the graph, we can extract
the required implant energy for a boron implant with a peak depth of $0.3\,\mu m = 3000\,\mathring{A} = R_p$

$\hookrightarrow$ $\underline{E \approx 90\,keV}$

From the same graph, we find the standard deviation to be $\Delta R_p \approx 680\,\mathring{A}$.

The profile is approximated by a Gaussian:

$$N(x) = \underbrace{\frac{\phi}{\sqrt{2\pi}\,\Delta R_p}}_{=N_{max}} e^{-\frac{(x-R_p)^2}{2\Delta R_p^2}} \qquad (1)$$

From the maximum concentration $N_{max} = \frac{\phi}{\sqrt{2\pi}\,\Delta R_p}$, we can extract the required dose $\phi$

$$\phi = N_{max}\sqrt{2\pi}\,\Delta R_p = \sqrt{2\pi}\cdot 10^{17}\,cm^{-3}\cdot 680\,\mathring{A}$$
$$= \underline{\underline{1.7\cdot 10^{12}\,cm^{-2}}}$$

Using above equation (1) we can find the $x$ for which $N = 10^{15}\,cm^{-3}$

$\hookrightarrow$ $x_1 = 9.4\cdot 10^{-6}\,cm < R_p$

$x_2 = 0.50\cdot 10^{-4}\,cm = \underline{0.50\,\mu m}$

**Problem 23 (Specialized: Bio Eng-ECE4784) Code Number:_________**

$T = 6^{\circ}C = 279K$

$$\frac{RT}{F} = \frac{(8.314 J K^{-1} mol^{-1})(279K)}{96485\ C\ mol^{-1}} = 2.4\times10^{-3}\ J/C = 24mV$$

a) $V_K = \frac{RT}{z_K F} \ln\left\{\frac{[K^+]_e}{[K^+]_i}\right\} = 24 \ln\left\{\frac{150}{26}\right\} = 42mV$

Concentration conversion:

$$1mM = 10^{-3}M = 10^{-3}\frac{mol}{liter} = \frac{10^{-3} mol}{1000 cm^3} = 10^{-6}\frac{mol}{cm^3} \cdot 6.023\times10^{23} ions/mol$$

$$= 6.023\times10^{17} ions/cm^3$$

$[K^+]_i = (26mM)(6.023\times10^{17} ions/cm^3/mM) = 1.56\times10^{19} ions/cm^3$

$[K^+]_e = (150mM)(6.023\times10^{17}) = 9\times10^{19} ions/cm^3$

b) $V_{Na} = 24 \ln\left\{\frac{25}{90}\right\} = -30.7mV$

$[Na^+]_i = (90mM)(6.023\times10^{17}) = 5.42\times10^{19} ions/cm^3$

$[Na^+]_e = (25mM)(6.023\times10^{17}) = 1.5\times10^{19} ions/cm^3$

c) $V_{Cl} = \frac{RT}{z_{Cl} F} \ln\left\{\frac{70mM}{170mM}\right\} = -24(-0.887) = 21.3mV$

$[Cl^-]_i = 170mM\ (6.023\times10^{17}) = 1.02\times10^{20} ions/cm^3$

$[Cl^-]_e = 70mM\ (6.023\times10^{17}) = 4.2\times10^{19} ions/cm^3$

d) $V_{Ca} = \frac{24}{2} \ln\left\{\frac{200mM}{75mM}\right\} = 11.8mV$

$[Ca^{++}]_i = (75mM)(6.023\times10^{17}) = 4.5\times10^{19} ions/cm^3$

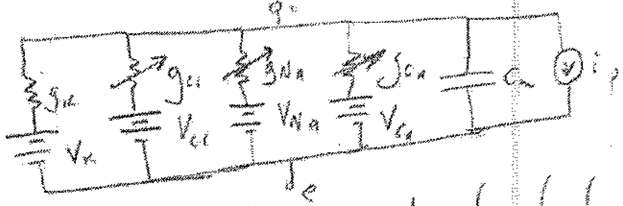
$[Ca^{++}]_e = (200mM)(6.023\times10^{17}) = 1.2\times10^{20} ions/cm^3$

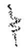
**Problem 23 (Specialized: Bio Eng-ECE4784) Code Number:_________**

e) $C_m = 2\mu F/cm^2 = \frac{\epsilon_0 \epsilon_r}{d} = \frac{\epsilon_0 \epsilon_r}{8nm}$

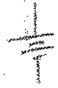
$$\epsilon_r = \frac{(2\times10^{-6}F/cm^2)(8\times10^{-7}cm)}{(8.85\times10^{-14}F/cm)} = \underline{18}$$

f)



 = membrane conductance – resting channels only

 = " " – voltage gated and resting channels

 = Nernst potentials

 $C_m$ = membrane capacitance due to the lipid bilayer

 $i_p$ = ion pump – maintains/pump restoring the ion concentrations

**Problem 24 (Specialized: Bio Eng-ECE4782) Code Number:________**

# ECE 4782 Answers

You have been hired as a consultant to design a medical device which can be used to determine when a person is "Brain Dead." The device may use any number of surface EEG electrodes.

A. What are the main problems that make it difficult to achieve a high level of confidence via surface EEG electrodes (20 points)?

The RMS Signal-to-Noise could be as bad as 1/10,000, and the resistance of the skin and skull is so high that it is difficult to detect neural activity at a depth greater than 5mm.

B. What if any stimuli would you use (20 points)?

Some sort of tactile or pain stimulus would be best, which could be produced by a strong mechanical or electrical stimulus. However, it will be important to verify that the patient does not have a spinal cord injury that would prevent sensory inputs from reaching the brain.

C. Draw a sketch showing where you would position the EEG electrodes (20 points)?

The sensory cortex is at the front edge of the Parietal Lobe, so electrodes over this area would be best. A sketch showing the Frontal and Parietal Lobes will be sufficient. Don't forget that tactile stimuli cross, i.e. a tactile stimulus on the right side of the body goes to the left brain, but the sensory input eventually passes to the right brain via the Corpus Callosum.

D. How could you improve the accuracy of your proposed protocol (20 points)?

A pulse stimulus could be repeated at random intervals to reduce the affects of periodic noise. Band-limited white-noise is another option, but the best stimulus might be a random binary input.

E. If you are fairly certain that a patient is Brain Dead, what types of other electrodes could you use to verify your conclusion (20 points)?

It would be terrible to conclude that a patient is Brain Dead when there is still neural activity deep in the brain. It would be possible to drill a hole in the scalp + skull and then insert a needle electrode into the brain. However, the electrode could pierce a major blood vessel that would be likely to kill a large portion of the brain. Some sort of imaging and electrode alignment should occur that identifies the location of the electrode with respect to major vessels. Another option would be to use a gradiometer to measure the MEG, which can resolve deeper activity.

## Problem 25 (Specialized: Bio Eng-ECE4781) Code Number:________

A. Write an equation that is linear and algebraic but does not obey the Law of Superposition (10 pts).

$$Y = a + bX$$

B. If a system is linear, time-invariant and causal, what is the best stimulus for characterizing the system and why (20 pts)?

The linear transfer function is equal to the pulse response, but watch out for noise.

C. What type of displacement sensor is described partly by a variable which is commonly referred to as Poisson's Ratio (10 pts)?

Strain gage

D. What is the biggest problem with semiconductor materials that are used to detect displacement (10 pts)?

They are very temperature dependent.

E. What does R represent in the Nernst Equation (10 pts)?

Gas constant

F. What type of neuron has multiple axons (10 pts)?

A motor neuron

G. What prevents an Action Potential from propagating backwards along a single neuron
(10 pts)?

Absolute refractory period

H. What is the lowest frequency band of the EEG called, which occurs during deep sleep
(20 pts)?

Delta