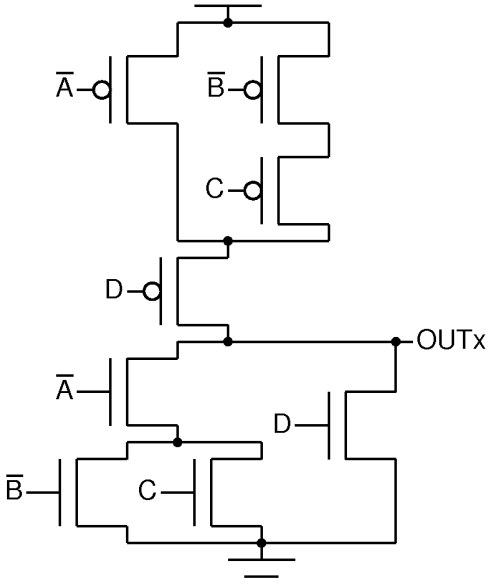
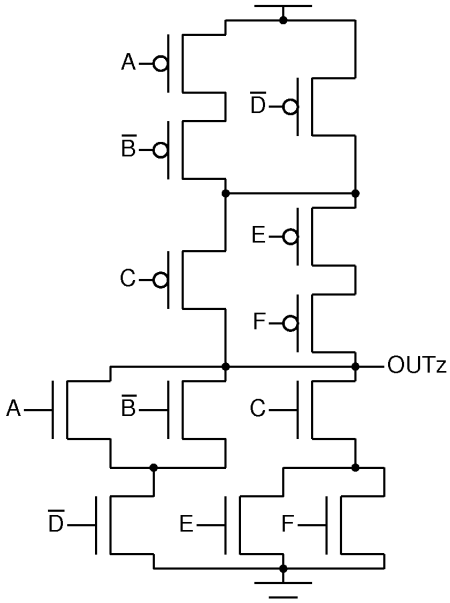
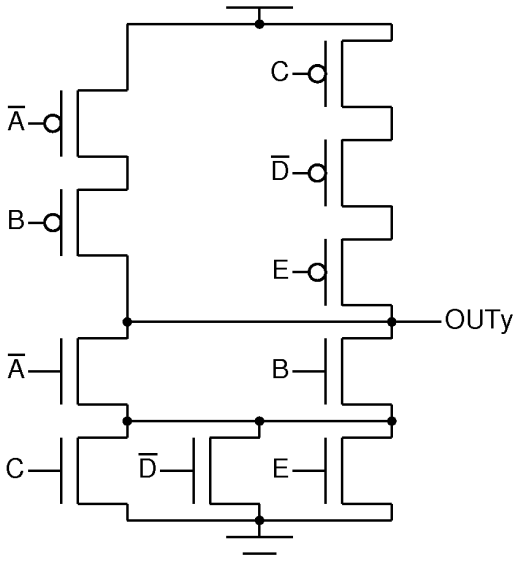
Problem 1 (3 parts, 30 points) Incomplete Circuits

The three parts below contain (A) a pull up network, (B) a pull down network, and (C) an expression to be implemented. For (A) and (B), complete the missing complementary switching networks so the circuit contains no floats or shorts and write the Boolean expression computed by the completed circuit. For (C), design the entire switching network. Assume the inputs and their complements are available.







OUTx = $(A + B \cdot \overline{C}) \cdot \overline{D}$

OUTy = $A \cdot \overline{B} + \overline{C} \cdot D \cdot \overline{E}$

OUTz = $\overline{(A + \overline{B}) \cdot \overline{D} + C \cdot (E + F)}$

Problem 2 (2 parts, 20 points) Boolean Algebra

Part A (12 points) Transform each of the following Boolean expressions to a form where they are ready for switch level implementation (i.e., there should only be bars over input variables, not over operations). The behavior of the expression should remain unchanged. **Do not implement**.

$OUT_X = \overline{(A \cdot B + \overline{C}) \cdot \overline{(D + \overline{E}) \cdot F}}$ $\quad (\overline{A} + \overline{B}) \cdot C + (D + \overline{E}) \cdot F$

$OUT_Y = \overline{\overline{M + \overline{N \cdot \overline{P}} + \overline{Q} \cdot R} + \overline{\overline{S} + T}}$ $\quad (M + \overline{N} + P) \cdot (Q + \overline{R}) + S \cdot \overline{T}$

Part B (8 points) Derive a canonical sum of products (using minterms) and a product of sums (using maxterms) expression for the truth table below.

A	B	C	$F_{(A,B,C)}$
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	1
0	1	1	1
1	1	1	0

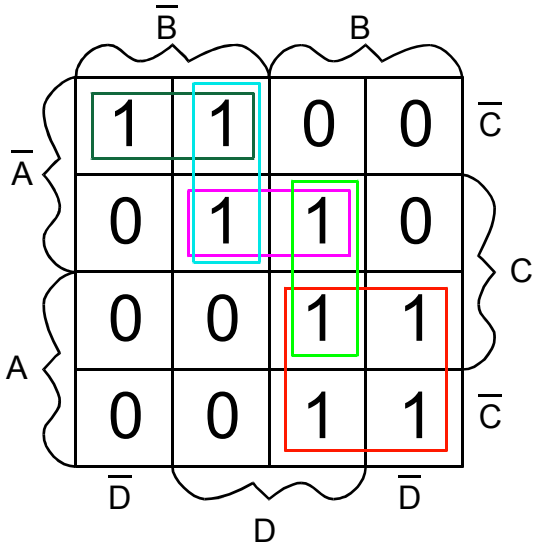
$SOP_{(MINTERMS)}$ = $A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot C + \overline{A} \cdot B \cdot C$

$POS_{(MAXTERMS)}$ = $(A + B + C) \cdot (A + \overline{B} + C) \cdot (\overline{A} + \overline{B} + \overline{C})$

Problem 3 (2 parts, 24 points) Karnaugh Maps

Part A (12 points) For the follow expression, derive a simplified ***sum of products*** expression using a Karnaugh Map. Circle and list *all* prime implicants, indicating which are essential.

$$Out = \overline{A}\cdot\overline{B}\cdot C\cdot D + B\cdot C\cdot D + \overline{A}\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot C\cdot\overline{D} + A\cdot B\cdot\overline{C}$$

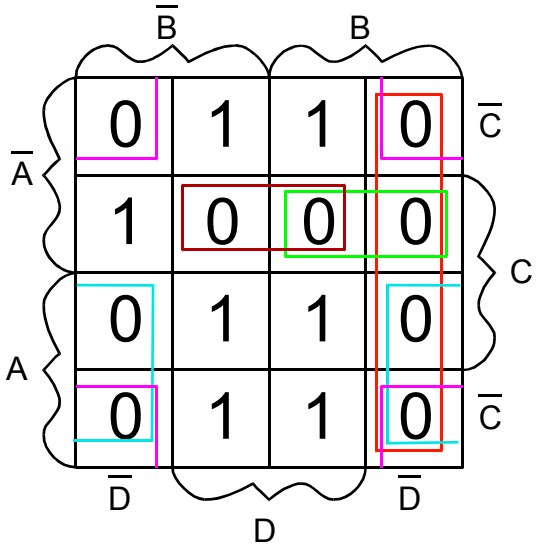


prime implicants	essential? yes	no
A B	☒	☐
B C D	☐	☒
$\overline{A}$ C D	☐	☒
$\overline{A}$ $\overline{B}$ D	☐	☒
$\overline{A}$ $\overline{B}$ $\overline{C}$	☒	☐
	☐	☐
	☐	☐
	☐	☐

simplified SOP expression $Out = A\cdot B + \overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot C\cdot D$

Part B (12 points) For the follow expression, derive a simplified ***product of sums*** expression using a Karnaugh Map. Circle and list all prime implicants, indicating which are essential.

$$Out = (B + C + D)\cdot(\overline{A} + \overline{B} + D)\cdot(A + \overline{C} + \overline{D})\cdot(A + \overline{B} + D)\cdot(\overline{A} + B + D)$$

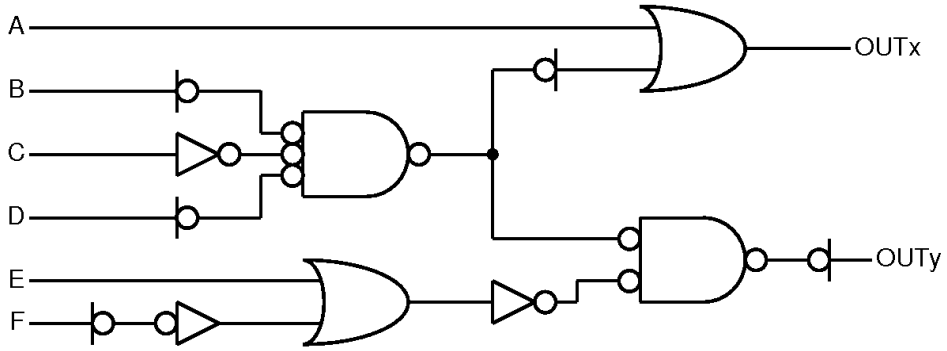


prime implicants	essential? yes	no
$\overline{B}$ + D	☐	☒
$\overline{A}$ + D	☒	☐
C + D	☒	☐
A + $\overline{B}$ + $\overline{C}$	☐	☒
A + $\overline{C}$ + $\overline{D}$	☒	☐
	☐	☐
	☐	☐
	☐	☐

simplified POS expression $Out = (\overline{B} + D)\cdot(\overline{A} + D)\cdot(C + D)\cdot(A + \overline{C} + \overline{D})$

Problem 4 (2 parts, 26 points) Mixed Logic Design

Part A (12 points) The following design has no supporting documentation. Derive the desired Boolean expression and the implementation cost (in CMOS transistors).

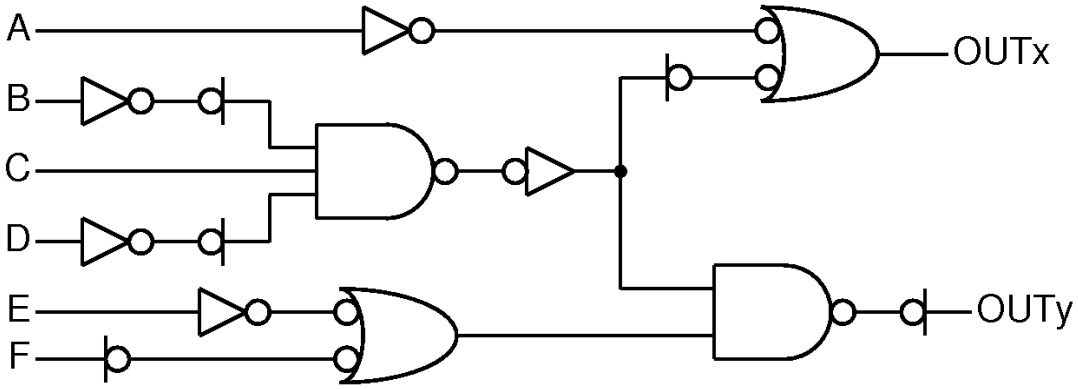


$OUT_X =$ $A + \overline{\overline{B} \cdot C \cdot \overline{D}}$

$OUT_Y =$ $\overline{\overline{B} \cdot C \cdot \overline{D} \cdot (E + \overline{F})}$

transistors = 1 x 8 + 3 x 6 + 3 x 2 = 8 + 18 + 6 = 32T

Part B (14 points) Now reimplement this expression using NAND gates and inverters. Use proper mixed logic notation. Determine the cost of this implementation (in transistors).



transistors = 1 x 6 + 3 x 4 + 5 x 2 = 6 + 12 + 10 = 28T