Ph.D. Preliminary Examination **Code Number ___________**

Fall 2011 Solutions

Instructions:

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Note that **Problem X occupies 2 pages** and **Problem Y occupies 2 pages**. Including the cover sheet, you should have **Z pages**. There should be no blank pages in the booklet.
2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.
3. All wireless devices must be turned off for the entire duration of the exam.
4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.
5. Your examination code number **MUST APPEAR ON EVERY SHEET**. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. **DO NOT** write your name on any of these sheets. Use the preprinted numbers whenever possible, or **WRITE LEGIBLY!!!**
6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. **DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!**
7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM.
8. When you hand in the exam:
   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in **EVERY** sheet you are turning in.
   (c) On the section at the bottom of this page, **CIRCLE** the problem numbers that you are turning in for grading.
   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

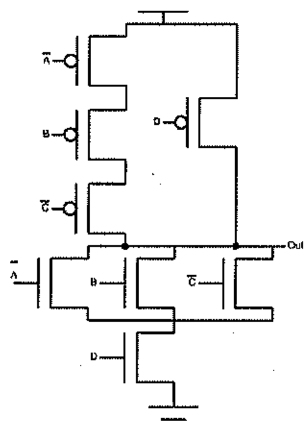
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**	**25**		

Problem 1 (Core: VSDD-ECE2030)   Code Number:________

ECE 2030 Prelim Problem:
Complete each design below. Be sure to label all signals.

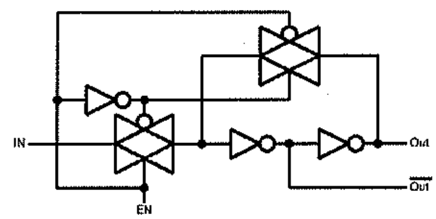
Part A: Implement the following expression using N and P type switches (NFETs and PFETs).

$$Out_X = A \cdot \overline{B} \cdot C + \overline{D}$$

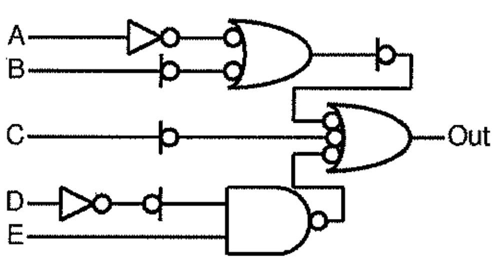


Part B: Implement a transparent latch using only pass gates and inverters.

IN	Y	OUT	$\overline{OUT}$
A	0	Q_o	$\overline{Q_o}$
A	1	A	$\overline{A}$



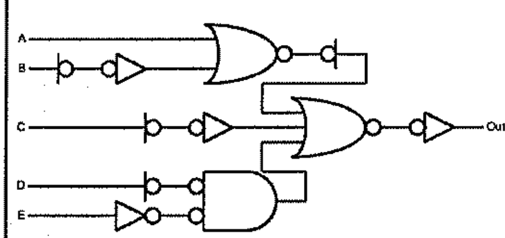
Part C: Determine the appropriate expression for this mixed logic design. How many transistors are required?



Out = $\overline{A+\overline{B}}+\overline{C}+\overline{D}\cdot E$

transistors = 18T

Part D: Reimplement the design in Part C using *only* NOR gates. Use proper mixed logic notation. How many transistors are required?



transistors = 22T

Problem 2 (Core: VSDD-ECE2030) Code Number:__________

(a) Consider the signed twos complement numbers X = 10110101 (8 bit signed twos complement) and Y = 1101 (4 bit signed twos complement). Compute Z = X+Y (write Z in 8 bit signed twos complement format)

Is there any overflow while computing Z ? No

If not, write the correct value of Z below, else leave blank.

Z = 10110010

What is the value of Z in decimal (if you gave a value for Z above) ?
Z = -77

(b) Consider X = 1011010 and Y = 1000101, where X and Y are 7 bit signed twos complement numbers. Compute Z = X+Y, where Z is 7 bit signed twos complement as well (write Z in binary).

Is there overflow while computing Z ? Yes

If not, write the correct value of Z below, else leave blank.

Z =

What is the value of Z in decimal (if you gave a value for Z above) ?

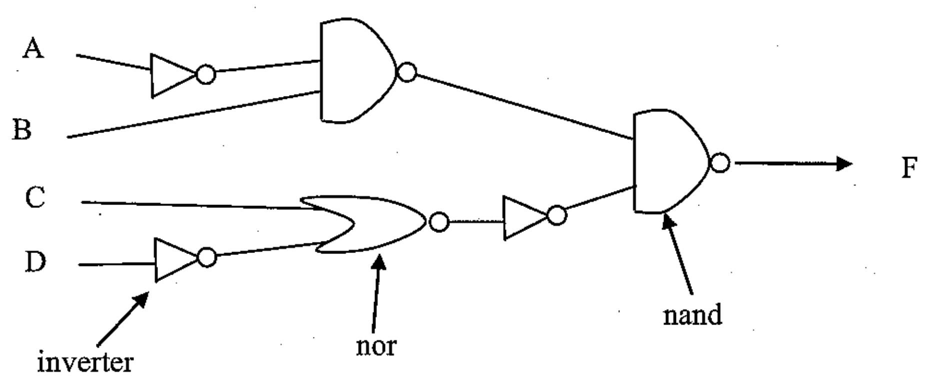
Problem 2 (Core: VSDD-ECE2030) Code Number:_________

(c) Give a minimal *product of sums* expression for the function f( ) below:

wx \ yz	00	01	11	10
00	1			1
01	1	1	1	
11	1		1	
10	1		1	1

f(wxyz) = $(W + X + \overline{Z}) \bullet (\overline{W} + Y + \overline{Z}) \bullet (\overline{X} + \overline{Y} + Z)$

(d) Using DeMorgan's Law, express F for the circuit below in terms of A, B, C and D in sum of products form. Simplify the expression for F to the maximum extent possible



F = $(\overline{A} \bullet B) + (\overline{C} \bullet D)$

Problem 3 (Core: CSS-ECE3055) Code Number:__________

SOLUTION

Prelim Question: Computer Systems and Software, ECE 3055 Material

Consider a virtual memory system with the below characteristics. For ease of analysis, assume that all memory operations are independent.

- inverted page table with a perfect hash function, i.e. each virtual page has a unique hash value
- TLB hit and cache hit together take 1 cycle
- cache access or TLB access by itself takes 1 cycle
- main memory access takes 50 cycles
- page fault takes 10^6 cycles
- page table entries are not cached, i.e. TLB misses go directly to main memory
- cache is indexed by physical addresses
- miss rates are as follows: TLB 0.02, cache 0.01, main memory 0.001

Calculate the expected number of cycles taken on a memory operation. Show your work.

There are 3 cases for translation and similar 3 cases for the memory access: C1) hit (TLB or cache), C2) miss (TLB or cache) + main mem. hit, C3) miss (TLB or cache) + main mem. miss. This yields 9 cases total when combining translation and access.

<u>Case 1:</u> TLB hit, cache hit

Prob. = 0.98 * 0.99 **Time = 1 cycle**

<u>Case 2:</u> TLB miss + main mem. hit, cache hit

Prob. = 0.02 * 0.999 * 0.99 **Time = 52 cycles**

<u>Case 3:</u> TLB miss + main mem. miss, cache hit

Prob. = 0.02 * 0.001 * 0.99 **Time = 10^6 + 52 cycles**

<u>Case 4:</u> TLB hit, cache miss + main mem. hit

Prob. = 0.98 * 0.01 * 0.999 **Time = 51 cycles**

<u>Case 5:</u> TLB miss + main mem. hit, cache miss + main mem. hit

Prob. = 0.02 * 0.999 * 0.01 * 0.999 **Time = 102 cycles**

<u>Case 6:</u> TLB miss + main mem. miss, cache miss + main mem. hit

Prob. = 0.02 * 0.001 * 0.01 * 0.999 **Time = 10^6 + 102 cycles**

<u>Case 7:</u> TLB hit, cache miss + main mem. miss

Prob. = 0.98 * 0.01 * 0.001 **Time = 10^6 + 51 cycles**

<u>Case 8:</u> TLB miss + main mem. hit, cache miss + main mem. miss

Prob. = 0.02 * 0.999 * 0.01 * 0.001 **Time = 10^6 + 102 cycles**

<u>Case 9:</u> TLB miss + main mem. miss, cache miss + main mem. miss

Prob. = 0.02 * 0.001 * 0.01 * 0.001 **Time = (2×10^6) + 102 cycles**

E[no. cycles] = $\sum (prob_i * time_i)$ = 32.50

Problem 4 (Core: VSDD-ECE3060) Code Number:________

Compare transistor sizes in NAND and NOR gates:

a) Size the transistors in a three-input, static complementary NAND gate so that the gate's rise and fall times are approximately equal.

Worst case pullup: one p-type. Worst case pulldown: 3 series n-types.
Let $k'_p\, W/L_p = 3\, k'_n\, W/L_n$
If $W/L_p = 6*(3/2)$, then $W/L_n = 3*(3/2)$

b) Size the transistors in a three-input, static complementary NOR gate so that the gate's rise and fall times are approximately equal.

Worst case pullup: three series p-types. Worst case pulldown: 1 n-type.
Let $3\, k'_p\, W/L_p = k'_n\, W/L_n$
If $W/L_p = 18*(3/2)$, then $W/L_n = (3/2)$

c) Find the ratio of total transistor area in the NAND gate vs. the NOR gate.

NAND area (in units of minimum-size transistors):

Total n-type area : 9
Total p-type area: 18
Total area: 27

NOR area (in units of minimum-size transistors):

Total n-type area: 9
Total p-type area: 54
Total area: 63

Problem 5 (Core: E&M-ECE3025) **Code Number:__________**

A parallel-plate capacitor is constructed from two metal plates of dimension 1 cm by 1 cm, held a distance of 5 mm apart by a material with $\varepsilon = 4\varepsilon_0$. The electric field between the plates is 400 V/m. Neglect fringing effects at the plate edges.

(a) What is the voltage between the plates?

$$V = \int \bar{E} \bullet d\bar{l} = 400 \text{ (V/m)} \times 0.005 \text{ (m)} = 2 \text{ (V)}$$

(b) What is the surface charge density magnitude on either plate?

$$\bar{D} = \varepsilon_0 \varepsilon_r \bar{E} = 4(400)\varepsilon_0 \hat{u}\ ;\ 1.417 \times 10^{-8} \hat{u} \text{ (C/m}^2\text{)}$$

$$\rho_S = \hat{n} \bullet \bar{D}\ ;\ 1.417 \times 10^{-8} \text{ (C/m}^2\text{)}$$

(c) How much energy is stored in the capacitor?

$$W = \iiint \frac{1}{2} \varepsilon_0 \varepsilon_r \left|\bar{E}\right|^2 dV = \frac{1}{2} 4\varepsilon_0 (400)^2 (0.01)(0.01)(0.005)$$
$$= 0.16\varepsilon_0\ ;\ 1.417 \times 10^{-12} \text{ (J)}$$

or, using the answers from parts (a) and (e),

$$W = \frac{1}{2} C V^2\ ;\ 1.417 \times 10^{-12} \text{ (J)}$$

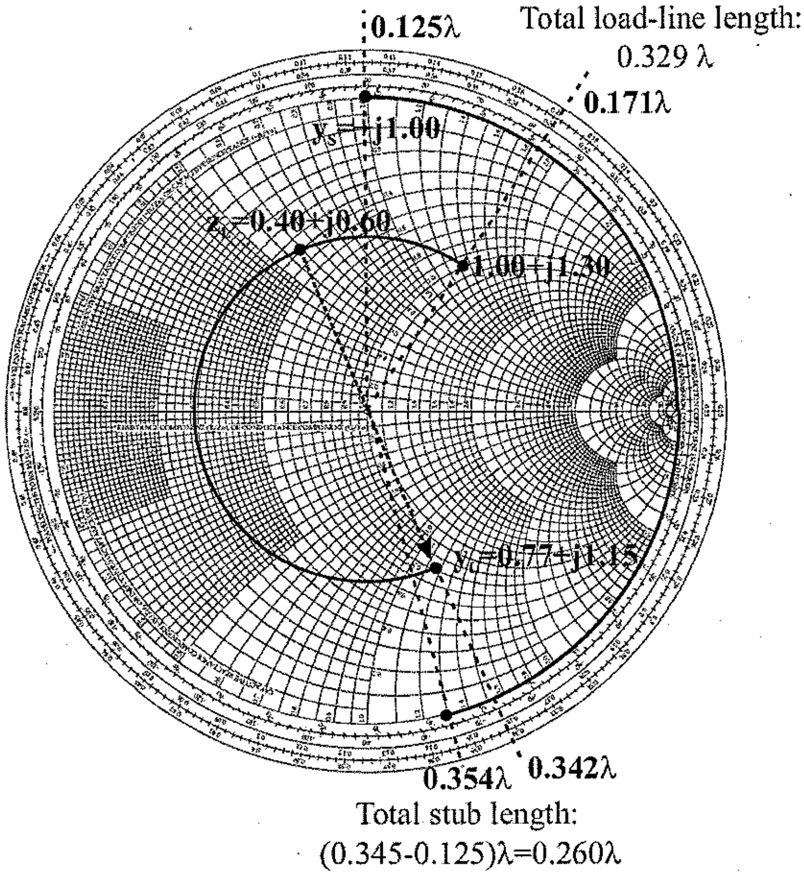
(d) What is the magnitude of the force, in Newtons, on each plate?

$$\left|\bar{F}\right| = \rho_S \left|\bar{E}\right| A / 2$$
$$= (1.417 \times 10^{-8})(200)(0.01)(0.01)\ ;\ 2.833 \times 10^{-10} \text{ (N)}$$

(e) What is the capacitance?

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0 (4)(0.01)(0.01)}{0.005} = 0.08\varepsilon_0\ ;\ 7.083 \times 10^{-13} \text{ (F)}$$

Problem 6 (Core: E&M-ECE3065) Code Number:__________



This problem follows the same formulaic approach of all parallel stub problems. Follow the algorithm:

(a) Calculate the normalized load impedance ($z_L = 0.40 + j0.60$) and plot on the Smith Chart.

(b) Rotate the load point 180° about the center to achieve the normalized *admittance* ($y_L = 0.77 - j1.15$).

(c) Translate this point down the transmission line until the equivalent load admittance has a matched real component. This occurs after a distance of $d = 0.329\lambda$, which results in a $1.0 + \text{j}1.3$ effective load.

(d) Since our stub terminates in a capacitor, we start out with a normalized admittance of $+j1.0$ (not on the $+j0.0$ point, which would be the admittance of an open circuit).

(e) Translate the stub down a length of line until the effective stub admittance is $-j1.3$ to cancel the reactive component of our load at point d. This occurs after a rotation of $w = 0.220\lambda$.

This clever maneuver does save some space compared to an open circuit stub design, which would have the same design except we would be required to start at $+j0.0$ – introducing an extra eight-wavelength to the stub length for a total of $w = 0.345\lambda$.

Problem 7 (Core: EDA-ECE2040) Code Number:_________

Solution:

Exploit the symmetry in the problem to the max. First if the current through the voltage source is I, then from the symmetry $I/2$ Amps must flow through both branches (by node A) on the top left, and $-I/2$ through the branches at the bottom left (by node B). Hence, we can "disect" the Möbius ladder and attach identical sources at the two ends to maintain the equivalence.

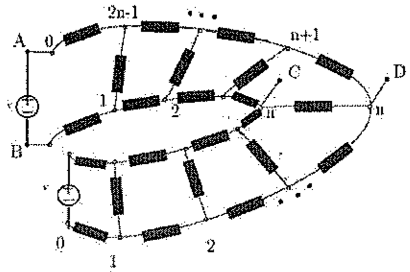


Figure 1: Möbius ladder opened

Open this up completely and give it a twist to get the ladder circuit of Figure 2.

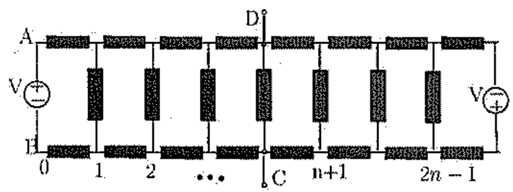


Figure 2: Möbius ladder twisted and stretched

i) Let's use superposition. Let i be the current from C to D in the resistor due to the voltage source V on the left, and zero the source on the right. From the symmetry, we see that the current from C to D due to the voltage source on the right and zeroing the source on the left must be $-i$. Consequently, no current will flow through the resistor connecting C and D in the Möbius ladder. If no current flows, the points C and D must have the same potential. Hence. $V_{oc} = 0$.

Now short the resistor from C to D. (This effectively replaces this resistor by a short.) The ladder is still symmetric, so also in this new configuration, no current will flow from C to D. Hence $I_{sc} = 0$.

2

Problem 7 (Core: EDA-ECE2040) **Code Number:__________**

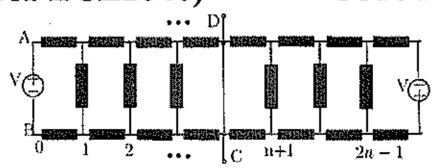


Figure 3: Configuration with CD shorted

The ratio V_{oc}/I_{sc} is undetermined,. Hence this will not yield the equivalent Thévenin resistance.

ii) In order to determine the Thévenin resistance, zero the source (it becomes a short circuit) and apply a test voltage V_{test} to CD. The corresponding current, I_{test}, through the test source determines R_{Th} by $R_{Th} = V_{test}/I_{test}$. From Figure 3, we se that this gives a configuration where the ladder on the left of CD is identical to the one on the right. The test source is further in parallel with a resistor R. We also know by the symmetry that the left and right ends are at the same potential, $V_{test}/2$, hence their connection can be severed, giving the configuration in figure 4. Let R_n be the internal resistance of the ladder on the right, then the total Thévenin resistance

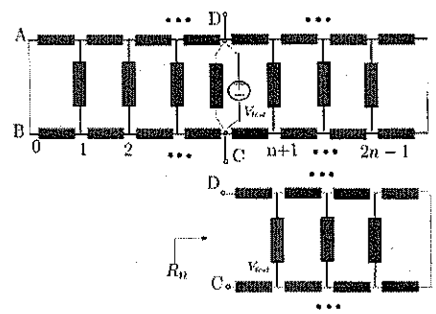


Figure 4: Determining the internal resistance

is

$$R_{Th,n} = R//R_n//R_n = R//\frac{1}{2}R_n = \frac{RR_n}{2(R+R_n/2)} = \frac{RR_n}{2R+R_n}. \tag{1}$$

See figure 5. For $n = 1$, we find $R_1 = 2R$, and with (1), $R_{Th,1} = \frac{2R^2}{4R} = \frac{R}{2}$.
For $n = 2$, we find $R_2 = (2R//R) + 2R = \frac{8}{3}R$, and with (1), $R_{Th,2} = \frac{8R^2}{16R} = \frac{4}{7}R$.
Thus $\rho_1 = \frac{1}{2}$ and $\rho_2 = \frac{4}{7}$.

iii) Of course, $R_{Norton}/R = R_{Th}/R = \rho$. See figure 6. It is readily seen from this figure that

$$R_{n+1} = 2R + R//R_n = 2R + \frac{RR_n}{R+R_n} = \frac{2R+3R_n}{R+R_n}R.$$

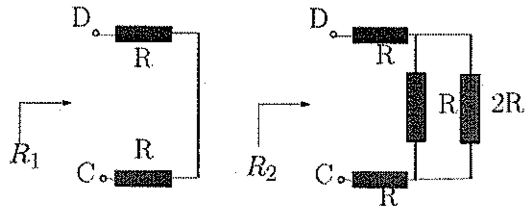


Figure 5: Determining the internal resistance for $n = 1$ and $n = 2$

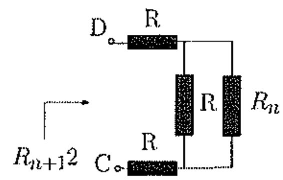


Figure 6: Setting up the iteration for the internal resistance

With (1), and its inverse relation, $R_n = \frac{2RR_{Th,n}}{R-R_{Th,n}}$, this yields

$$\begin{aligned} R_{Th,n+1} &= \frac{RR_{n+1}}{2R+R_{n+1}} \\ &= \frac{1+2\rho_n}{2+3\rho_n}R. \end{aligned}$$

Thus, finally,

$$\rho_{n+1} = \frac{1+2\rho_n}{2+3\rho_n}.$$

iv) Letting $n \longrightarrow \infty$, we get in the limit

$$\rho_\infty = \frac{1+2\rho_\infty}{2+3\rho_\infty},$$

or

$$3\rho_\infty^2 + 2\rho_\infty = 1 + 2\rho_\infty.$$

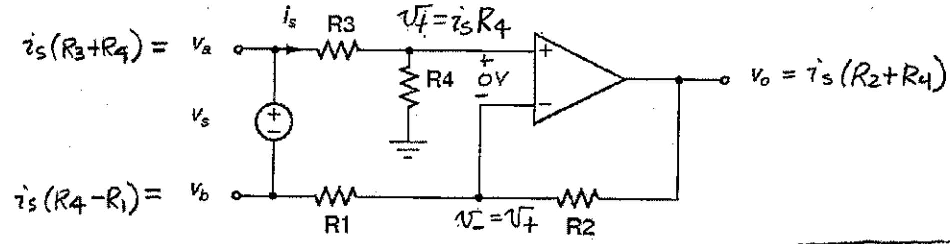
Solving for ρ_∞, we find, taking the requisite positive root

$$\rho_\infty = \frac{1}{\sqrt{3}}.$$

Problem 8 (Core: EDA-ECE3050) Code Number:__________

For the given circuit, calculate the values for R1, R2, R3, and R4 given the following specifications:

(1) $v_o / v_s = 10$
(2) $R_{in} = v_s / i_s = 10\text{k}\Omega$
(3) CMRR = ∞, i.e., ($v_o / v_a = - v_o / v_b$)
(4) Zero induced common mode input voltage, i.e, $v_a = - v_b$



(1) $i_s = \frac{v_s}{R_{in}}$ so $v_o = \frac{v_s}{R_{in}}(R_2+R_4) \Rightarrow$ $\boxed{R_2 + R_4 = 10\,R_{in} = 100\text{k}\Omega}$

(2) KVL around the loop including Vs and the virtual short:

$v_s = i_s R_3 + 0 + i_s R_1 \Rightarrow \frac{v_s}{i_s} =$ $\boxed{R_1 + R_3 = R_{in} = 10\text{k}\Omega}$

(3) To find v_o/v_a, set $v_b = 0$, then: $v_o = v_a \underbrace{\frac{R_4}{R_3+R_4}}_{v_+} \underbrace{\left(1+\frac{R_2}{R_1}\right)}_{\text{non-inverting gain}}$

To find v_o/v_b, set $v_a = 0$, then: $v_o = \underbrace{-\frac{R_2}{R_1}}_{\text{inverting gain}} v_b$

For CMRR $= \infty$, $\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right) = \frac{R_2}{R_1} \Rightarrow$ $\boxed{\frac{R_4}{R_3} = \frac{R_2}{R_1}}$

(4) For $v_a = -v_b$, $i_s(R_3+R_4) = i_s(R_1-R_4) \Rightarrow R_3 + 2R_4 = R_1$

Adding R_3 to both sides gives: $2(R_3+R_4) = R_1+R_3 = R_{in} = 10\text{k}\Omega$

$\Rightarrow$ $\boxed{R_3+R_4 = \frac{R_{in}}{2} = 5\text{k}\Omega}$

The solution satisfying all of the above is:

$R_3 = 455\Omega$, $R_4 = 4.55\text{k}\Omega$, $R_1 = 9.55\text{k}\Omega$, $R_2 = 95.5\text{k}\Omega$

Problem 9 (Core: Power-ECE3070) **Code Number:__________**

SOLUTION:

(a) The phase voltage of this generator at rated conditions is

$$V_\phi = \frac{13,800 \text{ V}}{\sqrt{3}} = 7967 \text{ V}$$

The armature current per phase at rated conditions is

$$I_A = \frac{S}{\sqrt{3}\ V_T} = \frac{10,000,000 \text{ VA}}{\sqrt{3}(13,800 \text{ V})} = 418 \text{ A}$$

Therefore, the internal generated voltage at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (1.5\ \Omega)(418\angle -36.87° \text{ A}) + j(12.0\ \Omega)(418\angle -36.87° \text{ A})$$
$$\mathbf{E}_A = 12,040\angle 17.6° \text{ V}$$

The magnitude of E_A is 12,040 V.

(b) The torque angle of the generator at rated conditions is δ= 17.6°.

(c) Ignoring R_A, the *maximum output power* of the generator is given by

$$P_{\text{MAX}} = \frac{3\ V_\phi\ E_A}{X_S} = \frac{3(7967 \text{ V})(12,040 \text{ V})}{12\ \Omega} = 24.0 \text{ MW}$$

The power at maximum load is 8 MW, so the maximum output power is three times the full load output power.

Under these conditions, the armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{R_A + jX_S} = \frac{12,040\angle 90° \text{ V - } 7967\angle 0° \text{ V}}{1.5 + j12.0\ \Omega} = 1194\angle 40.6° \text{ A}$$

The reactive power produced by the generator at this point is

$$Q = 3\ V_\phi\ I_A \sin\theta = 3(7967 \text{ V})(1194 \text{ A})\ \sin(0° - 40.6°) = -18.6 \text{ MVAR}$$

The generator is actually consuming reactive power at this time.

Problem 10 (Core: Power-ECE3070) **Code Number:__________**

a) $P_g = P_{in} - P_{stator} = \sqrt{3}(480)(50)(.85) - 1000 - 500 = 33.833\,\text{kW}$

b) $P_{em} = P_g - P_{rotor} = 33833\,\text{W} - 500\,\text{W} = 33.33\,\text{kW}$

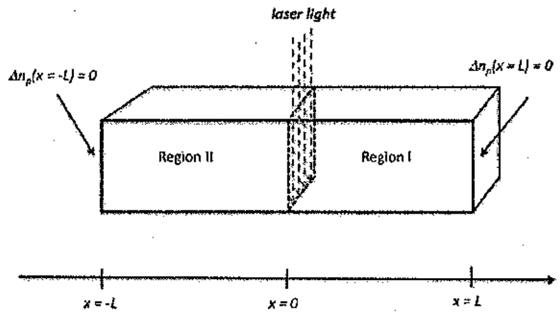
c) $P_{out} = P_{em} - P_{f\&w} = 33.33\,\text{kW} - 250\,\text{W} = 33.08\,\text{kW}$

$$\frac{33.08\,\text{kW}}{746\,\text{W}/\text{HP}} = 44.34\,\text{HP}$$

Problem 11 (Core: Microsystems-ECE3040) Code Number:__________

Problem 11: (Core: Microsystems ECE3040)   Code Number: ______________

Assume that a laser is used to illuminate a thin cross-section of a uniformly doped bar of silicon maintained at room temperature (i.e. T = 300°K). Assume that the laser-induced photogeneration occurs only at x =0 and is uniform across the cross-section as seen in the following diagram. At x=0, assume that the steady-state number of electron hole pairs (EHPs) is 10^{11} EHP/cm^3 and that there is no electric field inside the silicon bar. The dopant in this case is boron and the doping density is 10^{16} /cm^3. Also assume that the excess minority carrier concentration at the edges of this bar is zero at *x=L* and *x=-L*. The minority carrier lifetime for this bar can be assumed to be one microsecond. Also assume that the mobility for the minority carriers is approximately 800 cm^2/V-s, and that the intrinsic carrier concentration for silicon at room temperature is approximately 10^{10} /cm^3.



a.) What is the value of the diffusion constant for the minority carriers in this silicon bar?

$$D_n = \mu_n \frac{KT}{q} = 800 \frac{cm^2}{V \cdot s} \frac{1.38\times10^{-23} J/K \cdot 300K}{1.6\times10^{-19} C} = \boxed{20.7 \frac{cm^2}{s}}$$

b.) Assuming no significant variations of the carrier concentrations in the *y* and *z* directions, what is the differential equation that describes the excess minority carrier concentration in steady-state as a function of *x* in Region I and Region II of the semiconducting bar.

$\frac{\partial \Delta n_p}{\partial t} = 0$ in steady state. Using assumptions of low-level injection and no electric field in bar we can use the minority carrier diffusion equation

$$0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} \quad \text{(note that } G_L = 0 \text{ inside region I)}$$

$$\boxed{0 = \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{L_n^2}}; \text{ where } L_n = \sqrt{D_n \tau_n} = 4.55\times10^{-3} \text{ cm}$$

Problem 11 (Core: Microsystems-ECE3040) Code Number:__________

c) For the case where L is much greater than the average diffusion length, please give the approximate expression for the excess minority carrier concentration in Region I and Region II.

The general solution to the differential equation in part (b) is:

$$\Delta n_p = Ae^{-x/L_n} + Be^{x/L_n}$$

In region I, B must be zero so that solution can converge as $x \to \infty$; therefore,

$$\Delta n_p = Ae^{-x/L_n} \qquad \Delta n_p(x=0) = 10^{11} = Ae^{0}$$

Region I: $\Delta n_p(x) = 10^{11} e^{-x/L_n}$

Region II: $\Delta n_p(x) = 10^{11} e^{x/L_n}$

d) For the same assumptions in part c, please provide an expression for the quasi-Fermi level for the minority carriers (relative to the intrinsic Fermi level) in Region I.

$$n_p = n_i e^{(F_n - E_i)/kT}$$

$$\Delta n_p = n_p - n_{po}$$

$$F_n = E_i + kT\ln(\Delta n_p + n_{po})$$

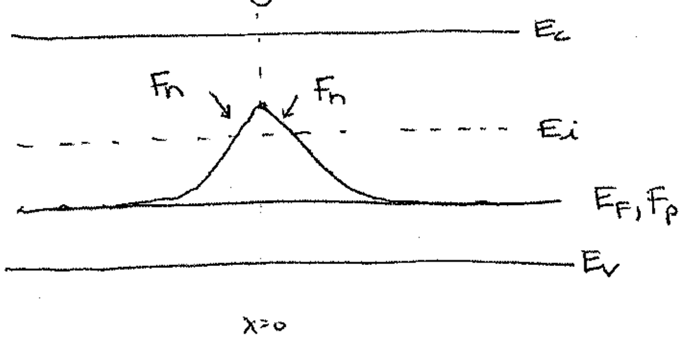
$$\boxed{F_n = E_i + kT\ln\left(10^{11} e^{-x/L_n} + \frac{n_i^2}{N_A}\right)}$$

$N_A = 10^{16}/cm^3$

e) For the same assumptions in part c, please show the band diagram for this bar and approximately show the quasi-Fermi levels for majority and minority carriers, the intrinsic Fermi level, the conduction band, and the valence band. For reference, show the equilibrium Fermi level on your diagram as well.

F_n = quasi-Fermi level for minority carriers $\quad E_F$ = equilibrium Fermi level

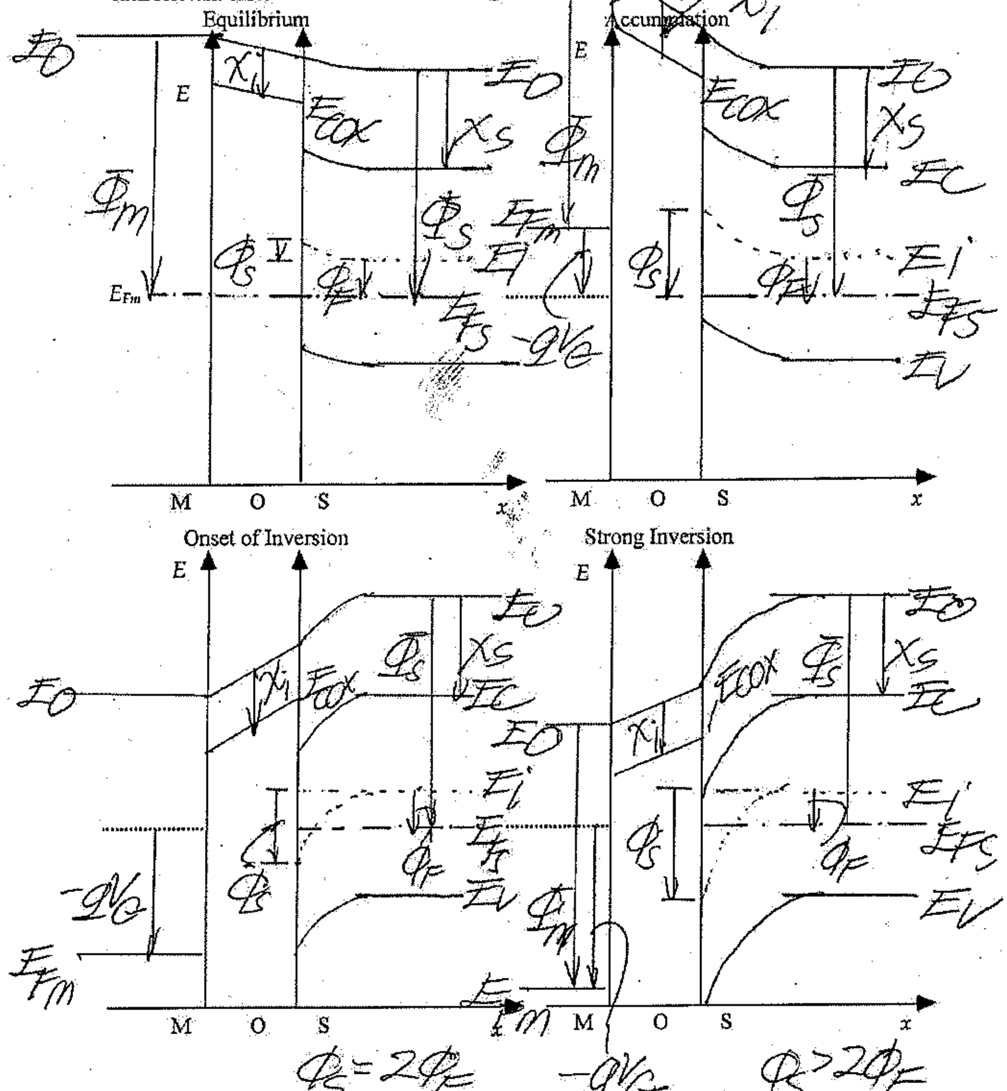
F_p = quasi-Fermi level for majority carriers $\quad E_i$ = intrinsic Fermi level



Problem 12 (Core: Microsystems-ECE3080) Code Number:__________

ECE 3040 QUALIFIER EXAM FALL 2011 DUPUIS

1a.-1d.: ***NMOS Capacitor with*** $\Phi_m > \Phi_s$: Show Φ_m, Φ_s, E_{cox}, E_c, E_v, E_{Fs}, E_{Fm}, E_i, E_o, ϕ_s, ϕ_F, and χ_s and draw and label an arrow showing the *magnitude of the applied gate voltage,* V_G, for each relevant case.



Problem 13 (Core: DSP-ECE2025) Code Number:_________

Problem 13 (Core: DSP-ECE2025)

Consider a continuous time signal $s(t) = 1 + 2\cos(0.2\pi t) + 3\cos(1.5\pi t + 0.4\pi)$.

(a) **Find the fundamental frequency f_0 and the Fourier series coefficients a_k's of $s(t)$, i.e., $s(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$. Note that you need to list all the non-zero coefficients a_k's and their corresponding indices k.**

$$f_0 = 0.05 \text{ Hz}$$
$$a_0 = 1, a_2 = a_{-2} = 1, a_{15} = \frac{3}{2}e^{j0.4\pi},\ a_{-15} = \frac{3}{2}e^{-j0.4\pi},$$
$$\text{otherwise}\ , a_k = 0$$

(b) **The signal $s(t)$ is filtered by a linear time-invariant system whose impulse response is $h(t)$, and whose frequency response is $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = (1 - e^{-|\frac{\omega}{2\pi}|})e^{-j0.1\omega}$. Find the resulting output $y(t) = s(t) \star h(t)$ with $\star$ denoting the linear convolution. Express your answer for $y(t)$ as a real-valued function of t.**

The frequency responses of the filter at $0, \pm 0.2\pi$, and $\pm 1.5\pi$ are

$$H(0) = 0,$$
$$H(j0.2\pi) = (1 - e^{-0.1})e^{-j0.02\pi},\ H(-j0.2\pi) = (1 - e^{-0.1})e^{j0.02\pi},$$
$$H(j1.5\pi) = (1 - e^{-0.75})e^{-j0.15\pi},\ H(-j1.5\pi) = (1 - e^{-0.75})e^{j0.15\pi}.$$

Thus, the output is

$$y(t) = (1 - e^{-0.1})2\cos(0.2\pi t - 0.02\pi) + 3(1 - e^{-0.75})\cos(1.5\pi t + 0.25\pi)$$
$$= 0.19\cos(0.2\pi t - 0.02\pi) + 1.58\cos(1.5\pi t + 0.25\pi).$$

(c) **The signal $s(t)$ is sampled by an ideal continuous-to-discrete (C-to-D) converter operating at a rate of $f_s = 1$ Hz. The resulting discrete-time signal $x[n]$ goes through an ideal discrete-to-continuous (D-to-C) converter operating at a higher rate of $f_s = 2$ Hz. Find the D-to-C output $z(t)$ as a real-valued function of t.**

$$x[n] = 1 + 2\cos(0.2\pi n) + 3\cos(1.5\pi n + 0.4\pi) = 1 + 2\cos(0.2\pi n) + 3\cos(0.5\pi n - 0.4\pi)$$
$$z(t) = 1 + 2\cos(0.4\pi t) + 3\cos(\pi t - 0.4\pi)$$

Here you can assume that the scaling factor from sampling is taken care of by the ideal low pass filter.

Problem 14 (Core: DSP-ECE3075) Code Number:__________

SOLUTION

Problem for ECE 3075:

For this problem, you may find the following formula useful:

$$\int_0^\infty x^n \exp[-ax]dx = \frac{n!}{a^{n+1}}$$

Max and Addie are engaged in a timed Sudoku tournament.

For Max, the probability density function for the time (in minutes) required to finish is:

$f_X(x) = \lambda^2 x e^{-\lambda x} u(x)$.

For Addie, the probability density function for the time required to finish is:

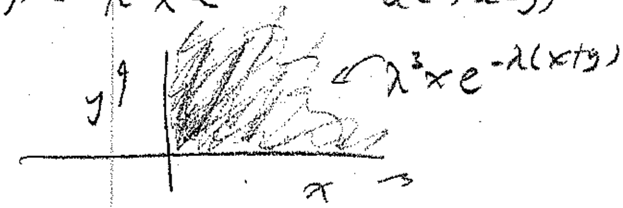
$f_Y(y) = \lambda e^{-\lambda y} u(y)$.

The times are statistically independent of each other.

(a) Find the joint density function for X and Y. A drawing will be helpful.

Since they are independent, the pdf's multiply:

$$f_{XY}(x,y) = \lambda^3 x e^{-\lambda(x+y)} u(x) u(y)$$



(b) Let Z be the difference between the two times: $Z = X - Y$. Find the means and variances for X, Y, and Z.

Using above formula:

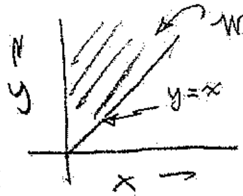
$\bar{X} = 2/\lambda$ $\qquad \sigma_X^2 = 2/\lambda^2$

$\bar{Y} = 1/\lambda$ $\qquad \sigma_Y^2 = 1/\lambda^2$

$\bar{Z} = \bar{X} - \bar{Y} = 1/\lambda$ $\qquad \sigma_Z^2 = \sigma_X^2 + (-1)^2 \sigma_Y^2 = 3/\lambda^3$

Problem 14 (Core: DSP-ECE3075)   Code Number:__________

(c) Let W represent the event {Max beats Addie} (i.e., $X < Y$). Find Prob(W).



$$Pr(W) = \int_0^\infty \int_x^\infty f_{XY}(x,y)\,dy\,dx$$
$$= \int_0^\infty \lambda^2 x e^{-\lambda x} \int_x^\infty \lambda e^{-\lambda y} dy\, dx$$
$$= \int_0^\infty \lambda^2 x e^{-2\lambda x} dx = \boxed{1/4}$$

- i.e. - X represents the time until the 2nd arrival in a Poisson process with rate λ. Y represents the time until the 1st arrival in an identical independent process. $X < Y \rightarrow$ two arrivals in the 1st process precede one arrival in second. Prob(·) = 1/4.

(d) Find $f_{X|W}(x|W)$, the conditional probability density function for Max's time, given that he beat Addie.

Using above, the conditional joint pdf for X, Y given $X<Y$ would be $4 f_{XY}(x,y)$ for $x<y$; $= 0$ otherwise.

$$f_{X|W}(x|W) = 4\int_x^\infty \lambda^3 x e^{-\lambda x} e^{-\lambda y} dy = 4\lambda^2 x e^{-\lambda x} \int_x^\infty \lambda e^{-\lambda y} dy$$
$$= 4\lambda^2 x e^{-\lambda 2x} u(x) = (2\lambda)^2 x e^{-(2\lambda)x} u(x)$$

(this is the pdf for the time until 2nd arrival of a process of rate 2λ. this rate = rate of process 1 + rate of process 2.)
$2\lambda = \lambda + \lambda$

(e) Independent of whether she wins or not, Addie will receive N dollars in prize money, where $N = e^{-Y}$. Find $f_N(n)$, the pdf for her winnings.

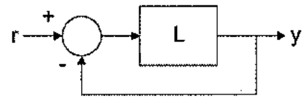
$Y = -\ln N$ range of N: $[0, 1]$

$\left|\frac{dN}{dY}\right| = \left|-\frac{1}{N}\right|$

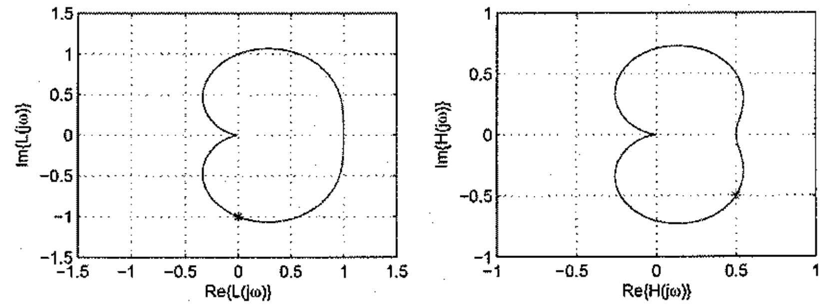
$$f_N(n) = \frac{1}{n}\lambda e^{-\lambda(-\ln n)} \quad \text{for } 0 \le n \le 1$$
$$= \frac{1}{n}\lambda e^{\ln n^\lambda}$$
$$= \frac{1}{n}\lambda \cdot n^\lambda = \lambda n^{\lambda - 1} \quad 0 \le n \le 1$$

Problem 15 (Core: S&C-ECE3085)   Code Number:_________

A control system has the following structure, where $L(s)$ is a ratio of two polynomials; $L(s)$ has more poles than zeros, and all its poles are located in the open left half plane. Let $H(s)$ denote the overall transfer function satisfying $Y(s) = H(s)R(s)$.



(a) The loop transfer function $L(s)$ is characterized in the diagram to the left and the overall transfer function $H(s)$ is characterized in the diagram to the right; the labeling of these diagrams establishes a unique correspondence between them. Precisely mark the diagram to the right with an asterisk corresponding to the asterisk found on the diagram to the left.



$$L(j\omega) = -j \quad \Rightarrow \quad H(j\omega) = \frac{L(j\omega)}{1 + L(j\omega)} = \frac{-j}{1-j} = \frac{-j}{1-j} \cdot \frac{1+j}{1+j} = \frac{1-j}{2}$$

(b) Prove that this system is internally stable.

The given plot of $L(j\omega)$ is at least a portion of the Nyquist plot of $L(s)$. Since $L(s)$ is strictly proper, the point $L(j\omega) = 0$ corresponds to $\omega = \pm\infty$. Since the coefficients of $L(s)$ are real, the point $L(j\omega) = 1$ corresponds to $\omega = 0$. As further confirmation that the entire $j\omega$ axis has been accounted for, note that the given plot is a closed curve symmetric about the real axis. Finally, since $L(s)$ has no poles in the open RHP, this system is internally stable if and only if the Nyquist plot of $L(s)$ does not pass through or encircle the critical point -1; by inspection, this property is satisfied in the given plot.

(c) Determine the phase margin and the gain margin of this system.

When $|L(j\omega)| = 1$ or 0 dB, $\angle L(j\omega) \geq -90°$. Therefore, PM is $90°$.

As $\angle L(j\omega) \to -180°$, $|L(j\omega)| \to 0$ or $-\infty$ dB. Therefore, GM is 1 or ∞ dB.

(d) Assume $r(t) = 1(t)$, where $1(t)$ denotes the unit step function.
Find α such that $\lim_{t\to\infty} y(t) = \alpha$.

At $\omega = 0$, $L(j0) = 1$ and hence $H(j0) = \frac{1}{2}$. Therefore, $\alpha = \frac{1}{2}$.

(e) Assume $r(t) = \cos(\omega_0 t) \cdot 1(t)$, where $1(t)$ denotes the unit step function.
Find α and θ such that $\lim_{t\to\infty} y(t) = \alpha\cos(\omega_0 t + \theta)$, where ω_0 corresponds to the asterisk in part (a).

At $\omega = \omega_0$, $L(j\omega_0) = -j$ and hence $H(j\omega_0) = \frac{1}{2}\sqrt{2}e^{-j\frac{\pi}{4}}$. Therefore, $\alpha = \frac{1}{2}\sqrt{2}$ and $\theta = -\frac{\pi}{4}$.

Problem 16 (Core: S&C-ECE3085)   Code Number:__________

1. We know that for steady-state tracking of a ramp there is a need for at least two integrators in the loop transfer function $G_c(s)G(s)$. One integrator is provided by the plant (the pole at 0), so all that is needed is an additional pole at 0. Let us try $G_c(s) = \frac{K}{s}$ for some $K > 0$. In this case the first column of the Routh table of the denominator polynomial of the closed-loop transfer function has the following entries, τ, 1, $-\tau AK$, AK. Consequently, the closed-loop system is unstable for any $K > 0$.

   Next, let us try a PI controller of the form $G_c(s) = K_p + \frac{K_I}{s}$ for some $K_p > 0$ and $K_I > 0$. The first column of the Routh table of the denominator polynomial of the closed-loop transfer function has the following entries, τ, 1, $A(K_p - \tau K_I)$, AK_I. Thus, the closed-loop system will be stable if $K_p - \tau K_I > 0$, and in this case the desired tracking will be achieved.

2. Let $T(s)$ denote the r-to-y transfer function, and let $T_v(s)$ denote the v-to-y transfer function. Then it is well known that $T_v(s) = -T(s)$. Now if the system yields steady-state tracking to a ramp input (r) then it tracks a step as well. Consequently, if $v = \beta \times step$ and $r = 0$ then $\lim_{t\to\infty} y(t) = \beta$, and disturbance rejection is not achieved.

Problem 17 (Specialized: CSS-CS3210) Code Number:______

Answer Key

- The program does not deadlock because the `pthread_cond_wait` at line 39 in the main is passed the address of the `activeMutex` as the second parameter. The `pthread_cond_wait` function is documented to *Unlock the mutex, Wait for the condition signal,* and then *Re-Lock the mutex.* Thus while the main is blocked at line 39 the `activeMutex` mutex is in fact unlocked.
- It is possible that threads 0, 1 and 3 all get scheduled before the main makes it to line 39 (which unlocks the `activeMutex`), and thus are blocked at line 16 (before decrementing `activeThreads`). No threads claim there are 4 active threads because two different threads could execute lines 16 - 22 (sequentially, not concurrently) decrementing `aciveThreads` twice before any other thread executed line 14.
- It is not possible to tell which thread executes line 20. Threads schedule in arbitrary order, so it could be any of the eight threads.

3

Problem 18 (Specialized: CSS - ECE3035) Code Number: _____

Consider a 32-bit system where the memory image of each process starts with the code at the bottom of the address space, followed immediately by the global data, and then the heap. The stack is on the top of the address space and grows downward. Any process can use up the entire 4GB of address space.

The program below consists of a series of recursive function calls. Suppose you can change the value of N defined in line 3. Estimate the maximum value you can define for N before the program will crash. You don't need to come up with an exact number but you need to show your steps.

Notes: 1. double data type is 8 bytes.

2. the code uses Variable Length Array, which is supported in the C99 dialect of C. For example, the GNU C Compiler gcc allocates memory for VLAs on the stack.

```
#include <stdio.h>
#include <stdlib.h>
#define N 7000

int dowork(int n)
{
   register int i,j;
   double A[n][n];

   if( n > 7000 ) {
      dowork(n-1);
   } else {
      for (i=0; i<n; i++)
        for (j=0; j<n; j++)
          A[i][j] = ((i*j)/3.452) + (N-i);
   }
   return 0;
}

int main(int argc, char *argv[])
{
   dowork(N);
   return 0;
}
```

Given the value of N, The program will require stack space of the following size:
The dominant part is the array allocation, which takes $[N^2 + (N-1)^2 + \ldots + 7000^2]*8$ **bytes.**
There are a small number of bytes for the input parameters, returning addresses, and the program itself.
The maximum value N can take is 7009 before the stack overflows the entire address space.

Problem 19 (Specialized: Telecom-ECE3076) Code Number:________

Given that the end-end delay between Host A and Host B is 10 mili-seconds, and a bottleneck link has bandwidth 12.8Mbps. Assume that the bottleneck link dominates the end-end delay which is half of the round trip delay (RTT). Assume maximum segment size (MSS) is 1000 bytes, and all segments are MSS.

(a) Assume TCP now uses a window size n to sent segments. What should n be to fill the "bit pipe" at the bottleneck link in one RTT? (In other words, what is the bandwidth-delay product when the delay is considered as RTT?)

A: n=2x12.8x0.001x10,000,000/(1000x8)=32

(b) Consider that TCP starts from slow-start. Assume that there is no loss. How many RTTs are needed for the TCP congestion window size to grow to be at least n? How many bytes are sent in all the RTTs from the beginning of the slow start to the window size n?

A: #RTT = log 32 =5
(1+2+4+16+32)x1000=55k bytes

(c) Now consider that two TCP flows are sharing the bottleneck link. Assume that each flow is "on" with probability p $(0 < p < 1)$ when an end-use is active and inactive with probability $1\text{-}p$. Users are either active or inactive independently. Assume other conditions remain the same, i.e., TCP still starts from slow start and there is no loss.

Assume TCp is fair. On the average, would one user send more bytes, or fewer bytes, or the same bytes, compared with your answer in Part (b), when the TCP congestion window grows sufficiently large to fill the bit pipe? Why?

A: When $0 < p < 0.5$, one user can send more bytes, since on the average, $0<2p<1$. Hence n can be larger than there is one use who is always active to fill the bit pipe.
When $p= 0.5$, one user sends the same bytes, since on the average, $2p=1$. Hence n is the same as that from Part (a).
When $0.5<p<1$, one user sends fewer bytes, since on the average, $2p>1$. Hence n is smaller than there is one use who is always active to fill the bit pipe.

Problem 20 (Specialized: Optics-ECE4500) Code Number:__________

Linear Polarization by Brewster Angle Incidence

Refractive indices

$$n_1 = 1.000$$

$$n_2 = 1.500$$

The Brewster angle

$$\theta_B = tan^{-1}\frac{n_2}{n_1} = 56.31°$$

The angle of refraction

$$\theta_2 = sin^{-1}\left(\frac{n_1\, sin\,\theta_1}{n_2}\right) = 33.69°$$

Fraction of TE polarized light reflected

$$r_{TE} = \left(\frac{E_r}{E_i}\right)_{TE} = \frac{n_1\, cos\,\theta_1 - n_2\, cos\,\theta_2}{n_1\, cos\,\theta_1 + n_2\, cos\,\theta_2} = -0.384615$$

Fraction of TM polarized light reflected

$$r_{TM} = \left(\frac{E_r}{E_i}\right)_{TM} = \frac{\tilde{n}_2\, cos\,\theta_1 - n_1\, cos\,\tilde{\theta}_2}{n_2\, cos\,\theta_1 + n_1\, cos\,\theta_2} = 0.000000$$

Fraction of TE power reflected

$$R_{TE} = r_{TE}^2 = (-0.384615)^2 = 0.147929$$

Percent of total incident power that becomes linearly polarized $= R_{TE}/2 = 7.3965\,\%$

The reflected light is linearly polarized normal to the plane of incidence.

Problem 21 (Specialized: Optics-ECE4501) Code Number:__________

a) $v = c/\lambda$ $c = 3x10^8$ m/s
$v = 193$THz

$\lambda_{fiber} = \lambda_o/n$ $n_{eff} \sim 1.470$ @lamda=1550nm
$\lambda_{fiber} = 1054$nm

b) $Vp = c/n_{eff}$
$Vp = 2.04x10^8$ m/s

$k = 2\, n_{eff}\, \pi$/lamda = 0.005959 1/nm = 59590 1/cm

c) Vgroup= c/(Ngroup)

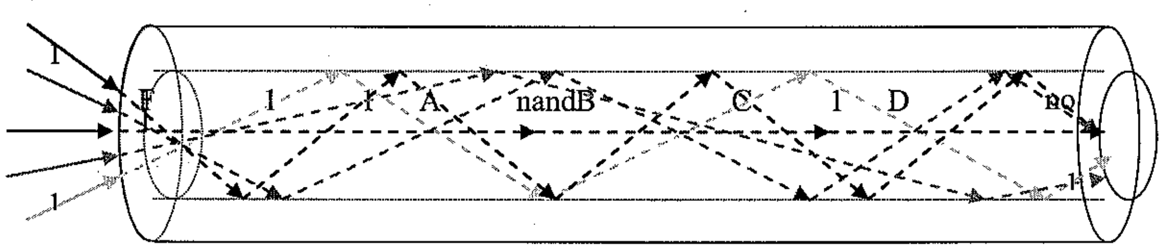
where Ngroup = n - λ x dn/dλ

dn/dλ = -2x10^(-5)

Ngroup = 1.470 – (1550nm(-2x10^{-5}))
Ngroup = 1.470 + 0.031 = 1.501

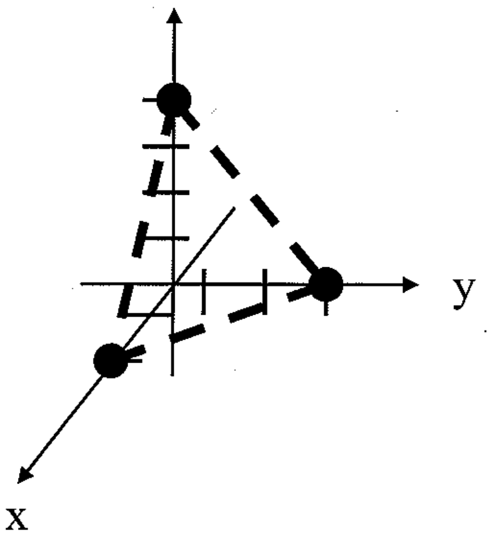
Vgroup = 1.999 x 10^8 m/s

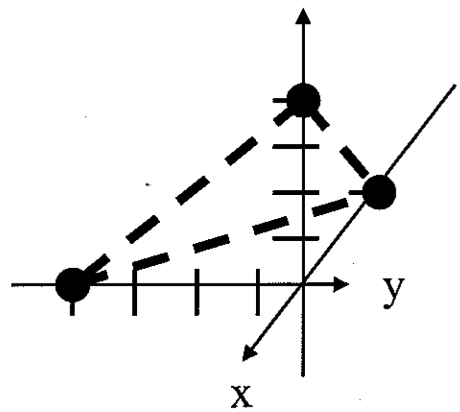
d) Straightest ray (fewest bounces per distance) has highest group velocity and lowest cutoff frequency



Problem 22 (Specialized: Microsystems-ECE4752) Code Number:____

A) Find the Miller indices for the planes illustrated in the figure below.





(2 3 4)	(-1 -4 4)
(1/2 1/3 1/4) 1/4)	(-1/1 -1/4
(6 4 3)	(-4 -1 1)

B) Calculate the angle between the (100) plane and the (111) plane in single crystal silicon material.

Use Dot Product

Cos (Theta) – (100) dot (111)

Theta = 54.7 degrees

Problem 23 (Specialized: Bio Eng-ECE4784) Code Number:_________

Signal : $I_K = g_K (V_m - E_K)$

$= 20pS(-40mV - 90mV)$

$I_K = 1 \times 10^{-12} A = 1 pA$

Noise $\equiv \overline{V_n} = \sqrt{\frac{4kT\Delta f}{R}}$

Want $\frac{S}{N} = 75 = \frac{I_K}{\overline{V_n}} \Rightarrow \overline{V_n} = 1.33 \times 10^{-14} A$

$75 = \frac{1 \times 10^{-12}}{\sqrt{\frac{4(1.38 \times 10^{-23})(298)(10 \times 10^{3})}{R}}}$

$\Rightarrow R = 9.25 \times 10^{11} \Omega$

Problem 24 (Specialized: Bio Eng-ECE4782) Code Number:_________

There could be diagnostic value in estimating a transfer function for the blink reflex in a patient with closed head injury. A "white-noise protocol" could be used.

A. (2 points) How would you create a Gaussian, band-limited, white-noise stimulus?
   A bright flash of light will cause the patient to blink. A Gausian amplitude distribution will be mostly small amplitudes that do not elicit a blink response, so the stimulus will need to reach amplitudes that do cause a blink response. There are lots of ways to create random modulations of light illuminations of the retina, but some light sources change color when the current level changes. For example an incandescent bulb becomes redder at higher current levels. A cluster of white LED's would be ideal for this study, but the light levels could be nonlinearly dependent on the current driving the LED's. Some sort of linearization scheme would be required.
B. (4 points) How would you calculate a first-order linear transfer function?
   The student will need to know the cross-correlation method for estimating the first-order Weiner kernel and must figure out a way to measure when blinks occur.
C. (4 points) What are the possible pitfalls in your diagnostic protocol that could lead to inaccurate results?
   There are a lot of potential pitfalls, e.g. the Power Spectral Density of the stimulus might not be very flat, which will distort all of the Weiner kernels. The all-or-none blink response will create lots of complications…. A random interval pulse stimulus would be much better for this study.

Problem 25 (Specialized: Bio Eng-ECE4781) Code Number:________

Spinal cord injured patients can often recover some level of motor control by practicing movements with EMG feedback.

A. (2 points) Describe how you would design and construct a surface electrode that mounts onto a finger.

   The electrode will need to attach to the finger without slipping or interfering with finger movements. Some sort of elastic/flexible band would be best that fits like a ring around the finger. Taping an EKG type electrode to the finger won't work. ECE4781 students learn about flexible, conductive electrodes that are used on babies….

B. (4 points) Show a block diagram that indicates all of the components you would need for a data acquisition system for a human EMG training protocol, i.e. to help the patient relearn how to control his/her finger. Also describe any software algorithms that would be needed.

   The first stage will need to be a preamp with low noise and high input resistance to reduce electrode polarization. The reference electrode could be at any nearby quiet location, e.g. the elbow. The next stages will need to be BP filters, and phase shifts are not very important. The next stage could be another amplifier stage, followed by some sort of isolation device (e.g. an opto-isolator). ECE4781 students learn that it is not safe to connect a patient to line ground. One way to quantify muscle activity is to rectify then integrate the EMG signal. This data processing could be performed using an A-to-D converter and software algorithms.

C. (2 points) Specify and justify the corners you would use for each analog filter and describe any phase shift considerations.

   Muscle spike noise will cause clipping and loss of data if the HP corner is set at a low frequency. Phase sifts are not very important when there is no stimulus, but it is possible to conceive a protocol that has some sort of stimulus….

D. (2 points) What sort of feedback would you provide to the patient to help the patient quantify the level of motor control of his/her finger.

   We could use sounds or plots to reveal when a high level of motor control has occurred. The student can be very creative here.