GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE 2030
QUIZ #1

Thursday, September 16, 1999

Name: _____
Last, First

- Closed book, closed notes.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- Clearly identify each answer.

Part	*pts*	*Score*
1	16	16
2	15	15
3	15	15
4	14	14
5	15	15
6	10	10
7	15	15
bonus		4
Total	100	104

Some useful Boolean identities.

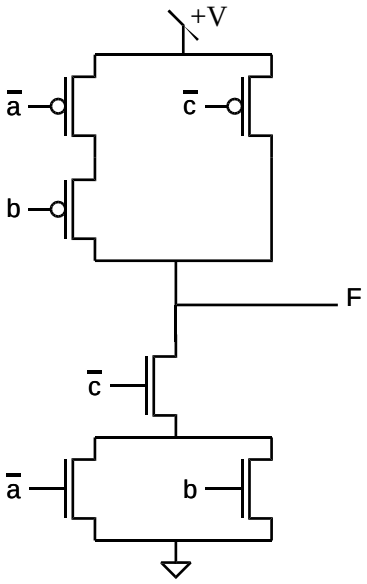
1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = (XY + XZ)$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Problem 1 (16 pts):

Implement the following expressions using n-type and p-type transistors in the standard CMOS configuration that we used in class.

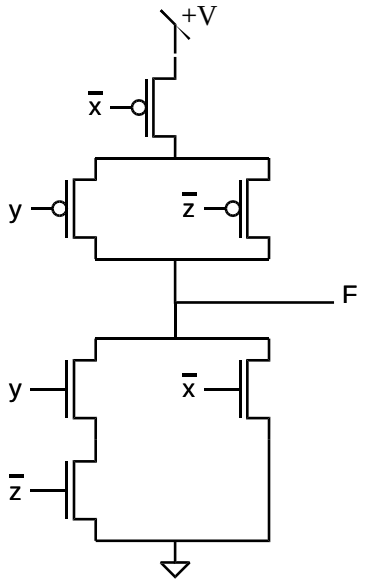
1. $F = a\,\overline{b} + c$

$$\overline{F} = (\,\overline{a} + b)\,\overline{c}$$



2. $F = x(\,\overline{y} + z)$

$$\overline{F} = \overline{x} + y\,\overline{z}$$



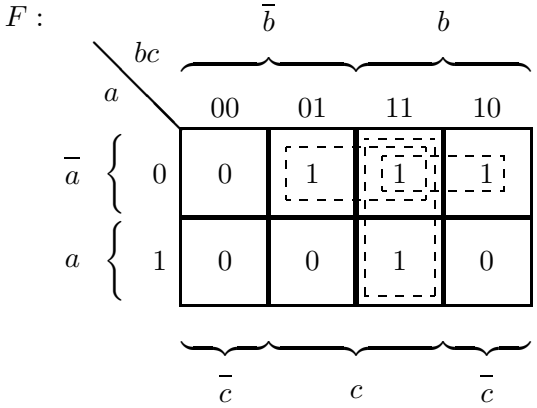
Problem 2 (15 pts):

Make a truth table for the following function:

$$F(a,b,c) = \sum m(1,2,3,7).$$

a	b	c	F
0	*0*	*0*	*0*
0	*0*	*1*	*1*
0	*1*	*0*	*1*
0	*1*	*1*	*1*
1	*0*	*0*	*0*
1	*0*	*1*	*0*
1	*1*	*0*	*0*
1	*1*	*1*	*1*

Create a K-map for simplifying the above function.



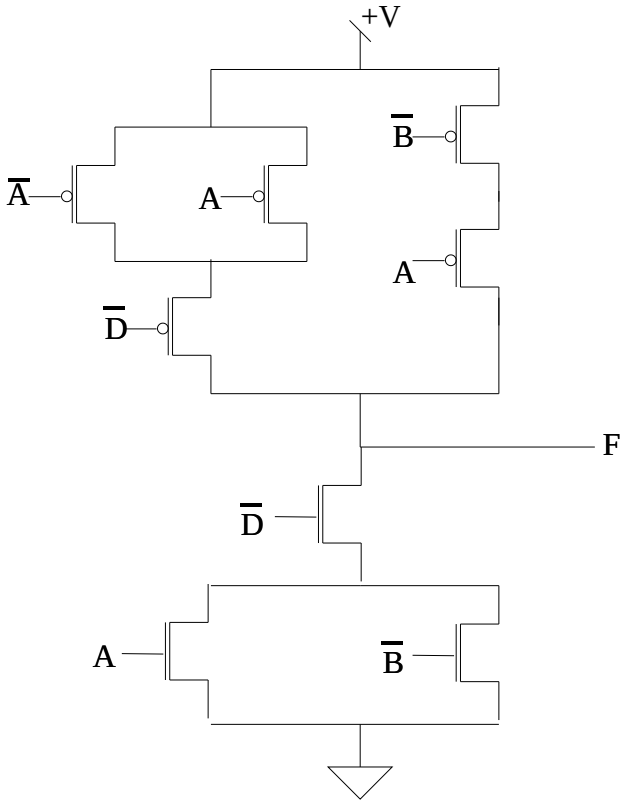
Write the simplified boolean expression for the above expression.

$$F = \overline{a}\,c + \overline{a}\,b + bc$$

Problem 3 (15 pts):

Complete the following switch level circuit by designing the bottom half (the pull-down network). Provide the expressions for F and $G = \overline{F}$.

$F = \quad (A + \overline{A})D + \overline{A}\,B = D + \overline{A}\,B$
$\overline{F} = \quad (A + \overline{B})\,\overline{D}$



Problem 4 (14 pts):

By using Boolean algebra manipulations, express the following function as a sum of products (SOP) and as a product of sums (POS). You are not required to simplify but you may if you want.

$$F = \overline{(A + \overline{BC})}\, D + \overline{A}\,\overline{D}\,(C + \overline{B}\,)$$

$$F = \quad \overline{A}\,BCD + \overline{A}\,C\,\overline{D} + \overline{A}\,\overline{B}\,\overline{D} = \overline{A}\,BC + \overline{A}\,\overline{B}\,\overline{D} \qquad \text{(SOP)}$$

$$F = \quad \overline{A}\,(\,\overline{B} + C)(B + \overline{D}\,) \qquad \text{(POS)}$$

There are several ways to convert an expression to a product of sums form. The direct way is to make a truth table and then extract the maxterms*. Another way is to simplify the SOP expression and then use DeMorgan's theorems as shown below. A method that we have not covered in class, but that was in your reading, is to use K-maps directly—working with the 0's.*

Note, a POS expression is of this form: $(A + B)(D + \overline{C}\,)$*,* **not** *this:* $(AB + C)(D + \overline{E}\,)$*, nor this:* $\overline{(A + B)(C + D)}$*.*

$$\begin{aligned}
F &= \overline{A}\,BC + \overline{A}\,\overline{B}\,\overline{D} \\
&= \overline{A}\,(\,\overline{(\overline{B} + \overline{C}\,)} + \overline{(B + D)}\,) \\
&= \overline{A}\,(\,\overline{(\overline{B} + \overline{C}\,)(B + D)}\,) \\
&= \overline{A}\,\overline{(\overline{B}\,D + B\,\overline{C} + \overline{C}\,D)} \\
&= \overline{A}\,\overline{(\overline{B}\,D + B\,\overline{C}\,)} \\
&= \overline{A}\,\overline{(\overline{B}\,D)}\,\overline{(B\,\overline{C}\,)} \\
&= \overline{A}\,(B + \overline{D}\,)(\,\overline{B} + C)
\end{aligned}$$

Problem 5 (15 pts):

A function, F, is defined by its truth table representation as shown below.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Express F as a sum of minterms.

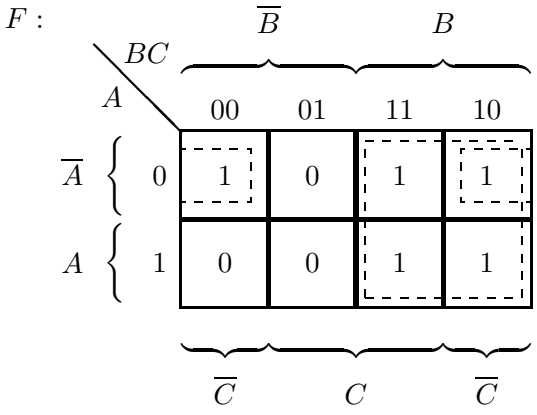
$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + \overline{A}\,BC + AB\,\overline{C} + ABC = \sum m(0,2,3,6,7)$$

2. Express F as a product of maxterms.

$$F = (A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C}) = \prod M(1,4,5)$$

3. Express F as a simplified sum of products (use any simplification method).

$$F = B + \overline{A}\,\overline{C} \qquad \text{(SOP)}$$



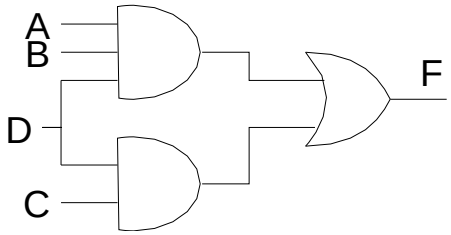
Problem 6 (10 pts):

Draw a gate implementation of the following function that exhibits a minimum amount of propagation delay.

$$F = D(A(\overline{B}\,C + B) + C\,\overline{A}\,)$$

Expressions in POS or SOP forms translate to gate implementations with the minimum amount of delay because they have only two levels. Note, the expression does not need to be simplified but it does need to be in POS or SOP form.

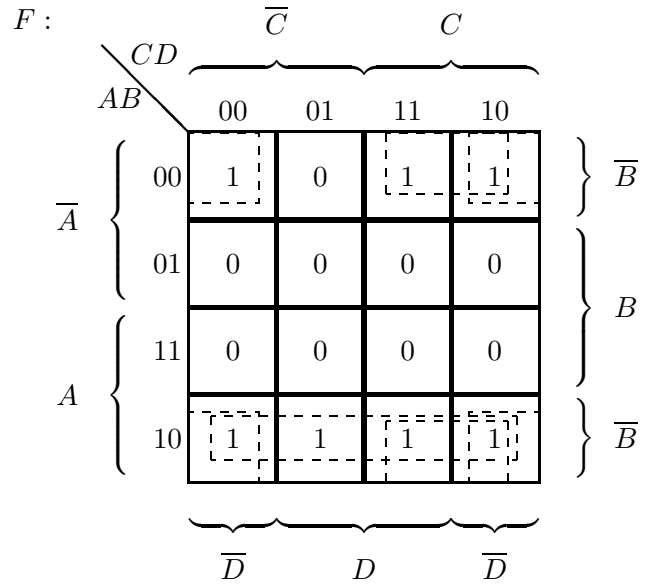
$$\begin{aligned} F &= D(A\,\overline{B}\,C + AB + C\,\overline{A}\,) \\ &= A\,\overline{B}\,CD + ABD + \overline{A}\,CD \\ &= CD + ABD \end{aligned}$$



Problem 7 (15 pts):

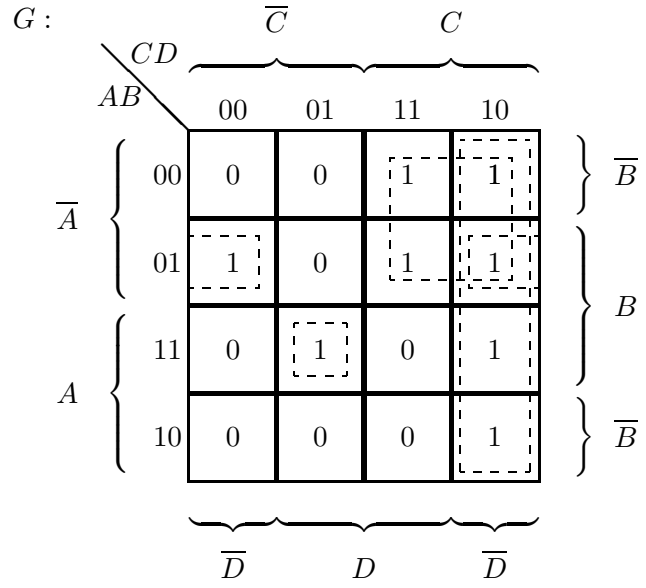
For the following functions, derive a simplified *sum of products* (SOP) expression using a Karnaugh map. Circle the prime implicants used in the simplified expression.

F :



$F =$ $A\,\overline{B} + \overline{B}\,\overline{D} + \overline{B}\,C$

G :



$G =$ $C\,\overline{D} + \overline{A}\,C + \overline{A}\,B\,\overline{D} + AB\,\overline{C}\,D$

Bonus (4 pts): How many transistors are required to implement the following functions using CMOS circuits? (Assume the complemented inputs are available.)

1. $F = AB$ 4

2. $F = A + B$ 4

3. $F = \overline{AB}$ 4 or 6

4. $F = \overline{A}\,B + A\,\overline{B}$ 8